The **LOCAL** Model

**Model:**
- O(1) communication rounds
- Unbounded message sizes
- Unbounded computational power

**Focus:**
Locality of computational problems

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### Independent Sets/Colorings

**Hardness.** The maximum independent set and the minimum vertex coloring problems are NP-hard, and they are even hard to approximate within a factor of n^{1-\epsilon}.

**Exponential Time.** Under the assumption that P \neq NP, every local algorithm with non-trivial approximation ratio for either problem has to use exponential time computations.

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### Related Work

Most works on distributed independent sets and colorings consider the maximal independent set problem and the (\Delta + 1)-coloring problem. These problems can easily be solved sequentially. The work of Barenboim [ICALP, 2012] is closest to our work and presents a O(n^{1/2+\epsilon})-approximation local algorithm for the minimum vertex coloring problem (using exponential time computations).

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### Results

**Upper Bounds:** We present local randomized O(n^{\epsilon})-approximation algorithms for the maximum independent set and the minimum vertex coloring problems, for any \epsilon > 0, which run in O(3^d) rounds.

**Lower Bounds:** We prove that both algorithms are optimal in that no local algorithm can achieve n^{\epsilon\Omega(1)}-approximations for either problem.

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### Distributed Maximum Independent Set Approximation

Suppose that every node computes a maximum independent set in its k-neighborhood. How can we combine these locally optimal solutions to a coherent global solution?

1. **New Vertex Decomposition**
   For a constant k = O(\frac{1}{\epsilon}), partition V into disjoint sets V_1, V_2, \ldots, V_k so that v \in V_j if j is the smallest i such that
   \[
   \max IS(N^{c_{i-1}}(v)) \geq n^{i-1}/k, \quad \text{and} \quad \max IS(N^{c_i}(v)) \leq n^i/k,
   \]
   where (c_i)_i is an exponentially increasing sequence, and N^d(v) denotes the d-neighborhood of v.

2. **Ruling Set Algorithm**
   For i = 1, \ldots, k, we treat the sets V_i separately. Using an algorithm by Gfeller and Vicari [PODC 2007], in O(1) rounds, we compute a (2c_{i-1} + 1)-independent subset V_i' \subseteq V_i which essentially c_i-dominates V_i.

   \[
   \text{dist}(u, v) \geq 2c_{i-1} + 1
   \]

   Then, a large independent set I_i = \bigcup_{v \in V_i'} \max IS(B^{c_{i-1}}(v)) is established. Let I* be a a maximum independent set in the input graph. We show that:
   \[
   n'|I*| \geq |I* \cap V_i|,
   \]

3. **Merging the Independent Sets**
   From the sets I_1, \ldots, I_k, we compute an independent set I so that |I| \geq |I_i| for every i. Since there exists an i such that |I* \cap V_i| \geq |I*|/k, and using Inequality 1, I is a \langle k \cdot n^\epsilon \rangle-approximation.

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### Distributed Minimum Vertex Coloring Approximation

We make use of the following connection between minimum vertex coloring and network decompositions.

**Definition** A \langle d, c \rangle-network decomposition is a partitioning of the vertices of the input graph into clusters of maximal diameter d so that the graph obtained when contracting the clusters into vertices can be colored with at most c colors.

**Theorem (Barenboim [ICALP, 2012])** Suppose that nodes of a graph G = (V, E) know their color in a \langle d, c \rangle-network decomposition. Then, there is an O(d)-rounds distributed algorithm that computes a c-approximate minimum vertex coloring.

Barenboim [ICALP, 2012] showed that there is a sampling-based, local algorithm that computes a \langle O(1), n^{1/2+\epsilon} \rangle-network decomposition, implying a local O(n^{1/2+\epsilon})-approximation algorithm for minimum vertex coloring.

**Our Result** We show that via a recursive sampling-based approach similar to Barenboim’s method, a \langle O(1), n^\epsilon \rangle-network decomposition can be computed, leading to a local O(n^\epsilon)-approximation algorithm for minimum vertex coloring.