Exercise Sheet 4
COMS10007 Algorithms 2019/2020

25.02.2020

1   Algorithm Design

Describe an $\Theta(n \log n)$ time algorithm that, given an array A of n integers and another integer x, determines whether or not there are two elements in A whose sum equals x (Hint: Sorting!).

2   Bubblesort

Bubblesort is a popular, but inefficient, sorting algorithm. It works by repeatedly swapping adjacent elements that are out of order:

Algorithm 1 BUBBLESORT

Require: Array A of n integers
1: **for** $i = 0$ **to** $n - 2$ **do**
2:     **for** $j = n - 1$ **downto** $i + 1$ **do**
3:         **if** $A[j] < A[j-1]$ **then**
4:             exchange $A[j]$ with $A[j-1]$
5:         **end if**
6:     **end for**
7: **end for**
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1. What is the worst-case runtime of BUBBLESORT?

2. Consider the loop in lines $2 - 6$. Prove that the following invariant holds at the beginning of the loop:
$$A[j] \leq A[k], \text{ for every } k \geq j \ .$$
   Give a suitable termination property of the loop.

3. Consider now the loop in lines $1 - 7$. Prove that the following invariant holds at the beginning of the loop:
$$\text{The subarray } A[0, i] \text{ is sorted.}$$
   Give a suitable termination property that shows that A is sorted upon termination.

3   Proofs by Induction (optional and difficult!)

Let n be a positive number that is divisible by 23, i.e., $n = k \cdot 23$, for some interger $k \geq 1$. Let $x = \lfloor n/10 \rfloor$ and let $y = n \ \% \ 10$ (the rest of an integer division). Prove by induction on k that 23 divides $x + 7y$.

Example: Consider $k = 4$. Then $n = 92$, $x = 9$ and $y = 2$. Observe that the quantity $x + 7y = 9 + 7 \cdot 2 = 23$ is divisible by 23.