

# High Scalability Multipole Method. Solving Half Billion of Unknowns

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- **Introduction**
- **FMM-FFT algorithm**
- **Parallel implementation**
- **HEMCUVE. Challenge history**
- **Experimental Results**
- **Conclusions**

- Numerical solution of the integro-differential electromagnetic equations -> important in industry
- High frequency in large objects -> scalability limits
  - 0.78 m<sup>2</sup> at 3 GHz -> 15000 unknowns
  - real car at 79 GHz -> 400 million unknowns
- Great effort reducing computational time of MoM
- Fast Multipole Method (FMM) ->  $O(N^{3/2})$  good scalability
- Multilevel FMM (MLFMA) ->  $O(N \log N)$  poor scalability
- Modern HPC systems have thousands of cores
- FMM-FFT reduces complexity preserving scalability

- **FMM**

- The geometry is partitioned into  $M$  groups using an oct-tree decomposition algorithm
- Parallelization:
  - $k$ -space samples are independent from one another
  - The workload can be partitioned in the *k-space*

- **MLFMA**

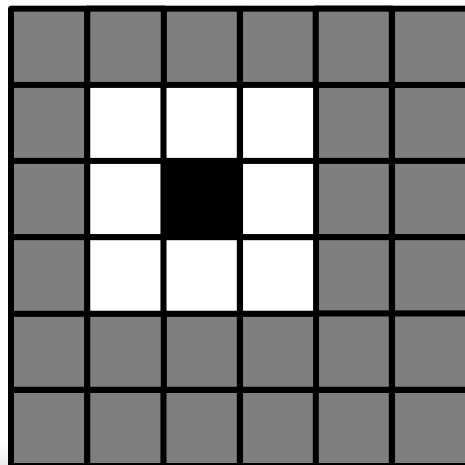
- The same algorithm can be recursively applied in a hierarchical multilevel oct-tree decomposition
- Computational cost reduced to  $O(N \log N)$
- Parallelization:
  - $k$ -space samples are not independent due to interpolation/interpolation of fields across levels
  - Usually workload distributed by groups



- Full domain is divided in groups
- The translation stage in the FMM can be seen as a 3D circular convolution
- Use the FFT to speedup the translation stage
- Reduce complexity from quadratic to logarithmic
- The FMM-FFT combines a low complexity like MLFMA with good parallel scalability of the original FMM
- It constitutes a real alternative to the MLFMA to take advantage of modern supercomputers

## Parallel implementation

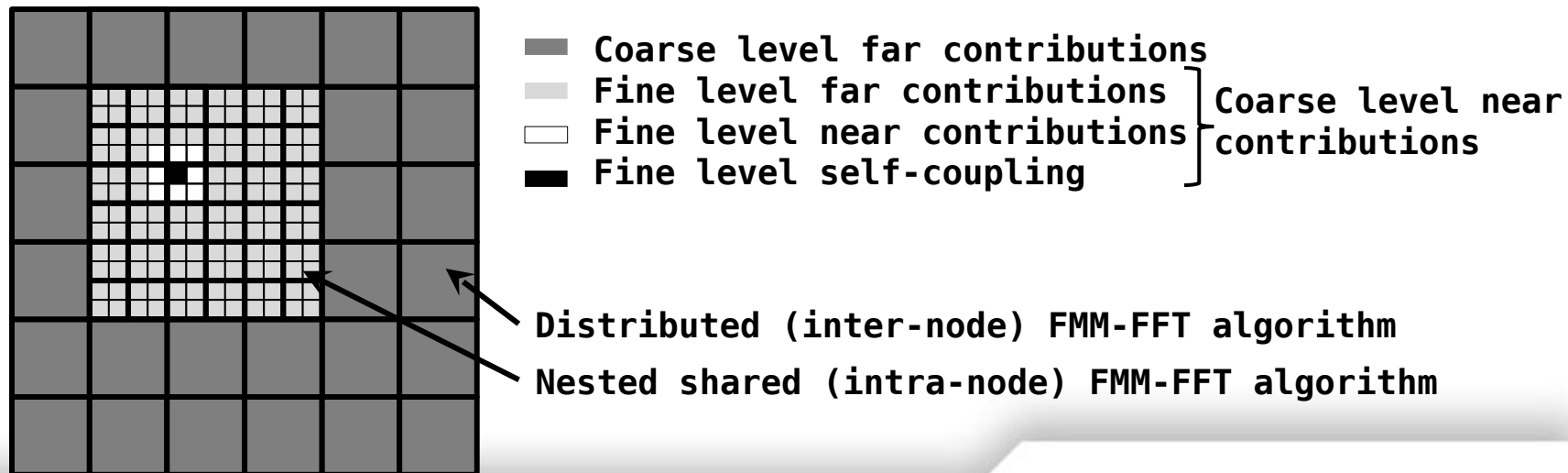
- Three stage parallelization strategy
  - **Far contributions:** distribution of fields (*k-space* samples) among processors
  - **Near contributions:** distribution of oct-tree groups
  - **Iterative solver:** distribution of unknowns
- Optimal load balance and data locality, while minimizing memory footprint and communication requirements
- A single communication step is required at the end of the MVP



