Abstraction, Induction and Abduction in Scientific Modelling

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1 Introduction

The development of scientific knowledge consists in two major components. The first component involves the construction of the calculus of a theory, that I choose to refer to as 'theory formulation', and the second involves the attempt to relate this calculus to experimental reports, that I choose to refer to as 'theory application'. Distinguishing the two is, in my view, important and useful both epistemologically and methodologically.

Philosophers of science, notably [10, 1, 6, 5, 8, 3, 4], have explicitly recognised that theory formulation involves the conceptual processes of abstraction and idealisation. Suppes' view is couched in the jargon of the Semantic Conception of scientific theories, but without committing to the latter we could still make use of his general idea, which could be spelled out as follows. Assuming that we begin with the universe of discourse, by selecting a small number of variables and parameters abstracted from the phenomena we are able to formulate what we generally refer to as the general laws of a theory. For example, in classical mechanics we select position and momentum and establish a relation amongst the two variables, which we call Newton's second law or Hamilton's equations. By abstracting a set of parameters we thus create a sub-domain of the universe of discourse, which we call the domain of a scientific theory. Thus, Newton's laws signify a conceptual object of study that we call the domain of classical mechanics. Similarly Maxwell's equations signify the domain of classical electromagnetism, the Schrödinger equation signifies the domain of quantum theory, and so forth. Scientific domains, viewed from this perspective, are clearly distinct from physical domains, which they could represent only if they are expanded by or integrated with other conceptual resources (see [9]. Hence theory formulation abstracts a scientific domain from the universe of discourse and thus groups together different phenomena based on the particular aspects dictated by the particular domain.

In all the above general laws something is left unspecified: the force function in Newton's 2nd law, the electric and magnetic field vectors in Maxwell's equations, and the Hamiltonian operator in the Schrödinger equation. Scientific methodology demands that these are specified in order to establish a link between the assertions of the theory and physical systems. The theory application component enters in the process of specifying those elements of scientific theories that need to be filled-out if the theoretical assertions are to be linked to empirical phenomena, such as force functions, electric and magnetic field vectors, Hamiltonian operators, etc. The aim of these specifications are not to extend the theoretical assertions all the way to phenomena but it is to construct a model that resembles as many of the features of its target physical system. My aim in this paper is to suggest a meta-algorithm that captures the ways by which we specify force functions, Hamiltonian operators, etc. To be more precise, my attempt is to establish a logical framework (i.e. to rationally reconstruct) that captures the ways by which scientific models are constructed for the representation of physical systems.

The process of specification can be understood to involve two distinct aspects, both of which, each in its own way, play a crucial role in improving the accuracy or the representational capacity of the model. The first aspect concerns the question of how the degree of resemblance of a model to its target physical system is increased. This aspect comprises in the amalgamation inside the model of different descriptions about different aspects of the physical system, so that a more detailed and refined representation of the former is achieved. Let me refer to this aspect as the process of concretisation (or de-idealisation). The second aspect involves discovering (or inventing) the different descriptions that enter in the process of concretisation. It is, I claim, in the latter aspect of model construction that induction and abduction are vital.

2 A Reconstruction of Modelling Processes

In trying to use the theoretical assertions to model physical systems, we usually start from a highly abstract description of an ideal-type, which we attempt to concretise by reintroducing into the description all the abstracted features. Concretisation may involve a careful study of the physical system in question and of all its peculiarities and it is something that often takes an entire scientific research program to achieve (e.g. the structure of the nucleus research program). What is important in my discussion is the question of how the theory-dictated 'primary' description of a physical system is supplemented by what within the theory is considered of 'secondary' importance. Concretisation is involved at three levels, firstly in distinguishing what factors are necessary for achieving an acceptable representation of the physical system, I refer to these as the primary factors of the theoretical description. Secondly, what factors are required in bringing every individual primary term of the description closer to reality, *as if it functions alone*. And thirdly what is required in bringing closer to reality the interacting terms, thus compensating for the assumption that the separate terms are disjoint and autonomous. The logical schema I want to suggest, to capture this thought process, is a multi-dimensional improvement of Nowak's 1980 account [8]. Nowak's idealisation account was meant to capture the logic of theories in the social sciences and economics. I believe that the complexities involved in the physical sciences, especially in the application of Quantum Mechanics, require the multi-dimensional more generalized account that I urge, and that could be formulated as follows:

$$T^{\alpha\beta}: \ If \ R(x) \ and \ S_{11}(x)=0,\ldots,S_{\alpha\beta}(x)=0, \\ and \ if \ P_{m1}(x),\ldots,P_{mn}(x) \ act \ on \ the \ physical \ system \\ autonomously \ from \ P_{k1}(x),\ldots,P_{kl}(x), \ then \\ H(x)=f_1(P_{11}(x),\ldots,P_{1\gamma}(x))+f_2(P_{21}(x),\ldots,P_{2\xi}(x))+ \\ \ldots+f_\delta(P_{\delta1}(x),\ldots,P_{\delta\epsilon}(x))$$

The statement $T^{\alpha\beta}$ says that in a realistic description $R(x)$ of a physical system we abstract in two distinct ways. Firstly we abstract by categorising the factors of influence into primary, $P's$, and secondary, $S's$, and by subtracting all the secondary factors of influence from our initial theoretical description (i.e. by assuming that they do not act on the system in question). Secondly we abstract by grouping the primary factors into separate terms, $f_i's$, each of which is assumed to act autonomously in the physical system, and by categorising the secondary factors into their corresponding groups. Each f_i represents a mathematical function of different primary and secondary factors of influence, and the subscripts (indices) are only meant to state distinctions between different factors and groupings among factors. For instance, f_1 is a function conceptually distinct from f_2 because the influencing factors of which it is a function are assumed to act autonomously on the physical system from the respective factors of which f_2 is a function. Also, each P_{ij} (or S_{ij}) are indexed so that the modelling assumption that each factor of influence can be described distinctly from other factors is captured in the logical schema. The first index in the primary (and secondary) factors refers to the grouping to which the factor belongs and the second index is its name. The overall model description is represented by H, which is the sum of mathematical terms each of which is functionally related only to different primary factors of influence. The step-by-step process of concretisation of our hypothesis, that would improve the representational capacity of our model, involves the gradual addition of the secondary factors related with each and every one of the individual primary terms. A first step concretisation would be the following:

$$T^{\alpha\beta-1}: \ If \ R(x) \ and \ S_{11}(x)=0,\ldots,S_{\alpha\beta-1}(x)=0, \\ and \ S_{\alpha\beta}(x)\neq 0, and \ if \ P_{m1}(x),\ldots,P_{mn}(x) \\ act \ on \ the \ physical \ system \ autonomously \ from \\ P_{k1}(x),\ldots,P_{kl}(x), \ then \ H(x)=f_1(P_{11}(x),\ldots,P_{1\gamma}(x))+ \\ \ldots+g_{\alpha\beta-1}[f_\alpha(P_{\alpha1}(x),\ldots,P_{\alpha\eta}(x)), \ h_{\alpha\beta}(S_{\alpha\beta}(x))]+ \\ \ldots+f_\delta(P_{\delta1}(x),\ldots,P_{\delta\epsilon}(x))$$

Where, I have added the influence of just one secondary factor ($S_{\alpha\beta}$) in just one of the g_{ij} terms (namely, $g_{\alpha\beta-1}$). The g_{ij} terms are simply new names to the grouping-function that is altered by the introduction of one secondary function of influence, the first index i signifies the name of the grouping and the second index j signifies the number of factors introduced into the particular grouping. The h_{ij} terms are the names of the mathematical expressions through which the secondary factors of influence are represented. The addition of just one secondary factor of influence into the logical schema goes only to show that concretisation factors are added only to individual primary terms, it does not portray the actual practice in science, where concretisation factors may be added simultaneously or after significant theoretical and experimental developments. It must be noted that this logical schema allows for the regrouping of the terms in a description, as well as for the introduction of new terms as correction factors or as addenda. In other words, it allows for radical improvements to representational models in a particular physical domain that usually come about after a breakthrough is accomplished. A final concretised assertion would have the following form:

$$T^{00}: \ If \ R(x) \ and \ S_{11}(x)\neq 0,\ldots,S_{\alpha\beta}(x)\neq 0, \\ and \ if \ P_{m1}(x),\ldots,P_{mn}(x) \ act \ on \ the \ physical \ system \\ autonomously \ from \ P_{k1}(x),\ldots,P_{kl}(x), \ then \ H(x)= \\ g_{10}[f_1(P_{11}(x),\ldots,P_{1\gamma}(x)), \ h_{11}(S_{11}(x)),\ldots,h_{1\theta}(S_{1\theta}(x))]+ \\ g_{20}[f_2(P_{21}(x),\ldots,P_{2\psi}(x)), \ h_{21}(S_{21}(x)),\ldots,h_{2\chi}(S_{2\chi}(x))] \\ +\ldots+ \\ g_{\delta0}[f_\delta(P_{\delta1}(x),\ldots,P_{\delta\epsilon}(x)), \ h_{\delta1}(S_{\delta1}(x)),\ldots,h_{\delta\phi}(S_{\delta\phi}(x))]$$

The final statement T^{00} says that in a theoretical description of a physical system, in which all known factors of influence that were initially abstracted from the realistic description R are now reintroduced, we have an expression that breaks down the impact of all influencing factors into several terms each of which is assumed to act autonomously in the physical system. I believe that this account captures well the construction process of many applications of Classical and Quantum Mechanics. It also sheds some light on how representational models relate to the theory (a task that is beyond the scope of the present work). Moreover, it explicates one other important element of scientific model construction. Each different term of the description carries its own separate, and frequently independent, assumptions, which is a much more accurate understanding of scientific practice than regarding all as assumptions bound to the overall model description.

The claim I want to urge is that inductive and abductive procedures are operative in discovering (or inventing) how each term in the overall description is to be represented. That is to say, that we need either inductive or abductive arguments in order to justify the introduction of the individual terms $P'_{ij}s$ and $S'_{ij}s$ in the logical schema above, but that such arguments on their own do not justify the overall model description H, the latter is something that is determined by the process of abstraction/idealisation and its converse process of concretisation. In other words, induction and abduction are processes that piggyback on the processes of abstraction/idealisation and concretisation. I will proceed to briefly sketch two examples that can help visualize the above modelling process and distinguish its two aspects.

3 Scientific Modelling from the Viewpoint of the Concretisation Logical Schema

The simple pendulum is probably one of the most successful scientific representations in the history of science. To model the actual pendulum apparatus we start by assuming a mass-point bob supported by a massless inextensible cord of length l performing infinitesimal oscillations about an equilibrium point. Thus the equation of motion of the simple harmonic oscillator can be used as the starting point for modelling a real pendulum and thus attempting to measure the acceleration due to the Earth's gravitational field: $\theta'' + (g/l)\theta = 0$. But the idealised assumptions underlying this model equation, do not describe how the apparatus is in the world but they dictate an ideal description of the apparatus. Hence it is obvious to physicists that if a reasonably accurate representation is demanded, the various influencing factors of the pendulum motion must be incorporated into the model. This is not something peculiar to the pendulum but it is a demand that is present in the majority of cases of modelling physical systems.

In the pendulum example a reasonably accurate representational model would involve the following influencing factors: (i) finite amplitude, (ii) finite radius of bob, (iii) mass of ring, (iv) mass of cap, (v) mass of cap screw, (vi) mass of wire, (vii) flexibility of wire, (viii) rotation of bob, (ix) double pendulum, (x) buoyancy, (xi) linear damping, (xii) quadratic damping, (xiii) decay of finite amplitude, (xiv) added mass, (xv) stretching of wire, (xvi) motion of support. To increase the degree of resemblance of the model to the pendulum apparatus mathematical descriptions of these factors are introduced into the model equation in a cumulative manner. Hence the aspects of modelling that were discerned above, i.e. concretisation, induction, and abduction, are clearly discerned in the pendulum case. To identify these influencing factors and to decide how they must be introduced into the model is a clear demonstration of what I have labelled the process of concretisation. In fact the above logical schema applies to the model of the pendulum in its most abstract and idealised form as follows:

$$T^{\alpha\beta}:\ If\ R(x)\ and\ S_{11}(x)=0,\ldots,S_{1\beta}(x)=0,\ then$$
$$H(x)=f_1(P_{11}(x)$$

