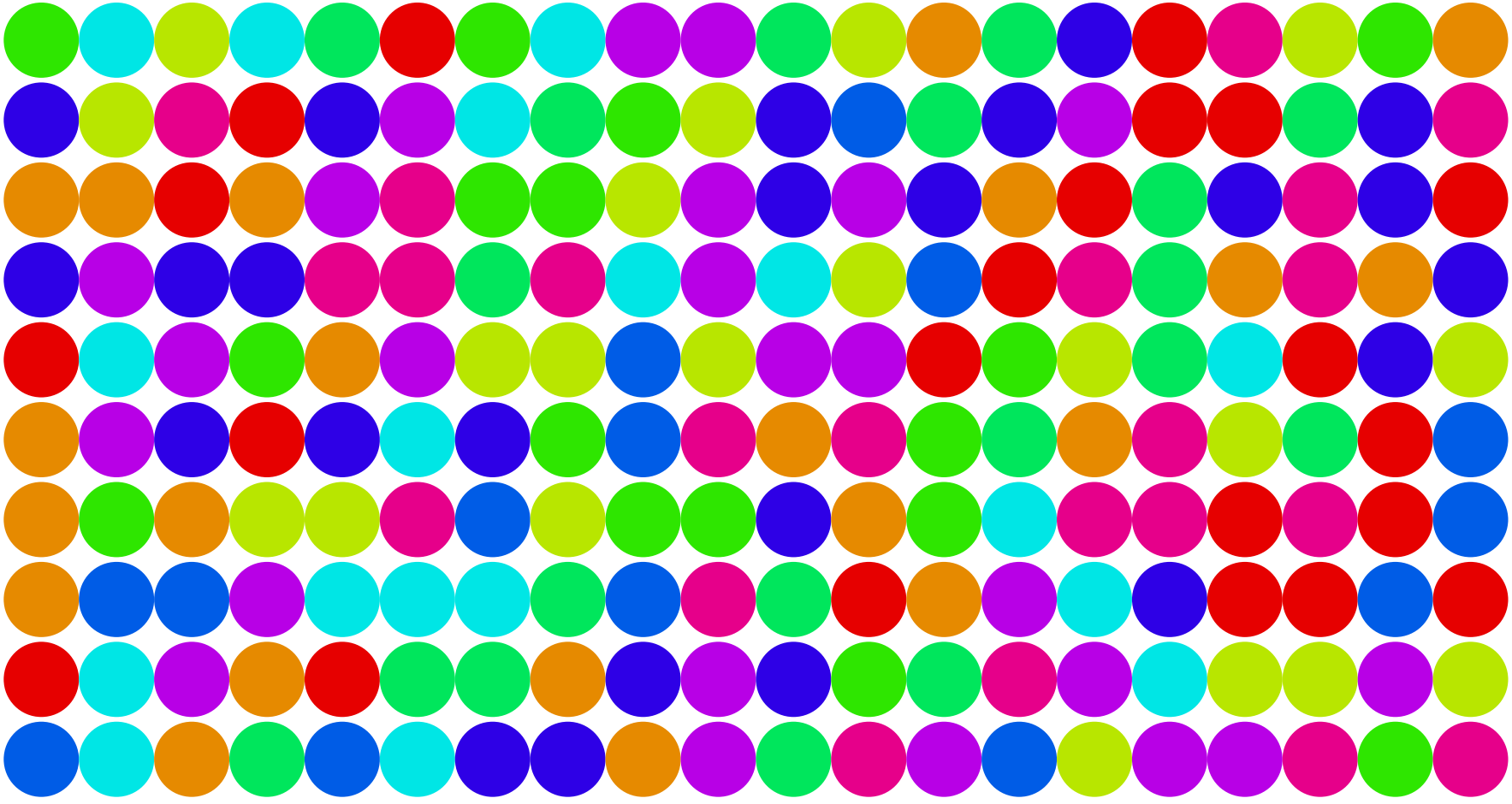
Topics in TCS

Counting distinct elements

Raphaël Clifford

Counting distinct elements



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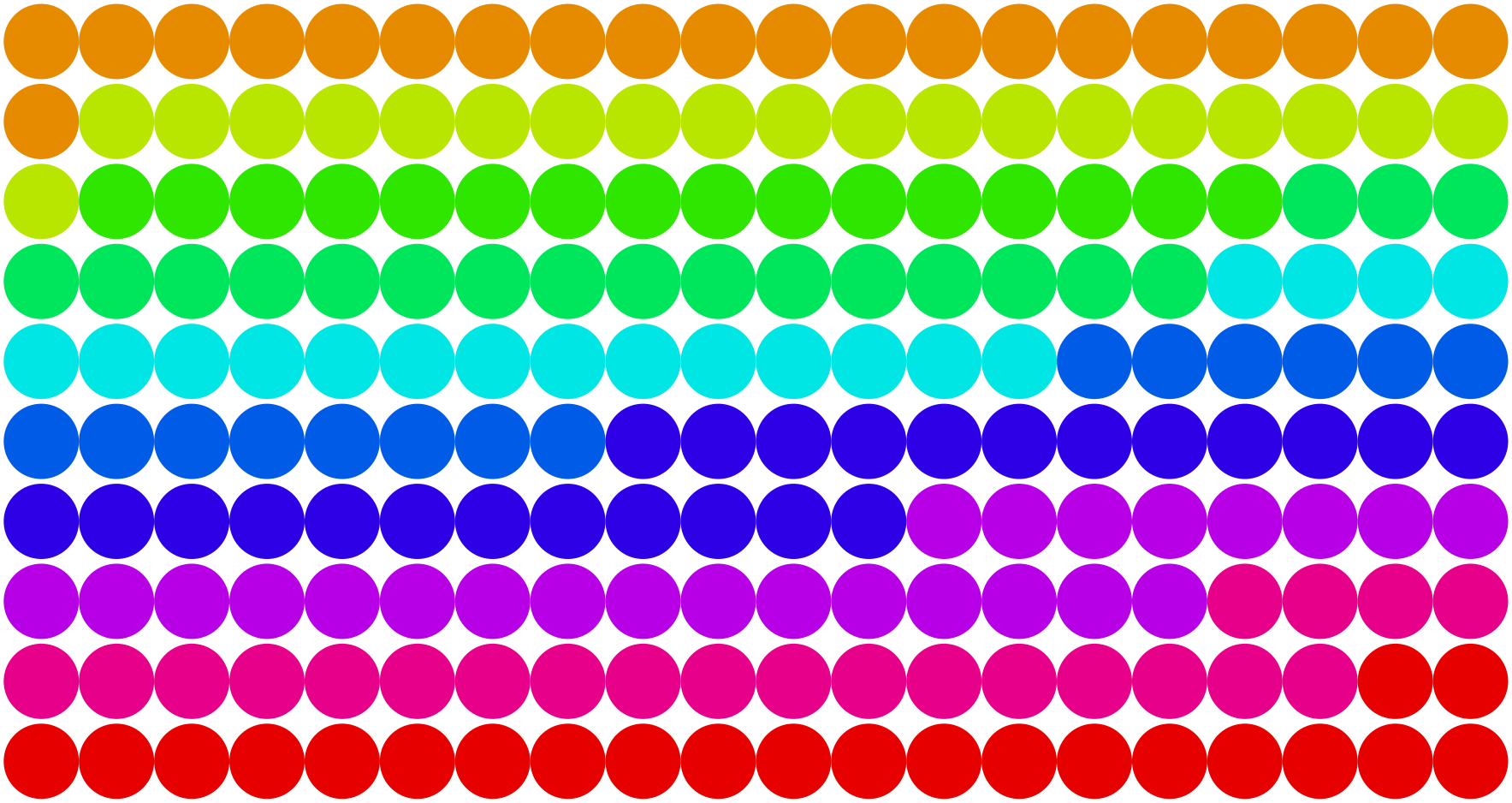

