University of BRISTOL

Advanced Algorithms – COMS31900

Pattern Matching part two

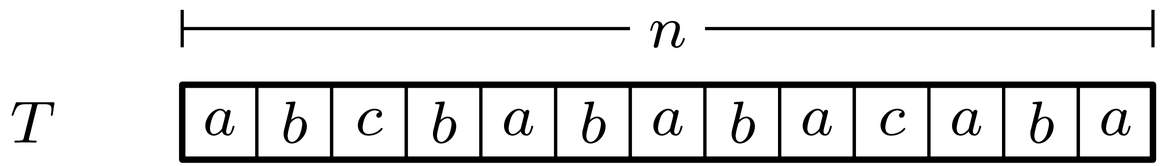
Suffix Arrays

Raphaël Clifford

Slides by Benjamin Sach

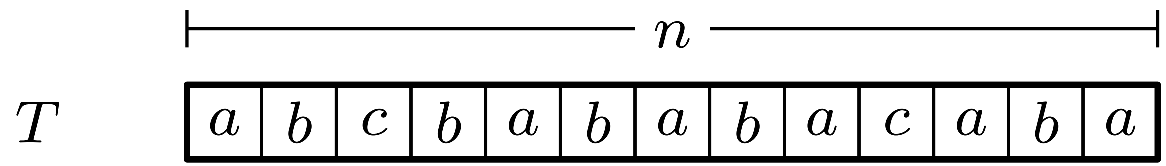
Text indexing

Preprocess a text string T (length n) to answer pattern matching queries. . .

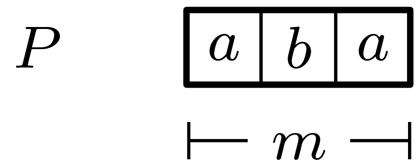


Text indexing

Preprocess a text string T (length n) to answer pattern matching queries...



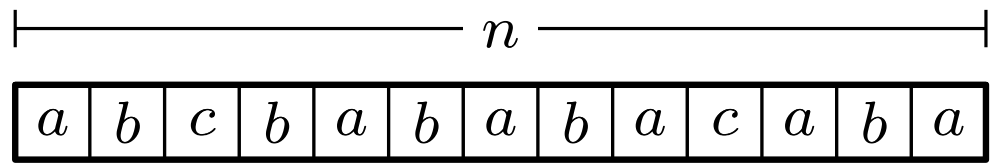
After preprocessing, a **query** is a pattern P (length m),



Text indexing

Preprocess a text string T (length n) to answer pattern matching queries...

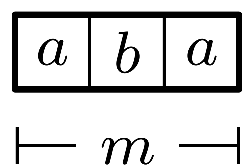
T



After preprocessing, a **query** is a pattern P (length m),

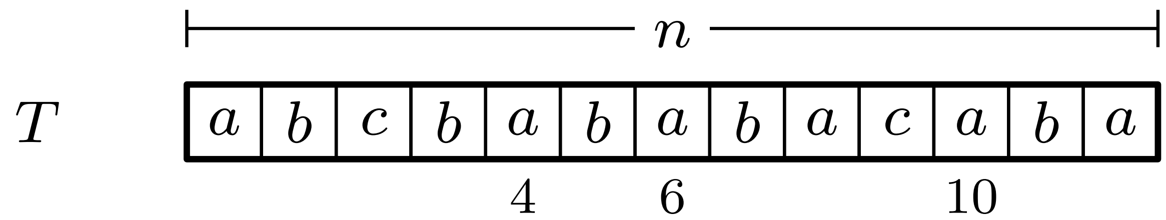
the output is a list of all matches in T.

P

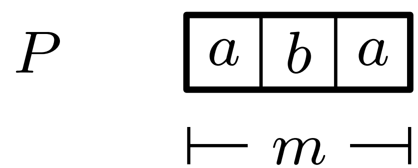


Text indexing

Preprocess a text string T (length n) to answer pattern matching queries...