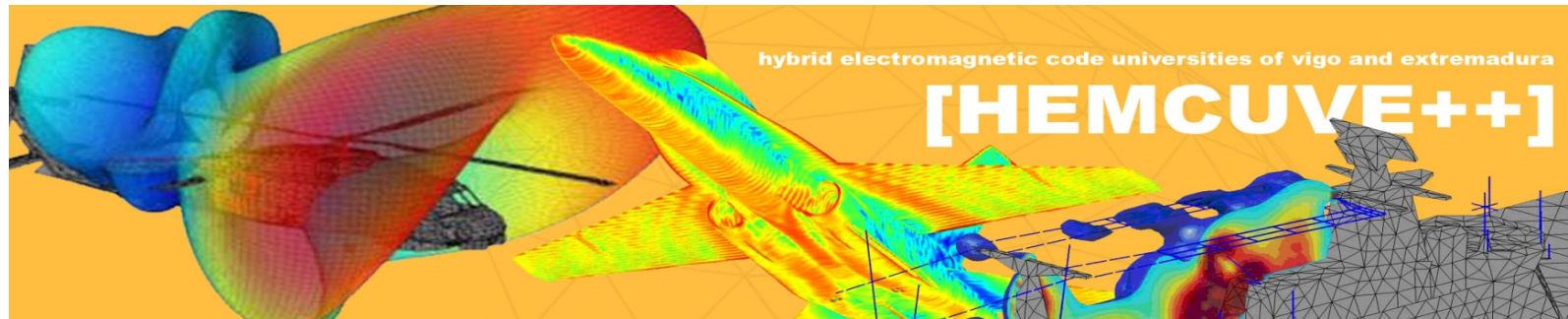
- Far contributions
- Near contributions
- Self-coupling

## Nested FMM-FFT

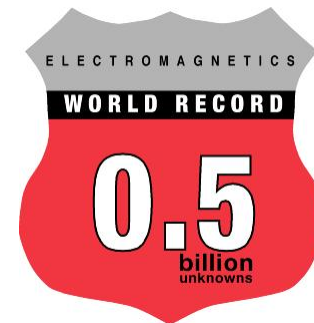
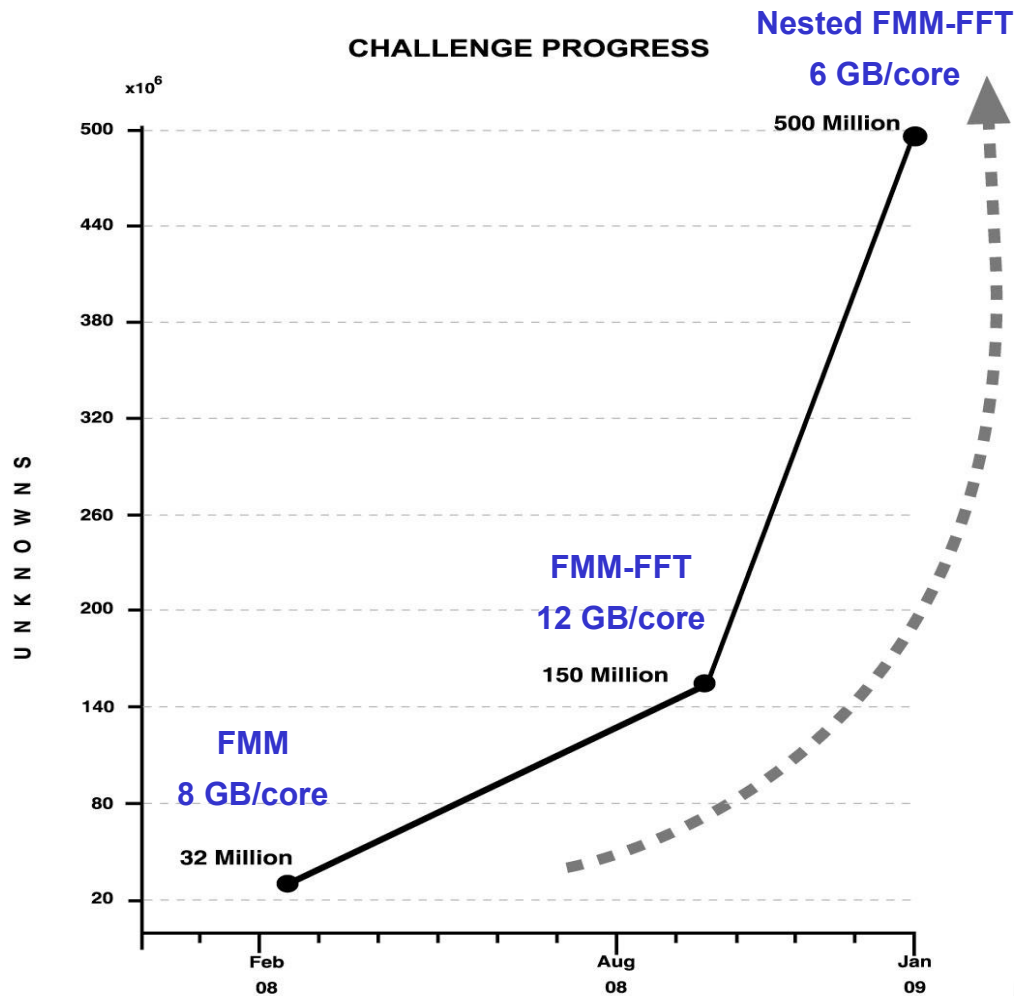
- A refinement of the oct-tree decomposition is applied (one or more refinement steps)
- The far contributions are obtained at the coarse level of the oct-tree using a distributed FMM-FFT
- The near contributions are obtained at the fine level using a local shared memory FMM-FFT in each compute node
- **Dramatically reduction of memory requirements**