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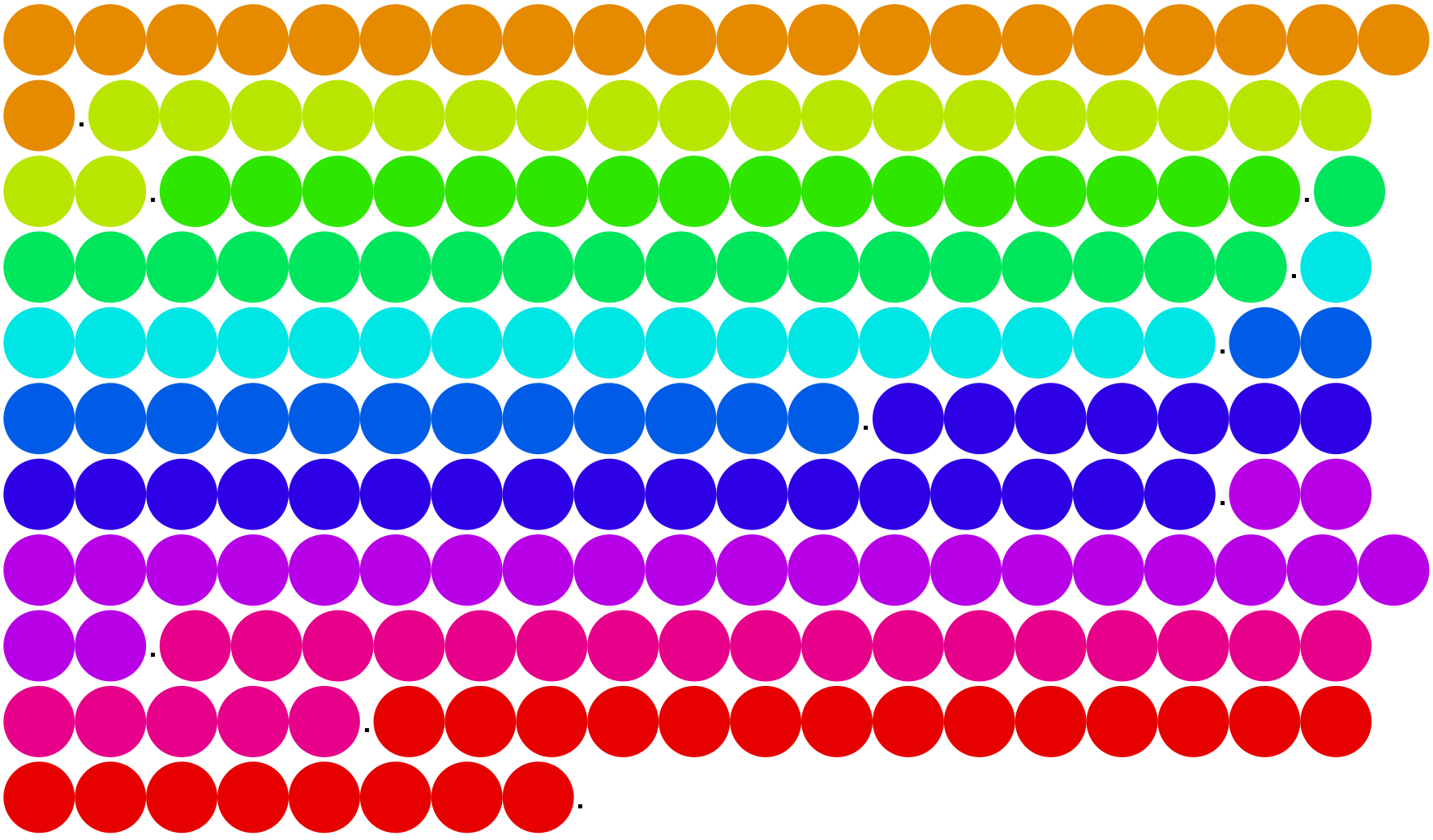
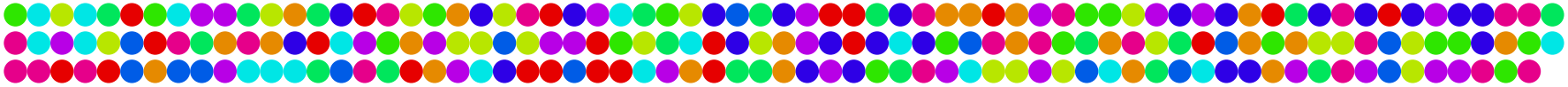


10 distinct colours

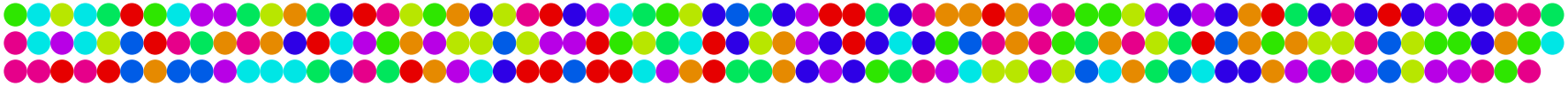
Counting distinct elements



Naive counting solution

Sort the elements. $O(m \log m)$ time.

Counting distinct elements



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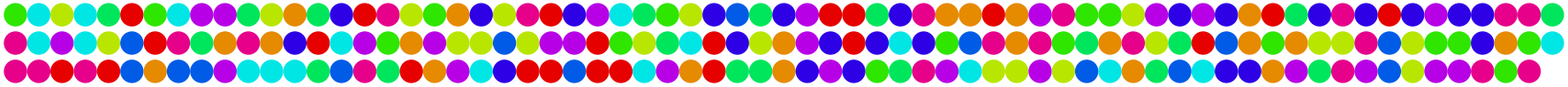

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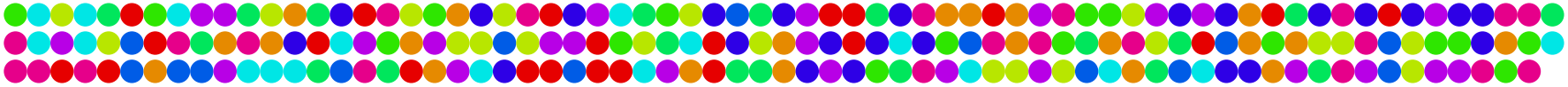

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NOT one-pass.

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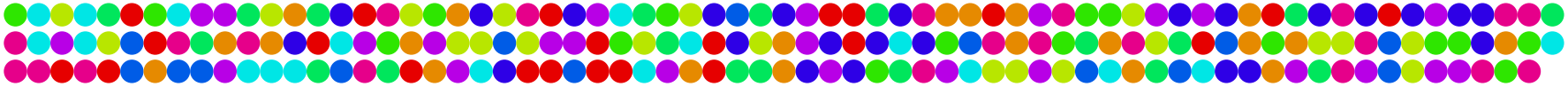
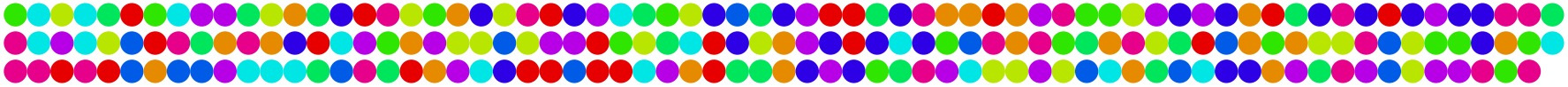


Slightly less naive counting solution

Use a balanced binary search tree.