After preprocessing, a **query** is a pattern P (length m),

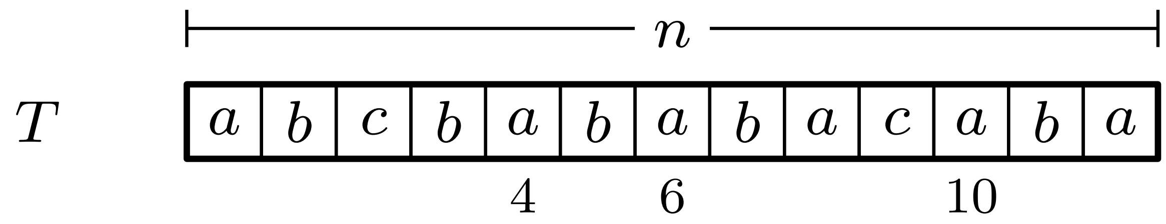


the output is a list of all matches in T.

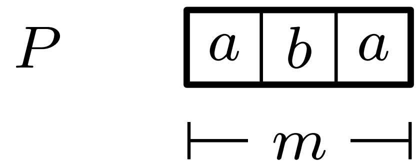
e.g. $4, 6, 10$

Text indexing

Preprocess a text string T (length n) to answer pattern matching queries. . .



After preprocessing, a **query** is a pattern P (length m),



the output is a list of all matches in T.

e.g. $4, 6, 10$

- Last lecture we saw the that **text indexing** problem can be solved using a suffix tree which uses $O(n)$ space *(when it's stored compacted)*

Text indexing

Preprocess a text string T (length n) to answer pattern matching queries...

n

T													
	a	b	c	b	a	b	a	b	a	c	a	b	a
					4		6				10		

After preprocessing, a **query** is a pattern P (length m),

P			
	a	b	a

m

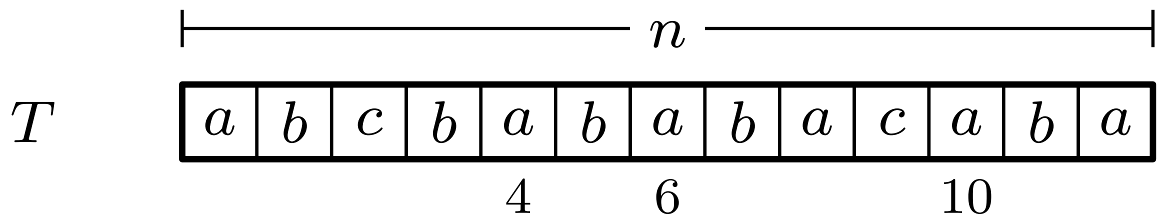
the output is a list of all matches in T.

e.g. $4, 6, 10$

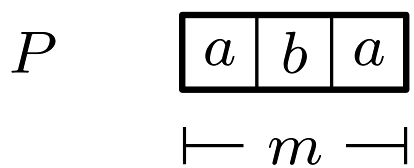
- Last lecture we saw the that **text indexing** problem can be solved using a suffix tree which uses $O(n)$ space *(when it's stored compacted)*
- Queries take $O(m + \text{occ})$ time when the alphabet size is constant
  - occ is the number of occurences (matches)

Text indexing

Preprocess a text string T (length n) to answer pattern matching queries. . .



After preprocessing, a **query** is a pattern P (length m),

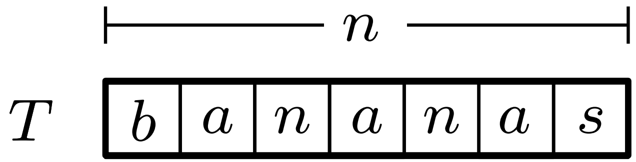


the output is a list of all matches in T.