- **Hybrid Electromagnetics: CESGA, University of Vigo and Extremadura**
- **C++ code for solving the previous algorithm**
- **In continuous development: increasing load balance, decreasing memory consumed, improving performance and scalability**

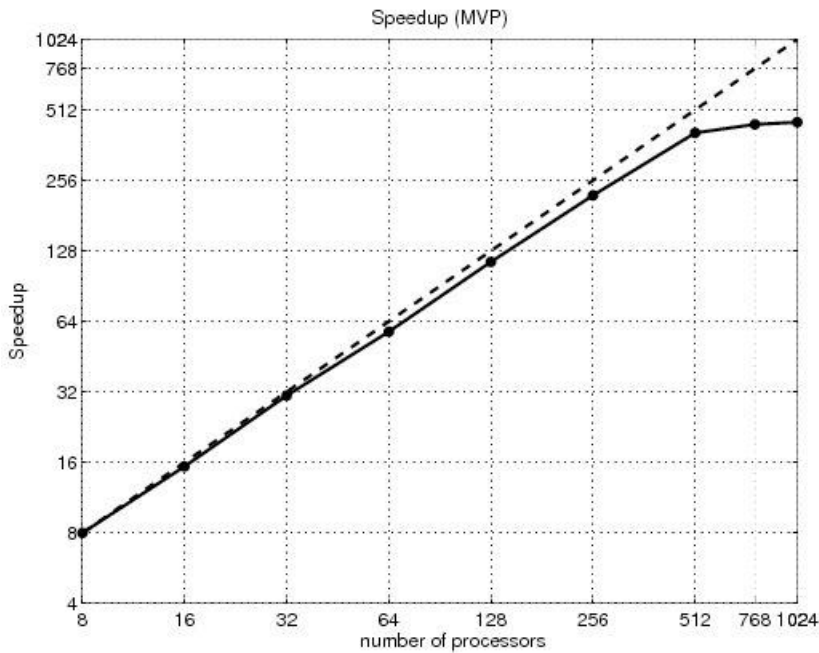


# HEMCUVE. Challenge history

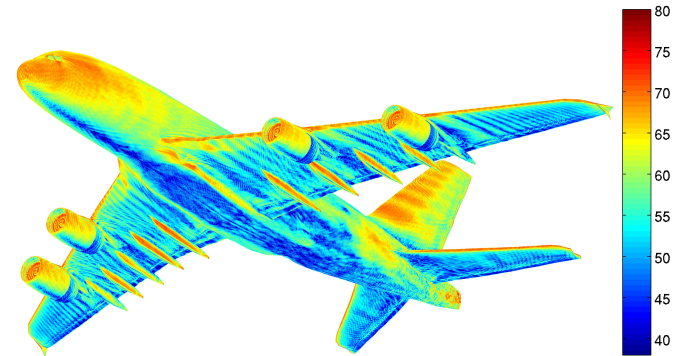




## Scalability

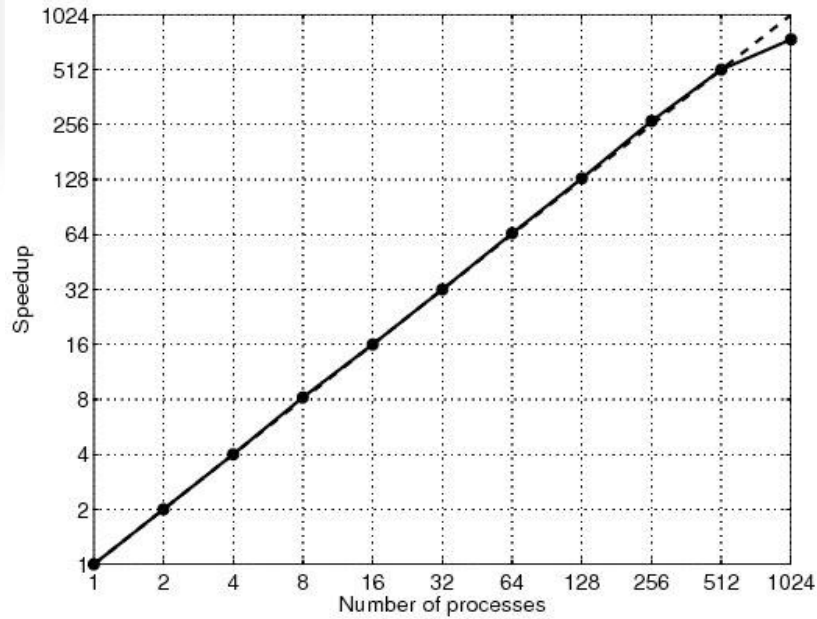