In this simple form the schema suggests that only one primary factor of influence is identified (that of the linear restoring force due to gravity), and all secondary factors of influence are corrections to the influence of gravity. Where $H(x) = f_1(P_{11}(x)$ is a metalinguistic description of the Newtonian equation of motion of the simple harmonic oscillator $\theta'' + (g/l)\theta = 0$, that is meant to model the pendulum at a high degree of idealization and abstraction.

To discover what descriptions must be used for each of the secondary influencing factors is a clear demonstration of either an induction or an abduction process (The modelling details of the real pendulum apparatus can be found in [7]). Here is a case of an abductive procedure in determining how the air resistance acts on the oscillating system (pendulum bob and wire) to cause the amplitude to decrease with time and to increase the period. The Reynolds number for each component of the system determines the law of force for that component. The drag force is hence expressed in terms of a dimensionless drag coefficient, which is a function of the Reynolds number. In the pendulum case it can be argued abductively that a quadratic force law should apply for the pendulum bob, whereas a linear force law should apply for the pendulum wire (both of these are clearly inferences to the best explanation). Hence, it makes sense to establish a damping force which is a combination of linear and quadratic velocity terms: $F = b\,|v| + cv^2$. To determine the physical damping constants b and c the work-energy theorem is employed, an appropriate velocity function $v = f(\theta_0, t)$ is assumed, and under the assumption of conservation of energy they are matched to experimental results. The final expression of the effect of air damping is introduced into the equation of motion of the model.

Here is a case based on an inductive procedure in determining how the length of the pendulum is increased by stretching of the wire due to the weight of the bob. By Hooke's law (which, being an empirical law, could be claimed that it is arrived at inductively) when the pendulum is suspended in a static position the increase is $\Delta l = mgl_0/ES$, where S is the cross-sectional area and E is the elastic modulus. The dynamic stretching when the pendulum is oscillating is due to the apparent centrifugal and Coriolis forces acting on the bob during the motion. This feature is modelled by analogy with the spring-pendulum system to the near stiff limit. When these features are introduced into the model equation it gives rise to a system of coupled equations of motion.

A more complicated modelling example is that of the nuclear unified model used in the representation of the nuclear structure [2]. The unified model is based on a highly complex hypothesis about the nature of the nucleus, which expresses our conception of the nuclear structure as it has been shaped by the successes and failures of predecessor models. The hypothesis asserts that the nucleus is a complex system of a collection of particles that exhibit some form of independent nucleon motion, but that this motion is constrained by a slow collective motion of a core of nucleons, and that the two modes of motion interact with each other. In addition it asserts that the collective mode of motion is constituted by three distinct kinds of motion (vibration, rotation and giant resonance), two of which demonstrate an interaction mode. These ideas are expressed in the formalism of Quantum Mechanics in terms of the Hamiltonian operator of the unified model that is used in the Schrdinger equation for the nucleus. This Hamiltonian operator takes the following form: $H_{TOT} = H_{SP} + H_{COL} + H_{INT}$. Where H_{SP} is the single-particle Hamiltonian term, H_{INT} is the interaction mode Hamiltonian term, and the collective Hamiltonian is divided into four distinct modes of motion: $H_{COL} = H_{ROT} + H_{VIB} + H_{ROT-VIB} + H_{GR}$. Each of these terms are, of course, constituted by complex expressions that represent the various factors involved in each particular mode of nuclear motion. Since there are six primary terms, the above logical schema applies to the unified model in its most abstract and idealised form as follows:

$$T^{\alpha\beta}:\ If\ R(x)\ and\ S_{11}(x)=0,\ldots,S_{\alpha\beta}(x)=0,$$
$$and\ if\ P_{m1}(x),\ldots,P_{mn}(x)\ act\ on\ the\ physical\ system$$
$$autonomously\ from\ P_{k1}(x),\ldots,P_{kl}(x),\ then$$
$$H(x)=f_1(P_{11}(x),\ldots,P_{1\gamma}(x))+f_2(P_{21}(x),\ldots,P_{2\xi}(x))+$$
$$\ldots+f_6(P_{61}(x),\ldots,P_{6\epsilon}(x))$$