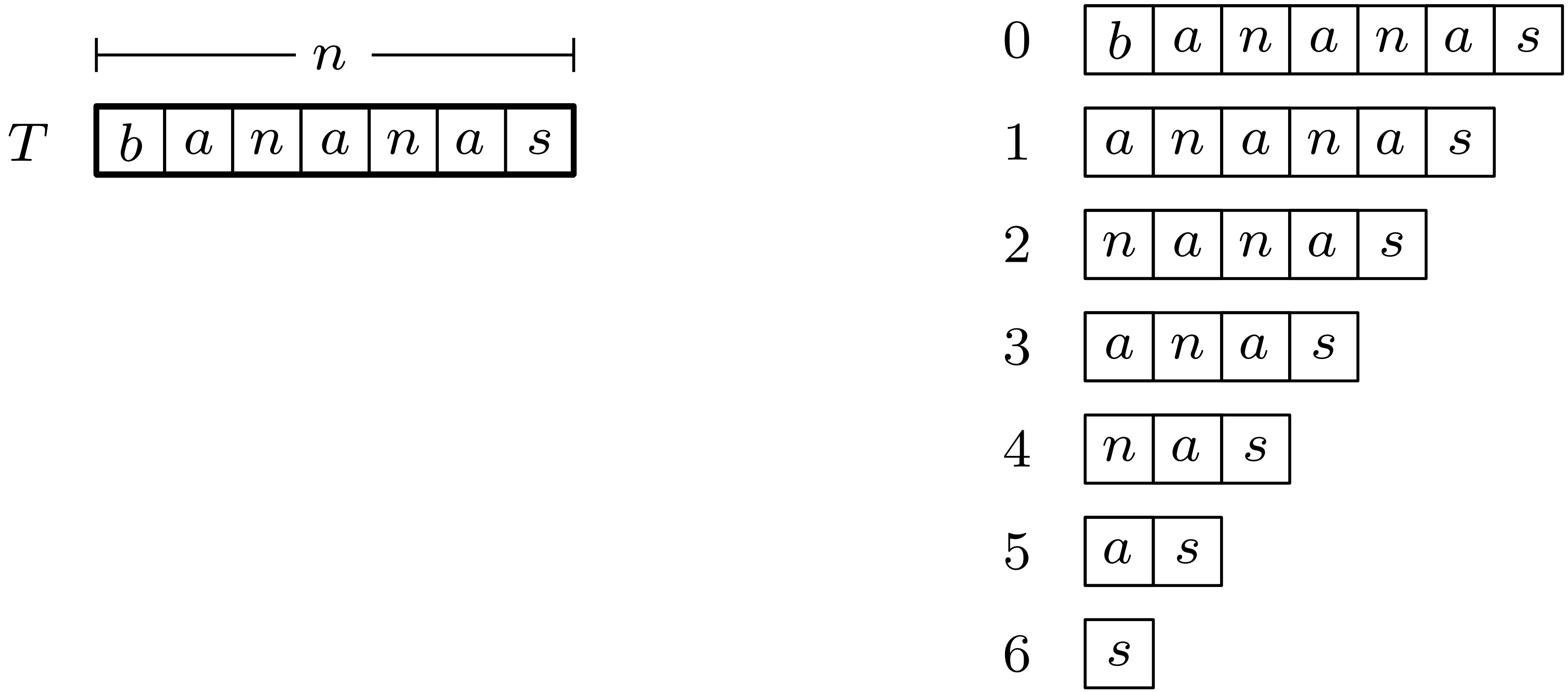
e.g. $4, 6, 10$

- Last lecture we saw the that **text indexing** problem can be solved using a suffix tree which uses $O(n)$ space *(when it's stored compacted)*
- Queries take $O(m + \text{occ})$ time when the alphabet size is constant
  - occ is the number of occurences (matches)
- Suffix trees can be constructed in $O(n)$ time (but we only saw how to achieve $O(n^2)$ time)