Counting distinct elements



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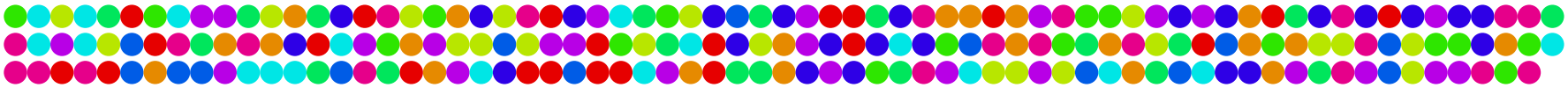

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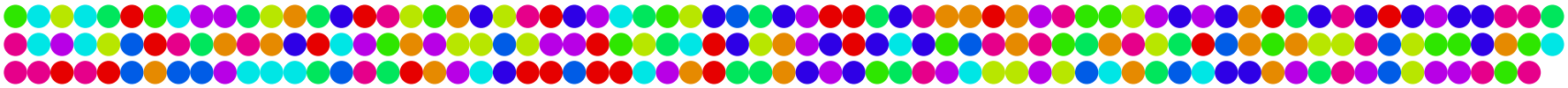

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At the conclusion the number of distinct values will be the number of nodes in tree.

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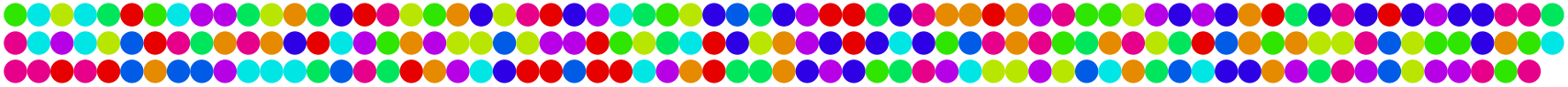

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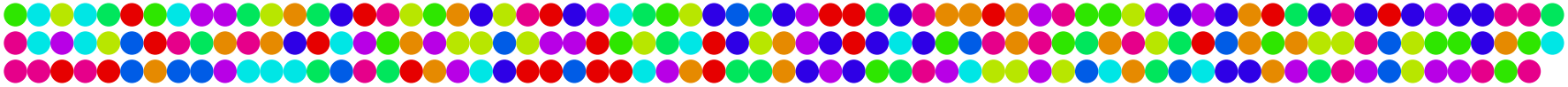

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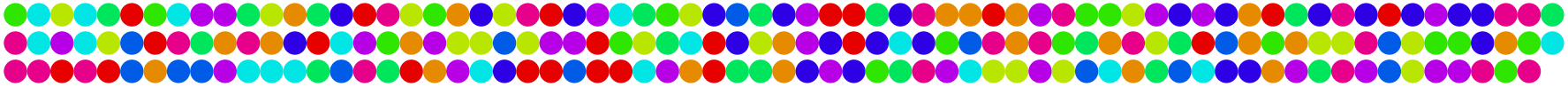

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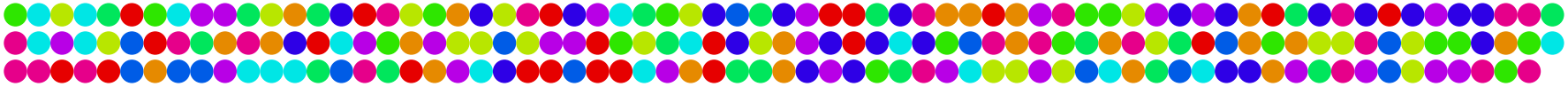

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No deterministic sub-linear space one-pass solution is possible.

We will need a randomised algorithm.

First randomised algorithm

stream $\langle a_1, \ldots, a_m \rangle, a_i \in [n]$
initialise
choose random $h : [n] \to [n]$

SIMPLE-COUNT
Set $M = h(a_1)$
For each $i \geq 2$
  if $h(a_i) < M$
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Worked example

- Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.

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- Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.
- Randomly hash to $\langle 10, 9, 10, 6, 9, 10, 6, 9 \rangle$
- $M = 6$ at termination.

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- Return $\hat{d} = 10/6 = 1\frac{2}{3}$

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- True answer is 3.

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Worked example. 2nd try

- Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.
- Rehash: $\langle 4, 3, 4, 8, 3, 4, 8, 3 \rangle$
-
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- Return $\hat{d} = 10/3 = 3\frac{1}{3}$
- True answer is 3.