**FMM**

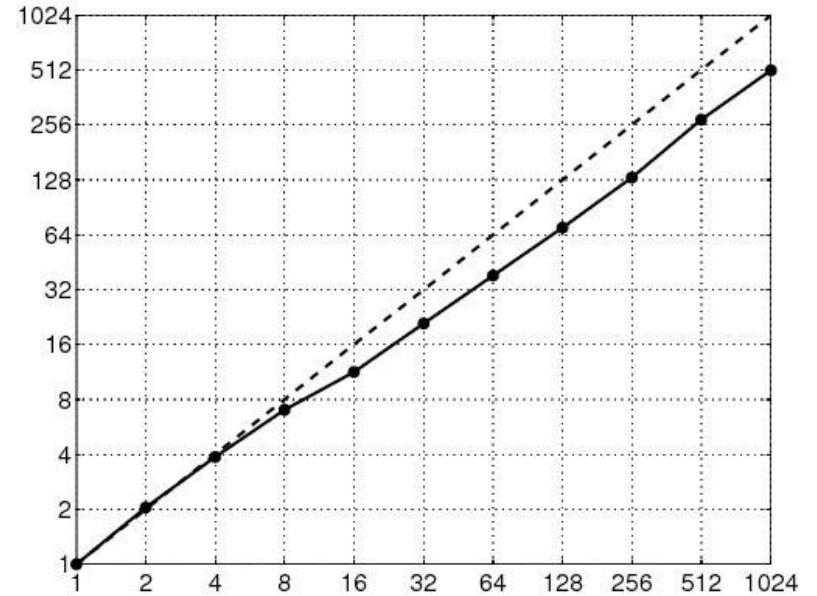


**Induced currents in an Airbus A380 for an axial incidence of 1,2 GHz (30M)**

## Scalability



**FMM-FFT**



**Nested FMM-FFT**

**Using a 10M unknowns problem**



## FinisTerra

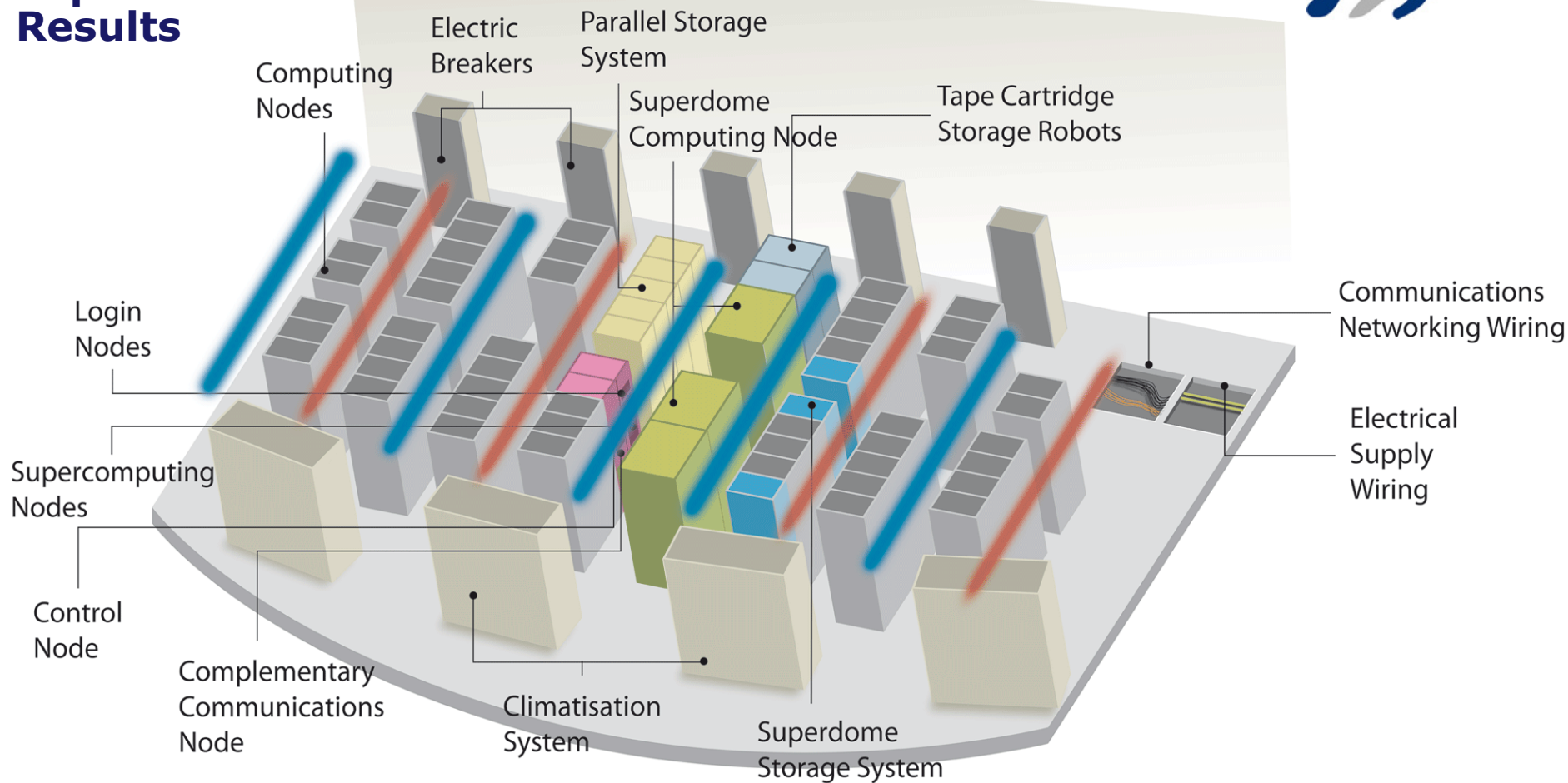
More than: **16,000 GFLOPS**

**2,580 CPUs**

**19,640 GB Memory**

SuSE Linux Enterprise Server 10

# Experimental Results



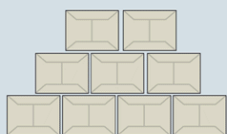
## Technical Specs.