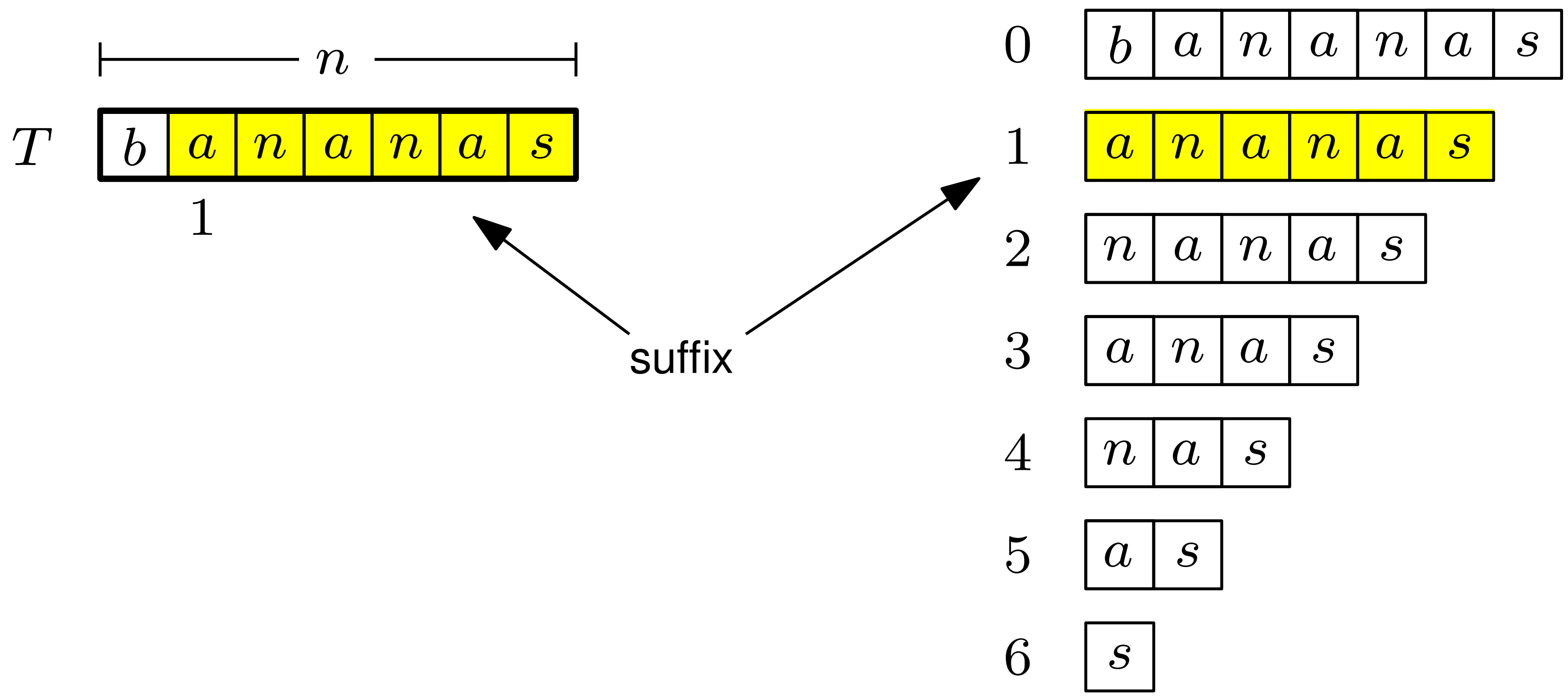
The suffix array



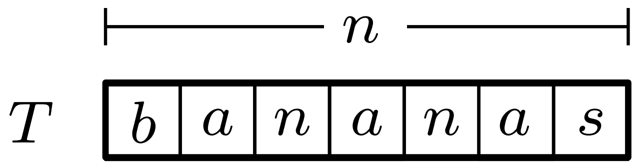
The suffix array

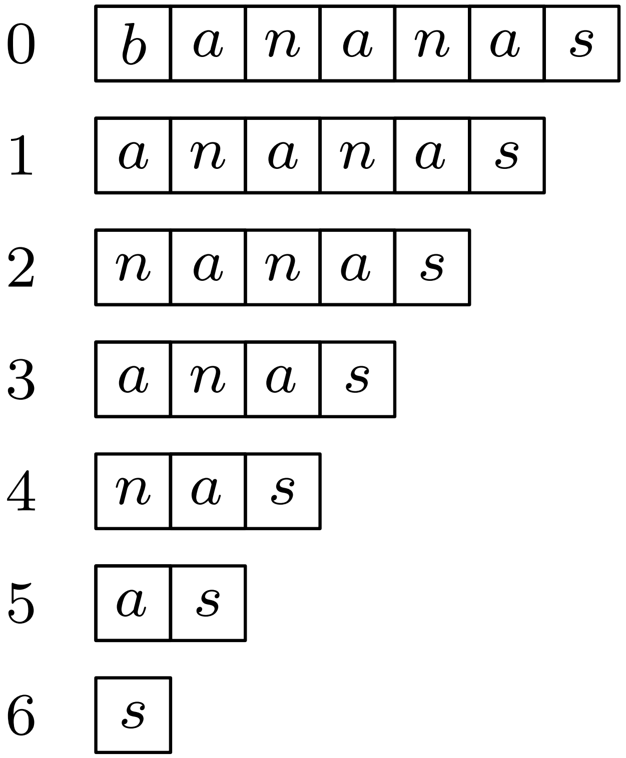


The suffix array

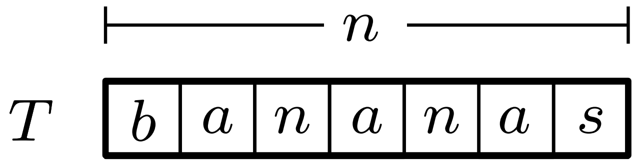


The suffix array

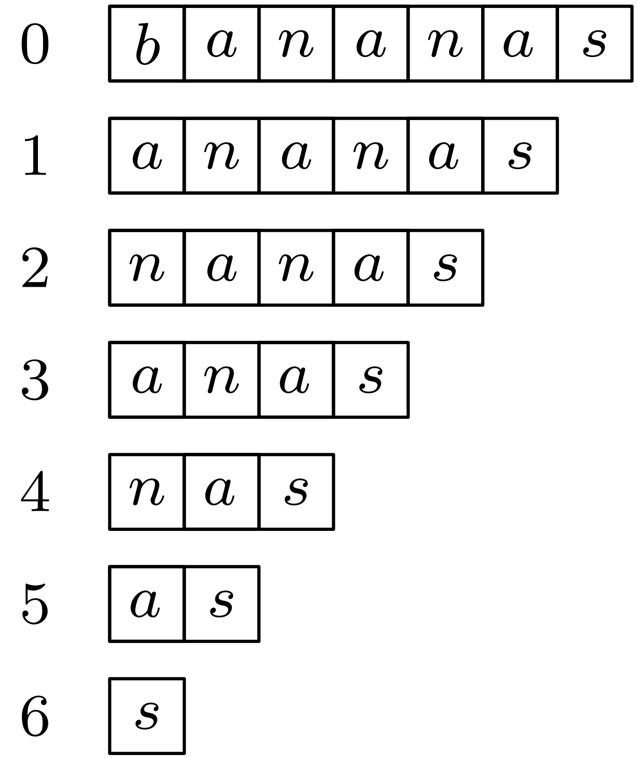




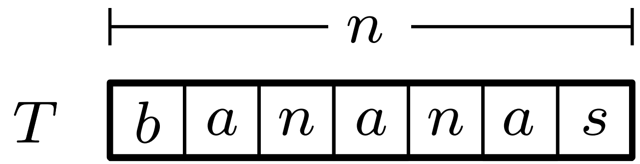