Probability lemma for counting distinct elements

Lemma

Let $(Z_1, \ldots, Z_N)$ *be an array of pairwise independent indicator random variables with* $\Pr(Z_i = 1) = p$ *and let* $W = \sum_{i=1}^{N} Z_i$*, then* $\mathbb{E}(W) = Np$, $\mathrm{var}(W) = Np(1-p)$ *and*

$$\Pr(W > 0) \leq Np \quad \textit{and} \quad \Pr(W = 0) \leq \frac{1}{Np} \tag{1}$$

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- $\Pr(W = 0) \leq \Pr(|W - \mathbb{E}(W)| \geq \mathbb{E}(W)) \leq \frac{\mathrm{var}(W)}{(Np)^2} \leq \frac{1}{Np}$

□

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Define $Z_i = 1$ if $h(a_i) \leq b$ and 0 otherwise
Let the random variable $Z = \sum_{i=1}^{d} Z_i$
From (1), $\Pr(M > b) = \Pr(Z = 0) \leq \frac{1}{\frac{db}{n}} = \frac{n}{db}$

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- Now let $da/n = 1/3$ and $n/db = 1/3$, so that $a = n/(3d)$ and $b = 3n/d$, then (2) becomes

$$\Pr(M \leq n/(3d)) \leq 1/3 \quad \text{and} \quad \Pr(M > 3n/d) \leq 1/3$$

or

$$\Pr(d \leq \hat{d}/3) \leq 1/3 \quad \text{and} \quad \Pr(d > 3\hat{d}) \leq 1/3$$

using $\hat{d} = n/M$.

Simple-Count space/time

Our random estimate $\hat{d}$ gives us:

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- Running time $O(m)$

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- Running time $O(m)$
- One-pass ✓

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- Space $O(\log n)$ ✓
- Can we use less space? (Sort of)

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- Running time $O(m)$
- One-pass ✓
- Space $O(\log n)$ ✓
- Can we use less space? (Sort of)
- The error probability is pretty bad. Can we fix that? (Yes)

The TIDEMARK algorithm

initialise
choose random $h : [n] \to [n]$
set $z = 0$

TIDEMARK(a_i)
if ZEROS($h(a_i)$) $> z$
  set z = ZEROS($h(a_i)$)

OUTPUT $2^{z+\frac{1}{2}}$

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- h chosen from a pairwise random family of hash functions.
- ZEROS counts the number of trailing zeros in the binary representation of a positive integer.
- Finds the maximum value of $\text{ZEROS}(h(a_i))$ overall.