Surface Area: 140 m<sup>2</sup>

Weight:



Storage:



2.200.000 GB on tape

390.000 GB on disk

Memory:



19.670 GB

2.528 Processing Cores

142 nodes, each with 16 cores & 128 GB memory

1 node with 128 cores & 1.024 GB memory

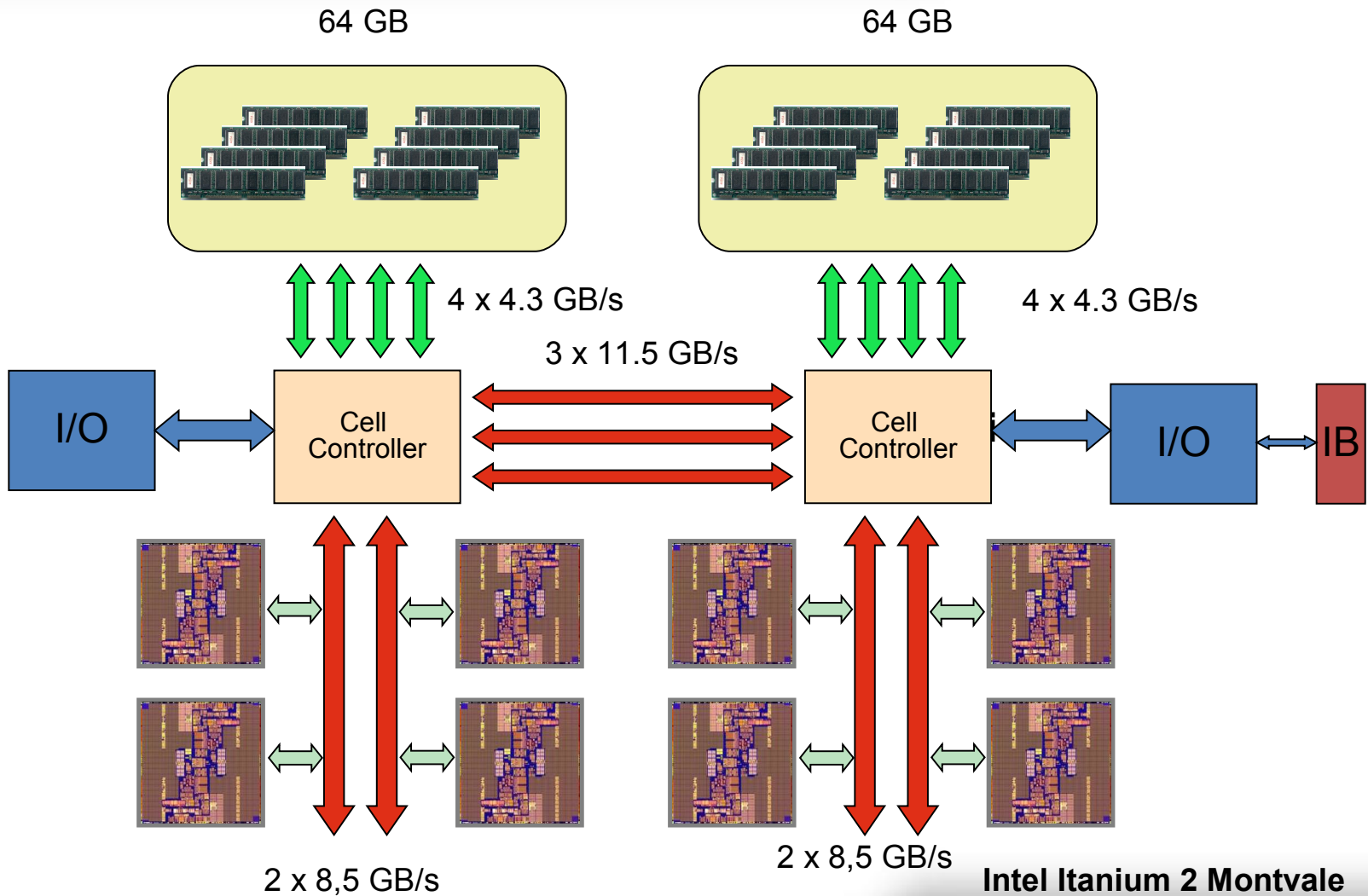
1 node with 128 cores & 384 GB memory

Node Interconnect INFINIBAND  
4x DDR at 20 Gbps

85 Km of interconnect cable

Open Software: Linux, Lustre, Globus...





