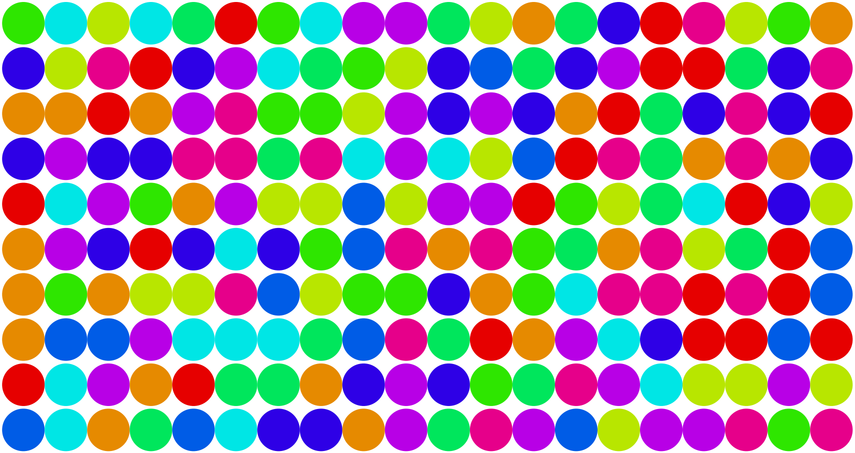
Topics in TCS

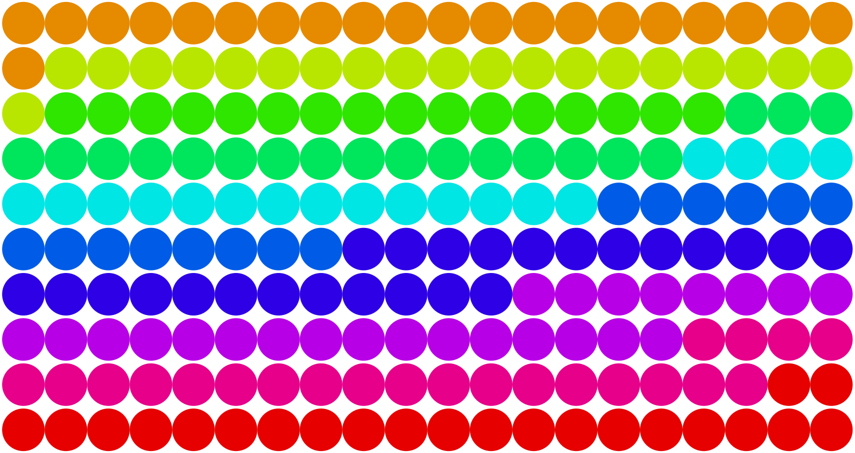
Counting distinct elements

Raphaël Clifford

Counting distinct elements



Counting distinct elements



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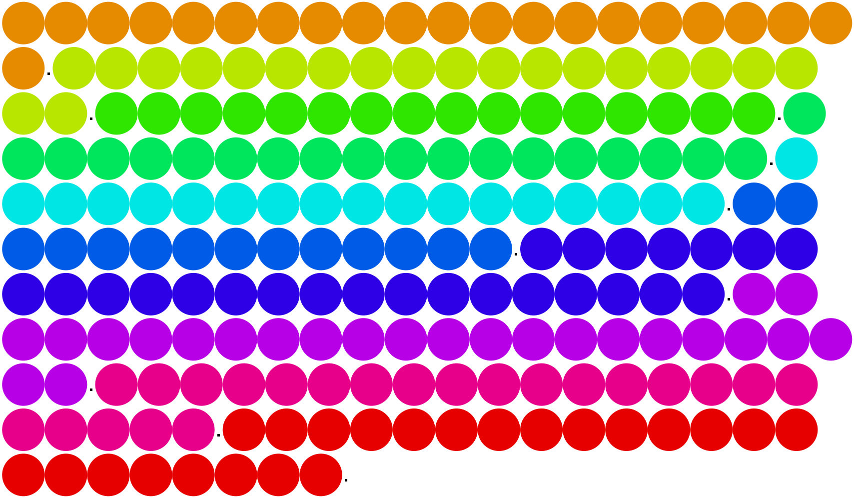

10 distinct colours

Counting distinct elements

Naive counting solution

Sort the elements. $O(m \log m)$ time.