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OUTPUT 2^{z+1/2}
```

- h chosen from a pairwise random family of hash functions.
- ZEROS counts the number of trailing zeros in the binary representation of a positive integer.
- Finds the maximum value of $\text{ZEROS}(h(a_i))$ overall.

Let's try it. Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.

The TIDEMARK algorithm

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initialise
choose random h : [n] → [n]
set z = 0

TIDEMARK(a_i)
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- Hashed to $\langle 8, 2, 8, 1, 2, 8, 1, 2 \rangle$
- In binary: $\langle 1000, 10, 1000, 1, 10, 1000, 1, 10 \rangle$

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- In binary: $\langle 1000, 10, 1000, 1, 10, 1000, 1, 10 \rangle$
- Max value of ZEROS is 3. Return $2^{3+1/2} \approx 11.3$

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- In binary: $\langle 1000, 10, 1000, 1, 10, 1000, 1, 10 \rangle$
- Max value of ZEROS is 3. Return $2^{3+1/2} \approx 11.3$
- True value 3. What happens if we pick another hash function?

The TIDEMARK algorithm

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Let's try it. Stream $\langle 5, 4, 5, 7, 4, 5, 7, 4 \rangle$. $n = 10$.

- Hashed to $\langle 4, 3, 4, 1, 3, 4, 1, 3 \rangle$
- In binary: $\langle 100, 11, 100, 1, 11, 100, 1, 11 \rangle$

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set z = 0

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- Hashed to $\langle 4, 3, 4, 1, 3, 4, 1, 3 \rangle$
- In binary: $\langle 100, 11, 100, 1, 11, 100, 1, 11 \rangle$
- Max value of ZEROS is 2. Return $2^{2+1/2} \approx 5.7$. We were luckier.

Understanding the TIDEMARK algorithm

$$\begin{aligned}
\textsc{Zeros}(x) &= 0 && x = 1, 3, 5, \ldots \\
\textsc{Zeros}(x) &\geq 1 && x = 2, 4, 6, \ldots \\
\textsc{Zeros}(x) &\geq 2 && x = 4, 8, 12, \ldots \\
\textsc{Zeros}(x) &\geq 3 && x = 8, 16, 24, \ldots \\
\textsc{Zeros}(x) &\geq 4 && x = 16, 32, 48, \ldots
\end{aligned}$$

