Advanced topics in TCS

Exercise sheet 5.
Weighted and unweighted matching

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Question 1. Weighted Matching Algorithm from the Lecture

Give an example of an input stream on which the algorithm for weighted matching discussed in the lecture produces an approximation ratio close to 1/6 (e.g., $< 1/5.9$ or similar). Such an example input stream demonstrates that our analysis is best possible.

Question 2. Greedy Matching on d-regular Graphs

We know that the Greedy matching algorithm has an approximation ratio of 1/2 for the Maximum Matching problem on arbitrary graphs. Let $G = (V, E)$ be a d-regular graph, i.e., a graph where every vertex has degree d. Suppose we run Greedy on an arbitrarily ordered sequence of the edges of G. Give an improved bound (better than 1/2) that depends on d on the approximation factor of Greedy when run on G. For example, the bound should yield 2/3 for $d = 2$.

Question 3. Weighted Matching with Restricted Edge Weights

Let $G = (V, E, w)$ be a weighted graph with $w : E \to \{1, 2\}$. Consider the following two algorithms, which can be implemented as semi-streaming algorithms, for computing matchings:

$\mathbf{A}_1$: Ignore the edge weights and use the Greedy matching algorithm to compute a maximal matching M. Return M with its edge weights.

$\mathbf{A}_2$: Run Greedy on the subgraph of edges of weight 1, which produces a matching M_1. In parallel, run Greedy on the subgraph of edges of weight 2, which produces a matching M_2. The output matching M is obtained by inserting every edge of M_1 into M_2 if possible.

1. What is the approximation guarantee of $\mathbf{A}_1$? Give a complete proof. Give a worst-case example that shows that your analysis is tight.

2. What is the approximation guarantee of $\mathbf{A}_2$? Give a complete proof. Give a worst-case example that shows that your analysis is tight.