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Traverse from left to right. $O(m)$ time.

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NOT one-pass.

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Slightly less naive counting solution

Use a balanced binary search tree.

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- If value not in tree, insert it

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At the conclusion the number of distinct values will be the number of nodes in tree.

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Running time $O(\log m)$ per FIND and INSERT operation making $O(m \log m)$ time overall BUT $O(m)$ words of space.

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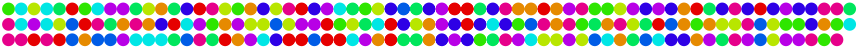

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No deterministic sub-linear space one-pass solution is possible.

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No deterministic sub-linear space one-pass solution is possible.

We will need a randomised algorithm.

First randomised algorithm

```
stream ⟨a_1, ..., a_m⟩, a_i ∈ [n]
initialise
choose random h : [n] → [n]

SIMPLE-COUNT
Set M = h(a_1)
For each i ≥ 2
   if h(a_i) < M
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return d̂ = n/M
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Worked example

- Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$.
  $n = 10$.

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Worked example

- Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.
- Randomly hash to $\langle 10, 9, 10, 6, 9, 10, 6, 9 \rangle$

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Worked example

- Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.
- Randomly hash to $\langle 10, 9, 10, 6, 9, 10, 6, 9 \rangle$
- $M = 6$ at termination.

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- True answer is 3.

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Worked example. 2nd try

- Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.
- Rehash: $\langle 4, 3, 4, 8, 3, 4, 8, 3 \rangle$
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- Rehash: $\langle 4, 3, 4, 8, 3, 4, 8, 3 \rangle$
- $M = 3$ at termination.
- Return $\hat{d} = 10/3 = 3\frac{1}{3}$
- True answer is 3.

Probability lemma for counting distinct elements

Lemma

Let $(Z_1, \ldots, Z_N)$ *be an array of pairwise independent indicator variables with* $\Pr(Z_i = 1) = p$ *and let* $W = \sum_{i=1}^{N} Z_i$*, then* $\mathbb{E}(W) = Np$, $\text{var}(W) = Np(1-p)$ *and*

$$\Pr(W > 0) \leq Np \quad \textit{and} \quad \Pr(W = 0) \leq \frac{1}{Np} \tag{1}$$

Proof.

□

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- $\Pr(W > 0) \leq Np$ is by the union bound and that $\Pr(Z_i = 1) = p$.
- $\Pr(W = 0) \leq \Pr(|W - \mathbb{E}(W)| \geq \mathbb{E}(W)) \leq \frac{\text{var}(W)}{(Np)^2} \leq \frac{1}{Np}$

□

SIMPLE-COUNT analysis

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- Y_a corresponds to W from (1) with $N = d$ and $p = a/n$. Therefore:

$$\Pr(Y_a > 0) = \Pr(M \leq a) \leq da/n \quad \text{and} \quad \Pr(M > b) \leq \frac{n}{db} \quad (2)$$

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Define $Z_i = 1$ if $h(S_i) \leq b$ and 0 otherwise
Let the random variable $Z = \sum_{i=1}^{d} Z_i$
From (1), $\Pr(M > b) = \Pr(Z = 0) \leq \frac{1}{\frac{db}{n}} = \frac{n}{db}$

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- Now let $da/n = 1/3$ and $n/db = 1/3$, so that $a = n/(3d)$ and $b = 3n/d$, then (2) becomes

$$\Pr(M \leq n/(3d)) \leq 1/3 \quad \text{and} \quad \Pr(M > 3n/d) \leq 1/3$$

or

$$\Pr(d \leq \hat{d}/3) \leq 1/3 \quad \text{and} \quad \Pr(d > 3\hat{d}) \leq 1/3$$

using $\hat{d} = n/M$.

Simple-Count space/time

Our random estimate $\hat{d}$ gives us:

$$\Pr(d \leq \hat{d}/3) \leq 1/3 \quad \text{and} \quad \Pr(d > 3\hat{d}) \leq 1/3$$

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- Running time $O(m)$
- One-pass ✓

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- Space $O(\log n)$ ✓
- Can we use less space? (Sort of)

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- Running time $O(m)$
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- Space $O(\log n)$ ✓
- Can we use less space? (Sort of)
- The error probability is pretty bad. Can we fix that? (Yes)

The TIDEMARK algorithm