Understanding the TIDEMARK algorithm

$\text{ZEROS}(x) = 0$	$x = 1, 3, 5, \ldots$	$p = 1/2$
$\text{ZEROS}(x) \geq 1$	$x = 2, 4, 6, \ldots$	$p = 1/2$
$\text{ZEROS}(x) \geq 2$	$x = 4, 8, 12, \ldots$	$p = 1/4$
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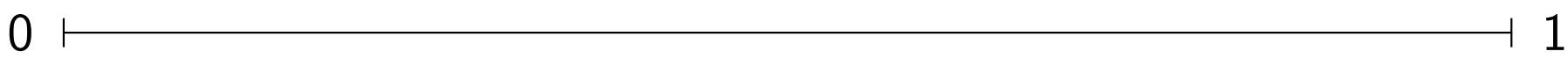
$\text{ZEROS}(x) \geq 4$	$x = 16, 32, 48, \ldots$	$p = 1/16$



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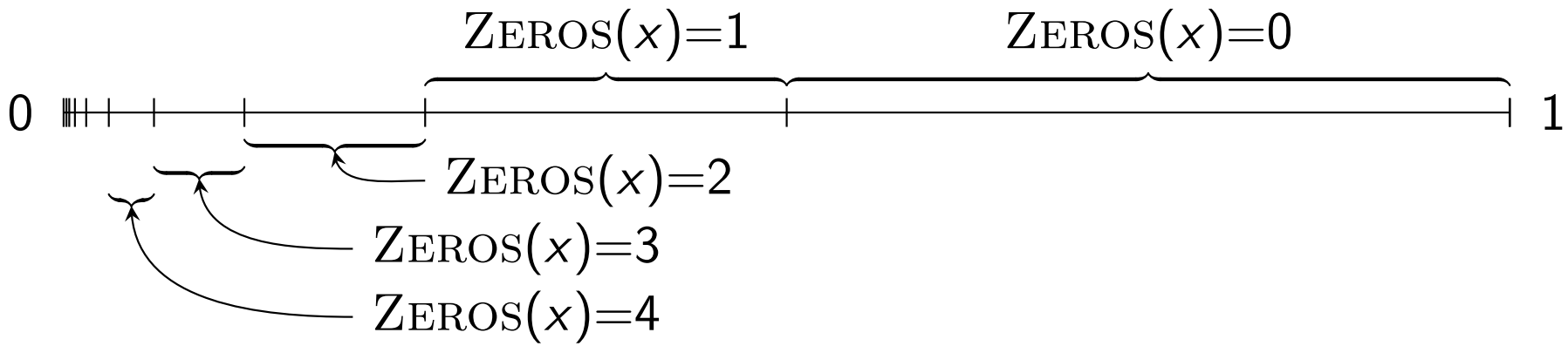

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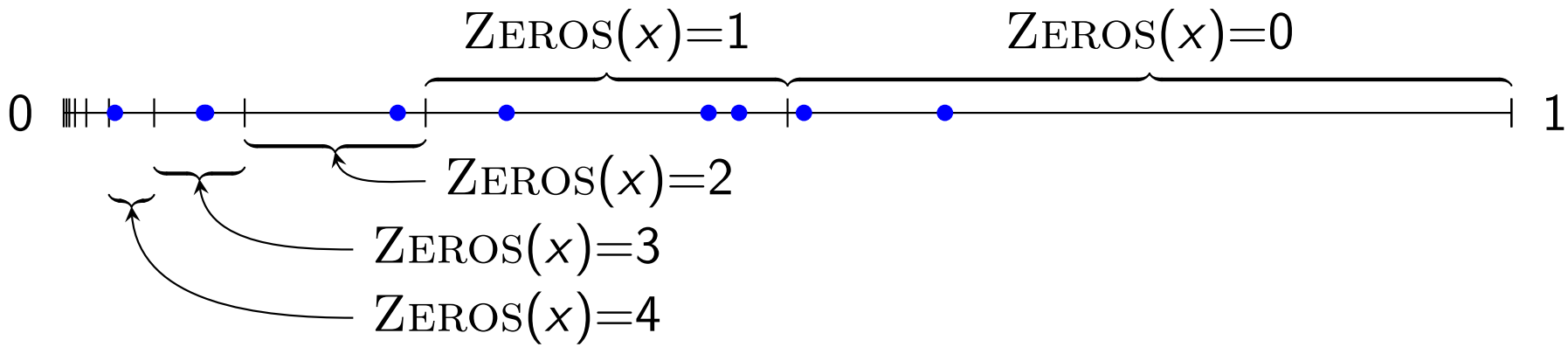

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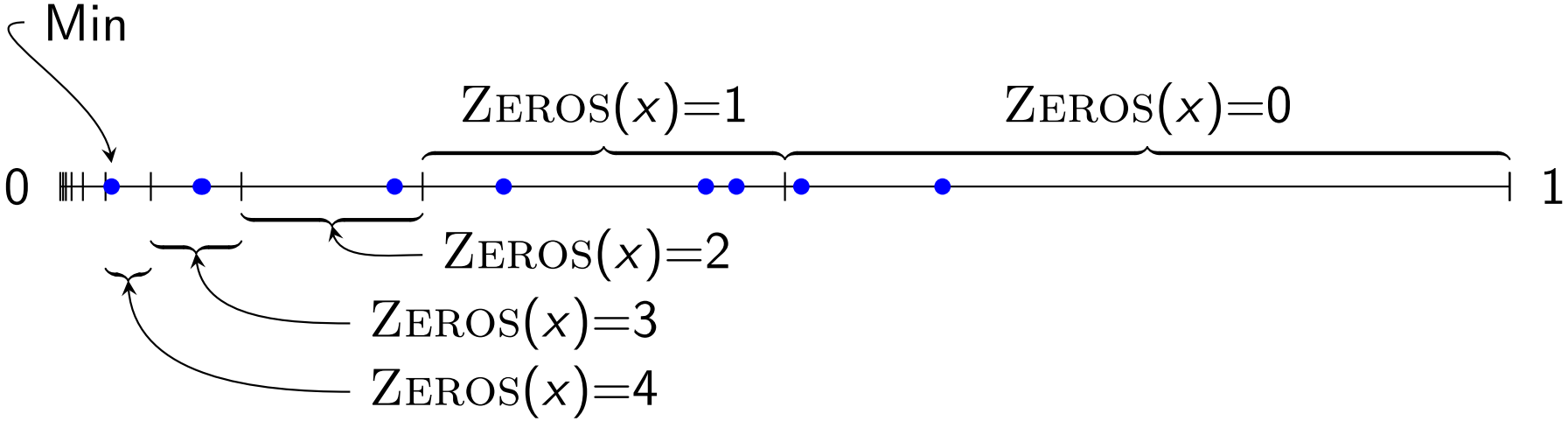

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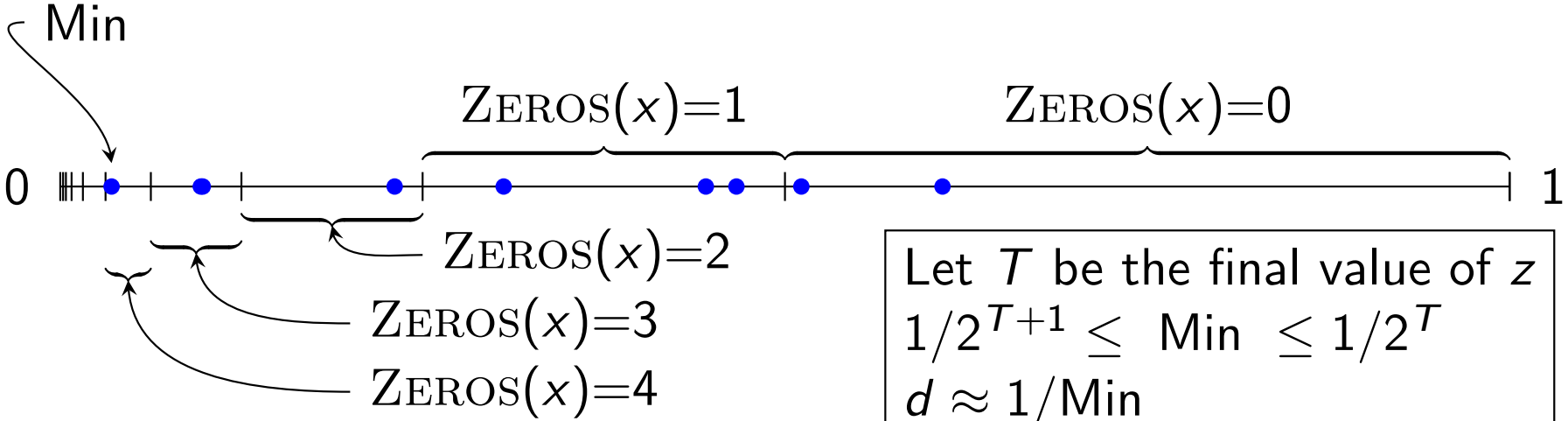

Let T be the final value of z

$1/2^{T+1} \leq \text{Min} \leq 1/2^{T}$

$d \approx 1/\text{Min}$

$\Longrightarrow \; d \approx 2^{T+1/2}$

The tidemark algorithm - analysis I

For $j \in [n]$, define indicator r.v. $X_{r,j} = 1$ if $\textsc{zeros}(h(j)) \geq r$.

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Define r.v. $Y_r = \sum_{j, f_j > 0} X_{r,j}$. That is the number of hashed values in the stream that have at least r trailing zeros.

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$$Y_r > 0 \text{ iff } T \geq r$$

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$$\mathbb{E}(X_{r,j}) = \Pr(\textsc{zeros}(h(j)) \geq r) = \Pr(2^r \text{ divides } h(j))$$

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If n is a power of 2, $\mathbb{E}(X_{r,j}) = 1/2^r$

The tidemark algorithm - analysis II

For simplicity, assume n is a power of 2.

$$\mathbb{E}(Y_r) = \sum_{j:f_j>0} \mathbb{E}(X_{r,j}) \quad = \frac{d}{2^r}$$

The tidemark algorithm - analysis II

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$$\Pr(Y_r > 0) = \Pr(Y_r \geq 1) \leq \frac{\mathbb{E}(Y_r)}{1} = \frac{d}{2^r}$$

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$$\Pr(Y_r = 0) \leq \Pr(|Y_r - \mathbb{E}(Y_r)| \geq d/2^r) \leq \frac{\text{var}(Y_r)}{(d/2^r)^2} \leq \frac{2^r}{d}$$

