BA: Distributed Minimum Vertex Coloring and Maximum Independent Set in Chordal Graphs

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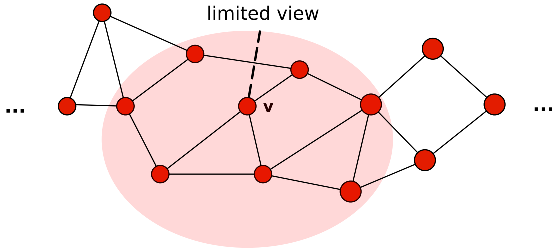
Minimum Vertex Coloring in the LOCAL Model

Input: Network $G = (V, E)$, $n = |V|$



- Synchronous communication, individual messages of unbounded sizes
- **Running time:** Number of communication rounds

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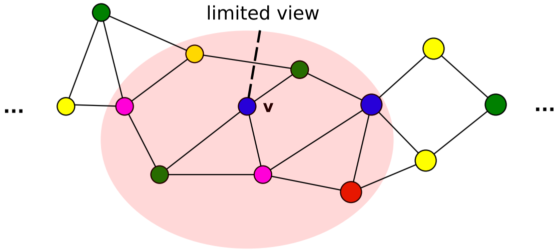

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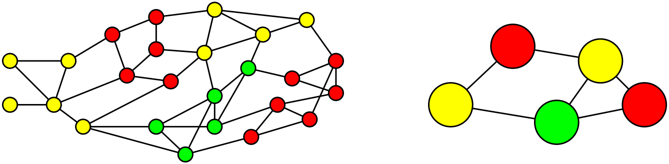
Minimum Vertex Coloring: (MVC)

- Find $\chi(G)$-coloring, $\chi(G)$: chromatic number
- Hard to approximate within factor $n^{1-\epsilon}$ **[Håstad, 1999]**

Distributed MVC via Linial-Saks

General Graphs: Network-decomposition **[Linial, Saks, 1993]**

- $\mathrm{O}(\log n)$-approximation in $\mathrm{O}(\log^2 n)$ rounds
- Nodes run exponential time algorithms



Open Question:

Constant factor approximation in $\mathrm{polylog}\, n$ rounds?

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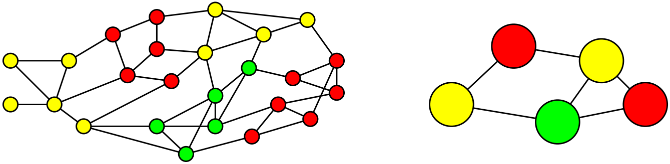

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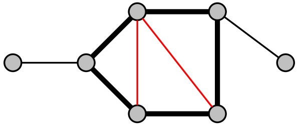
Constant factor approximation in $\mathrm{polylog}\, n$ rounds?
Difficult graph structure that prevents us from getting there?

Our Results

Chordal Graphs: Every cycle of at least 4 vertices contains a chord:



Our Results:

- $(1+\epsilon)$-approximation in $\mathrm{O}(\frac{1}{\epsilon} \log n)$ rounds
- Lower Bound: $\Omega(\frac{1}{\epsilon} + \log n)$ rounds
- Similar results for Maximum Independent Set

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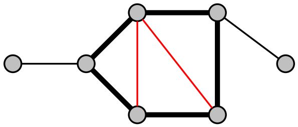

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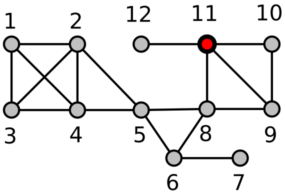
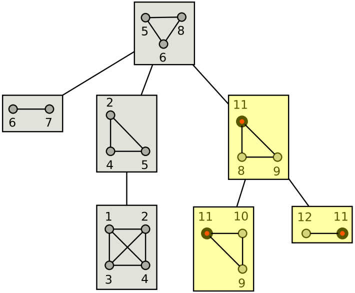
Technique: Tree Decomposition

Main Technique: Tree Decompositions

Chordal Graph: Clique Tree





Distributed Processing:

- Nodes compute local view of (global) clique tree
- Important property: Diameter of each bag is 1
- Peeling Process: In $O(\log n)$ iterations, peel off interval subgraph, color individual layers, correct coloring where layers meet

Interval Graphs: $(1 + \epsilon)$-approximation **[Halldórsson, Konrad, 2014, 2017]**

Summary

Distributed MVC:

- $\mathrm{O}(\log n)$-approximation in $\mathrm{O}(\log^2 n)$ rounds in general graphs
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Outlook:

- A graph has *tree-length* k if there is a tree-decomposition where every bag has diameter at most k
- Work in progress: 2-approximation in $\mathrm{O}(k \log n)$ rounds

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Thank you very much.