Where $H(x) = f_1(P_{11}(x),\ldots,P_{1\gamma}(x)) + f_2(P_{21}(x),\ldots, P_{2\xi}(x)) + \ldots + f_6(P_{61}(x),\ldots,P_{6\epsilon}(x))$ is a metalinguistic description of the total Hamiltonian operator of the unified model of nuclear structure, i.e. $H_{TOT} = H_{SP} + H_{ROT} + H_{VIB} + H_{ROT-VIB} + H_{GR} + H_{INT}$. The unified model is an example that demonstrates two fundamental elements of model construction in the application of quantum mechanics. Firstly, in the case of the unified model the hypothesis of the model is not asserted in a highly abstract form. It involves many of the significant features of the nuclear structure that are present in our description of the physical system. Nevertheless, in specifying a Hamiltonian we abstract by dividing these features into three separate terms, as if their contribution to the behaviour of the nucleus is distinct and autonomous. This procedure is very frequent in modelling in physics, but we must recognise that it is only a conceptual division. The three terms in the unified model Hamiltonian are not meant to act disjointedly nor to represent separately, we impel the division by abstracting. The abstraction involved is the foundation of the counterfactual assertion, implied by the Hamiltonian, that the overall nuclear motion is $as\ if$ it receives contributions from distinct and autonomous modes of motion. This way by which abstraction is used in our modelling is reflected in the above logical schema of model construction.

Secondly, the individual Hamiltonian terms of the model are not constructed in identical ways. The H_{SP} term is modelled by using the principles of Quantum Mechanics from the outset in a systematic manner, i.e. by using a stock model of the theory and postulating ways by which to concretise the abstractions involved. The collective motion terms, however, differ significantly in the method of construction. In fact the collective terms are first set up as if the system behaves in accordance to classical mechanics and at some appropriate stage its parameters are quantized, i.e. the classical functions are converted to quantum mechanical operators. This is a standard procedure in phenomenological modelling in quantum mechanics, which deserves its own analysis. But for the purposes of this work we must discern that in such cases no stock model of Quantum Mechanics is used, and no theoretical justification exists for the quantization of classical variables. In other words, part of the Hamiltonian of the unified model is in fact semi-classical. This gives rise to questions concerning the construction of representation models that are not outright products of quantum theory alone. This aspect of modelling, which is so common in the application of Quantum Mechanics, is also reflected in the above logical schema of model construction, since there is no restriction that the f_i's and the g_{ij}'s must be dictated by theory.

Abductive reasoning enters in the construction of the unified model in two levels. The first is in reaching the conclusion that although the individual motion term and the collective motion term are constructed in significantly different ways (i.e. the first by using quantum mechanical principles from the outset, and the second by semi-classical processes) the best way to achieve an explanation of the nuclear properties is by employing both terms in a unified Hamiltonian. The second is in reaching the conclusion of what contributes to each particular term of the Hamiltonian, i.e. in establishing the best possible description of each term that would most accurately represent the different modes of motion of the nucleus.

4 Conclusion

The logical schema of the concretisation process, I suggest, captures most of the elements of theory application. But most importantly, what underlies this way of looking at theory application is that inductive and abductive inferences are mainly present in determining specific factors that influence the behaviour of physical systems, and not in determining general unifying theories. Grouping these factors together in order to reach a theoretical representation of a target physical system is a process that is primarily guided by the abstraction and concretisation processes. This is, in my view, a more precise characterisation of scientific practice, and in particular 'theory application'. The importance of induction and abduction could be best understood if these processes are seen as operating together with the process of concretisation, and the logical schema above serves as a meta-algorithm for understanding how all three processes operate together in our attempt to construct representations of phenomena.

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