```
initialise
choose random h : [n] → [n]
set z = 0

TIDEMARK(a_i)
if ZEROS(h(a_i)) > z
   set z = ZEROS(h(a_i))

OUTPUT 2^(z+1/2)
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- h chosen from a pairwise random family of hash functions.

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- Hashed to $\langle 8, 2, 8, 1, 2, 8, 1, 2 \rangle$

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- Hashed to $\langle 8, 2, 8, 1, 2, 8, 1, 2 \rangle$
- In binary: $\langle 1000, 10, 1000, 1, 10, 1000, 1, 10 \rangle$

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- Hashed to $\langle 8, 2, 8, 1, 2, 8, 1, 2 \rangle$
- In binary: $\langle 1000, 10, 1000, 1, 10, 1000, 1, 10 \rangle$
- Max value of ZEROS is 3. Return $2^{3+1/2} \approx 11.3$

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- In binary: $\langle 1000, 10, 1000, 1, 10, 1000, 1, 10 \rangle$
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- True value 3. What happens if we pick another hash function?

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- Hashed to $\langle 4, 3, 4, 1, 3, 4, 1, 3 \rangle$
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TIDEMARK(a_i)
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OUTPUT 2^(z+1/2)
```

- h chosen from a pairwise random family of hash functions.
- ZEROS counts the number of trailing zeros in the binary representation of a positive integer.
- Finds the maximum value of ZEROS$(h(a_i))$ overall.

Let's try it. Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.

- Hashed to $\langle 4, 3, 4, 1, 3, 4, 1, 3 \rangle$
- In binary: $\langle 100, 11, 100, 1, 11, 100, 1, 11 \rangle$
- Max value of ZEROS is 2. Return $2^{2+1/2} \approx 5.7$. We were luckier.

Understanding the TIDEMARK algorithm

$$\textsc{Zeros}(x) = 0 \qquad x = 1, 3, 5, \ldots$$
$$\textsc{Zeros}(x) \geq 1 \qquad x = 2, 4, 6, \ldots$$
$$\textsc{Zeros}(x) \geq 2 \qquad x = 4, 8, 12, \ldots$$
$$\textsc{Zeros}(x) \geq 3 \qquad x = 8, 16, 24, \ldots$$
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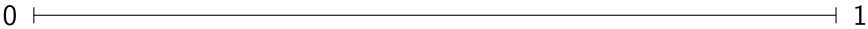

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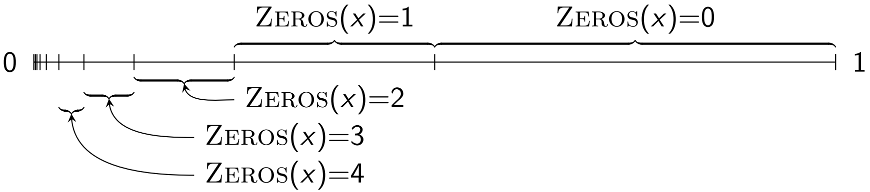

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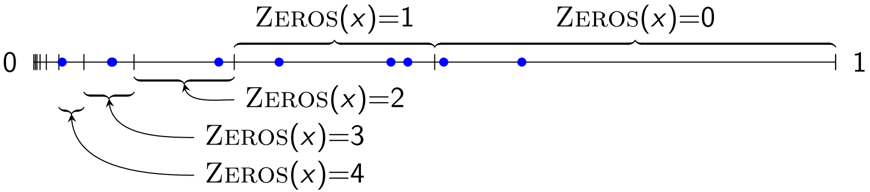

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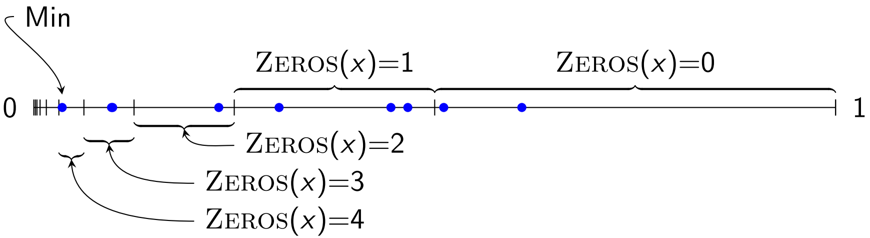

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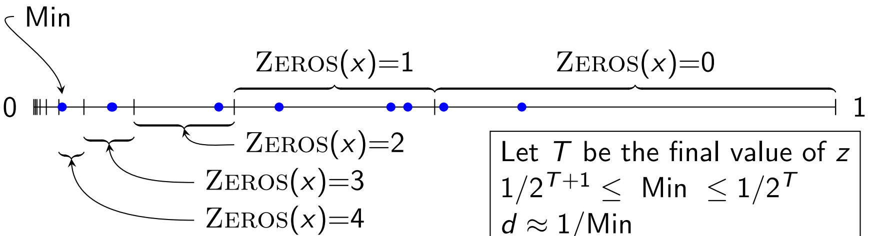