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The tidemark algorithm - analysis III

Recall: $\hat{d} = 2^{T+1/2}$, $\Pr(Y_r > 0) \leq \frac{d}{2^r}$ and $\Pr(Y_r = 0) \leq \frac{2^r}{d}$.

The tidemark algorithm - analysis III

Recall: $\hat{d} = 2^{T+1/2}$, $\Pr(Y_r > 0) \leq \frac{d}{2^r}$ and $\Pr(Y_r = 0) \leq \frac{2^r}{d}$.

- Let a be the smallest integer such that $2^{a+1/2} \geq 3d$. We have,

$$\Pr(\hat{d} \geq 3d) = \Pr(T \geq a)$$

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$$2^{a+1/2} \geq 3d \implies d \leq \frac{\sqrt{2} \cdot 2^a}{3}$$
$$\implies \frac{d}{2^a} \leq \frac{\sqrt{2}}{3}$$

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This gives us the probability that our estimate is too large. We now bound the probability it is too small.

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- For the probability that our estimate is too small let b be the largest integer such that $2^{b+1/2} \leq d/3$.

$$\Pr(\hat{d} \leq d/3) = \Pr(T \leq b) = \Pr(Y_{b+1} = 0) \leq \frac{2^{b+1}}{d} \leq \frac{\sqrt{2}}{3}$$

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Recall: $\hat{d} = 2^{T+1/2}$, $\Pr(Y_r > 0) \leq \frac{d}{2^r}$ and $\Pr(Y_r = 0) \leq \frac{2^r}{d}$.

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The bounds are not ideal in two ways.

1. We can't get an arbitrarily close approximation (yet).
2. The failure probably is high. $\sqrt{2}/3 \approx 0.47$!

Tidemark space/time

initialise
Choose random $h : [n] \to [n]$
Set $z = 0$

Tidemark(a_i)
if Zeros($h(a_i)$) $> z$
  set z = Zeros($h(a_i)$)

Output $2^{z+\frac{1}{2}}$

Tidemark space/time

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- One-pass and $O(m)$ time. ✓

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Why is it $O(\log \log n)$ bits of space?

TIDEMARK space/time

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Choose random h : [n] → [n]
Set z = 0

TIDEMARK(a_i)
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OUTPUT 2^{z+1/2}
```