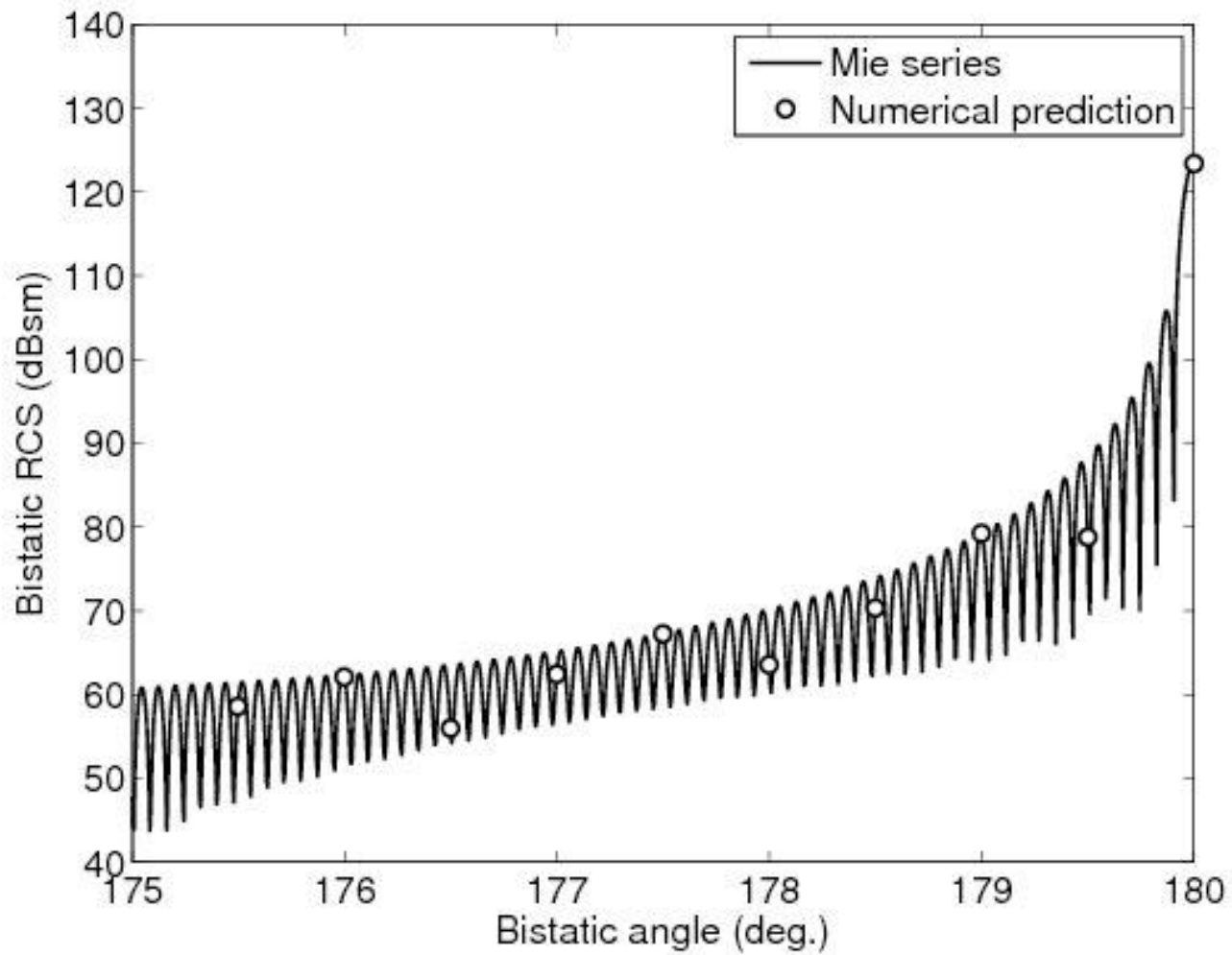


- Intel C++ Compiler version 11.0.069
- Intel MPI version 3.2.0.011
  - Limitation of 2 GB per message
- Intel Cluster MKL version 10.0.2.018