Let T be the final value of z
$1/2^{T+1} \leq$ Min $\leq 1/2^{T}$
$d \approx 1/\text{Min}$
$\implies d \approx 2^{T+1/2}$

The tidemark algorithm - analysis I

For $j \in [n]$, define indicator r.v. $X_{r,j} = 1$ if $\textsc{zeros}(h(j)) \geq r$.

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If n is a power of 2, $\mathbb{E}(X_{r,j}) = 1/2^r$

The tidemark algorithm - analysis II

For simplicity, assume n is a power of 2.

$$\mathbb{E}(Y_r) = \sum_{j: f_j > 0} \mathbb{E}(X_{r,j}) \quad = \frac{d}{2^r}$$

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Recall: $\hat{d} = 2^{T+1/2}$, $\Pr(Y_r > 0) \leq \frac{d}{2^r}$ and $\Pr(Y_r = 0) \leq \frac{2^r}{d}$.

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$$2^{a+1/2} \geq 3d \implies d \leq \frac{\sqrt{2} \cdot 2^a}{3} \implies \frac{d}{2^a} \leq \frac{\sqrt{2}}{3}$$

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The bounds are not ideal in two ways.

1. We can't get an arbitrarily close approximation (yet).
2. The failure probably is high. $\sqrt{2}/3 \approx 0.47$!

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- The maximum value of $\text{ZEROS}(h(a_i))$ is $\lfloor \log_2 n \rfloor \in O(\log n)$.
- Therefore only $O(\log \log n)$ bits are needed to represent the biggest possible value.

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Method: For each value in the input, run k independent copies of TIDEMARK in parallel and output the median.

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Define $X_i = 1$ if $\hat{d} \geq 3d$, 0 otherwise. $X = \sum_{i=1}^{k} X_i$, $\mu \leq \sqrt{2}k/3$.

Let $\delta = 3/(2\sqrt{2}) - 1 \approx 0.06$ so $(1+\delta)\mu = k/2$.

$$\Pr[X \geq (1+\delta)\mu] \leq \exp\left(-\delta^2\mu/3\right) \qquad \text{(Chernoff bound)}$$

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In other words, as k grows the probability that the median is at least $3d$ decreases exponentially.

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Recall: $\Pr(\hat{d} \leq d/3) \leq \frac{\sqrt{2}}{3}$

- If the median is less than d/3, then at least $k/2$ values are at most d/3.

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- SIMPLE-COUNT stores the minimum of the hashed values.
- SIMPLE-COUNT uses $O(\log n)$ bits of space.
- It gives us $\Pr(d \leq \hat{d}/3) \leq 1/3$ and $\Pr(d > 3\hat{d}) \leq 1/3$.

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