- One-pass and $O(m)$ time. ✓
- $O(\log \log n)$ bits of space. ✓

Why is it $O(\log \log n)$ bits of space?

- We store one value of $\text{ZEROS}(h(a_i))$ where $h(a_i) \in [n]$.

TIDEMARK space/time

```
initialise
Choose random h : [n] → [n]
Set z = 0

TIDEMARK(a_i)
if ZEROS(h(a_i)) > z
   set z = ZEROS(h(a_i))

OUTPUT 2^{z+1/2}
```

- One-pass and $O(m)$ time. ✓
- $O(\log \log n)$ bits of space. ✓

Why is it $O(\log \log n)$ bits of space?

- We store one value of $\text{ZEROS}(h(a_i))$ where $h(a_i) \in [n]$.
- The maximum value of $\text{ZEROS}(h(a_i))$ is $\lfloor \log_2 n \rfloor \in O(\log n)$.

Tidemark space/time

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- Therefore only $O(\log \log n)$ bits are needed to represent the biggest possible value.

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Method: For each value in the input, run k independent copies of TIDEMARK in parallel and output the median.

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Define $X_i = 1$ if $\hat{d} \geq 3d$, 0 otherwise. $X = \sum_{i=1}^{k} X_i$, $\mu \leq \sqrt{2}k/3$.

Let $\delta = 3/(2\sqrt{2}) - 1 \approx 0.06$ so $(1+\delta)\mu = k/2$.

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In other words, as k grows the probability that the median is at least $3d$ decreases exponentially.

The median trick - lower bound

Method: For each value in the input, run k independent copies of TIDEMARK in parallel and output the median.

Recall: $\Pr(\hat{d} \leq d/3) \leq \frac{\sqrt{2}}{3}$

- If the median is less than $d/3$, then at least $k/2$ values are at most $d/3$.

Define $X_i = 1$ if $\hat{d} \leq d/3$, 0 otherwise, $X = \sum_{i=1}^{k} X_i$, $\mu \leq \sqrt{2}k/3$.

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- It gives us $\Pr(\hat{d} \leq d/3) \leq \frac{\sqrt{2}}{3}$ and $\Pr(\hat{d} \geq 3d) \leq \frac{\sqrt{2}}{3}$
- By performing k parallel iterations the probability of being outside the range $[d/3, 3d]$ can be made exponentially small.