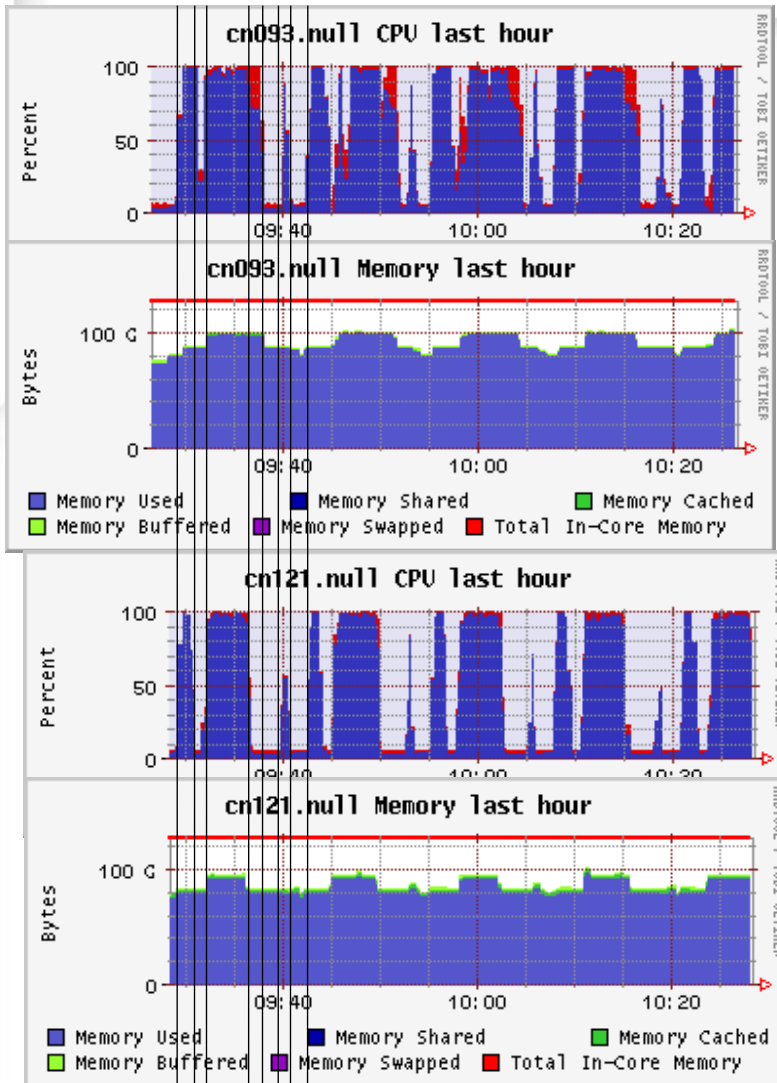
### **The Challenge problem:**

- RCS of a PEC sphere with  $728.36\lambda$  diameter
- 64 HP rx7640 nodes (1,024 cores) and 6TB of memory
- 10 GMRES iterations
- Residual error below  $5 \cdot 10^{-5}$
- More than 30 execution hours

Sphere diameter	$728.36\lambda$
Frequency	300 MHz
Number of Unknowns	500,159,232
Nodes / cores per node	64 / 16
Min / max memory per node	89.2GB / 99.9GB
Total memory	6 TB
Number of iterations	10
Setup / solution time	5h / 26 h

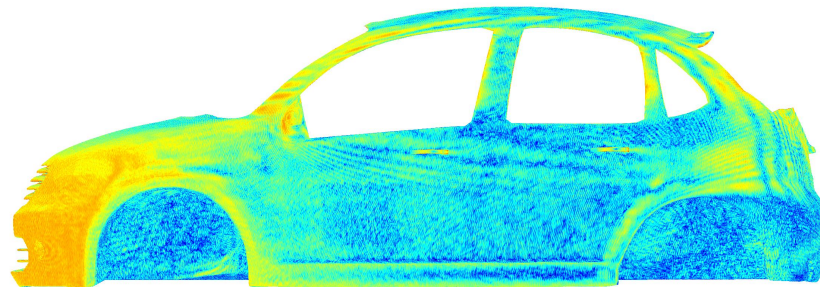
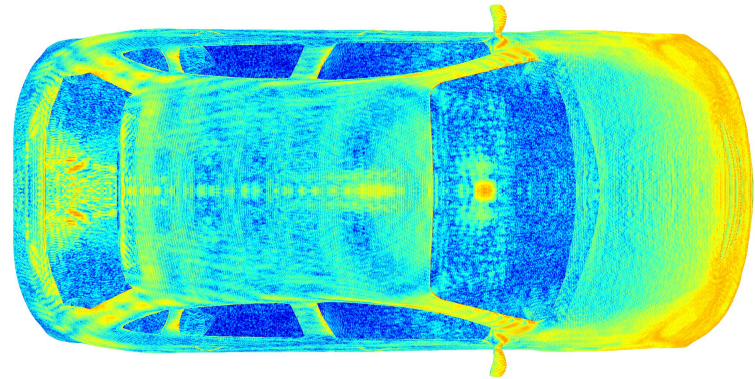
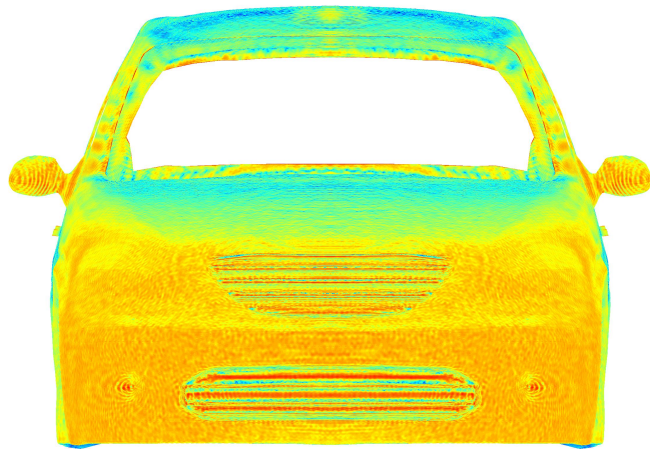


# Experimental Results



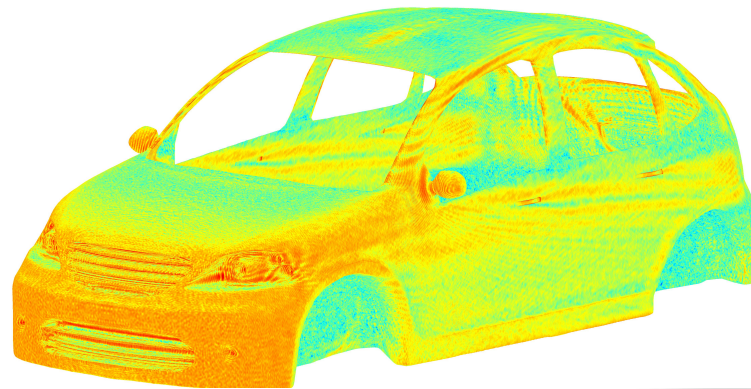
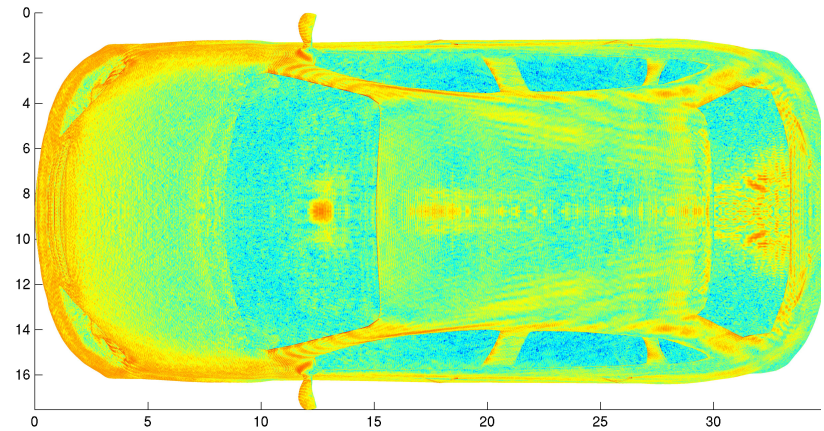
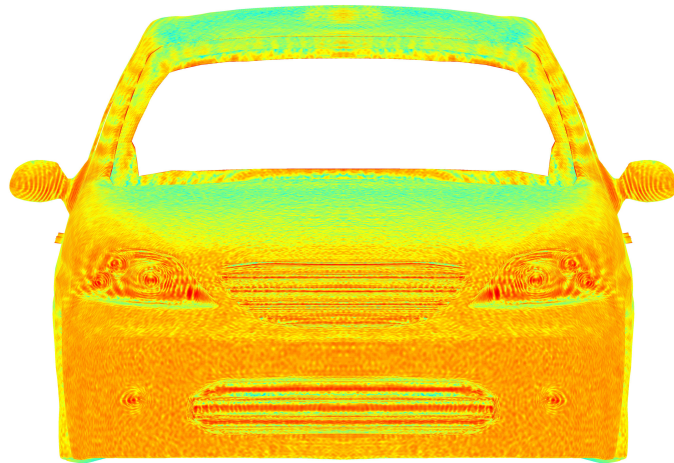
- 1: Aggregation, interpolation and near-field coupling
- 2: Communication Alltoallw
- 3: 3D-FFT translation
- 4: Restructuration of incoming fields
- 5: Communication Alltoallw
- 6: Disaggregation
- 7: Communication and GMRES (no OMP parallelization)

## Citröen C3 at 24.125 GHz (radar frequency) 40M unknowns





## Citröen C3 at 79 GHz (anti-collision frequency) < 400M

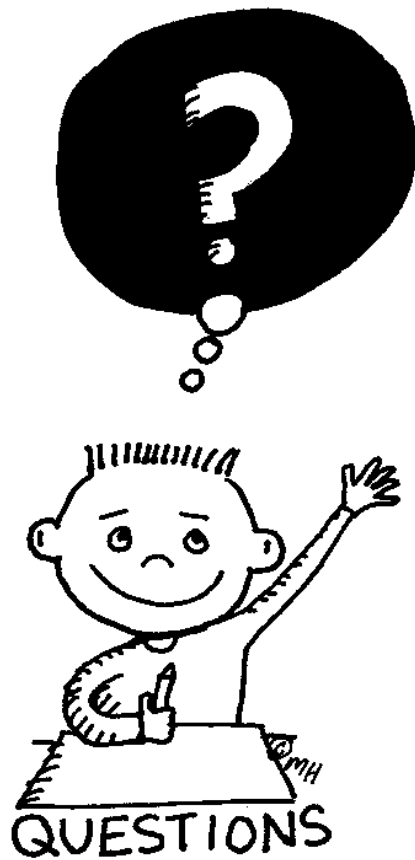


- The FMM-FFT is a real alternative for the analysis of very large general electromagnetic problems
- Very good scalability behavior up to 1,024 parallel processors
- Analysis of challenging problems up to 0.5 billion unknowns
- Suitable for calculating real structures at real frequencies

### **Future Work:**

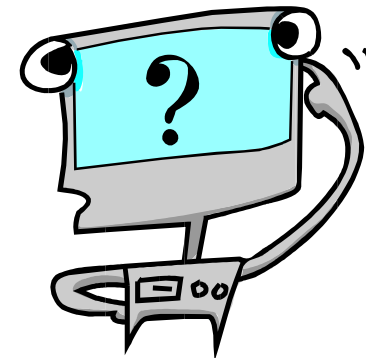
- new improved version: billion unknowns using 2,000 processors

- Spanish government
  - TEC2005-07355-C02-01
  - TEC2005-07355-C02-02
  - TEC2008-06714-C02-01
  - TEC2008-06714-C02-01
  - CONSOLIDER-INGENIO2010 CSD2008-00068
- Junta de Extremadura
  - 3PR05A002
- Xunta de Galicia
  - PGIDIT05TIC32201PR
  - INCITE08PXIB322250PR
- CESGA for their support in the use of Finis Terrae Supercomputer



**THANK YOU!**

**QUESTIONS?**



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