The suffix array



Sort the suffixes lexicographically



The suffix array

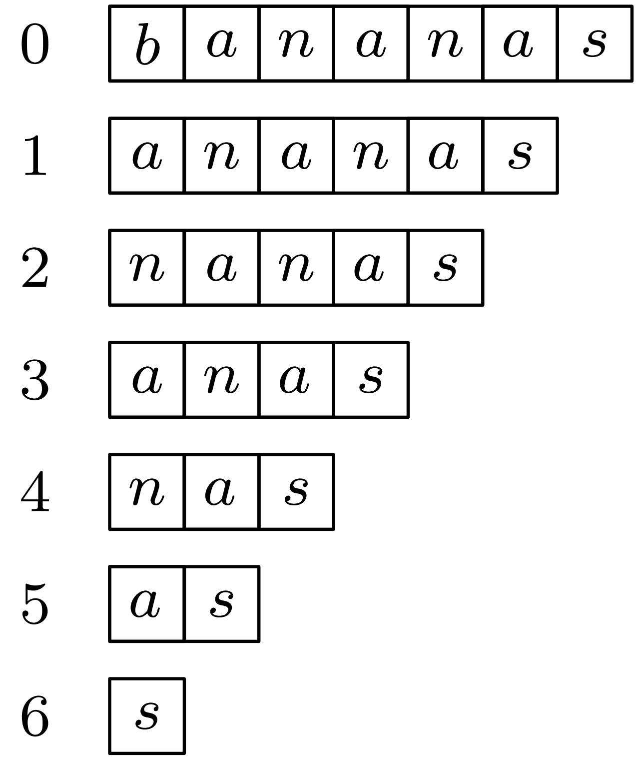


Sort the suffixes

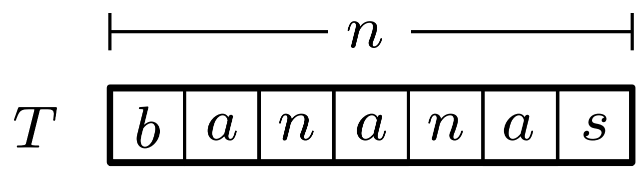
lexicographically

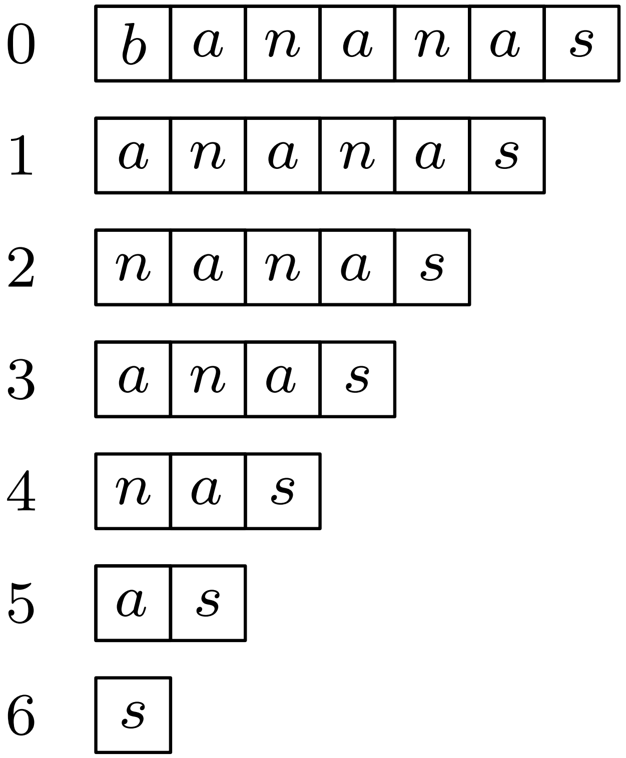
- The symbols themselves must have an order

  throughout we will use alphabetical order



The suffix array





Sort the suffixes

lexicographically

- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

The suffix array

$\longleftarrow n \longrightarrow$

T	b	a	n	a	n	a	s

0	b	a	n	a	n	a	s
1	a	n	a	n	a	s	
2	n	a	n	a	s		
3	a	n	a	s			
4	n	a	s				
5	a	s					
6	s						

Sort the suffixes

lexicographically

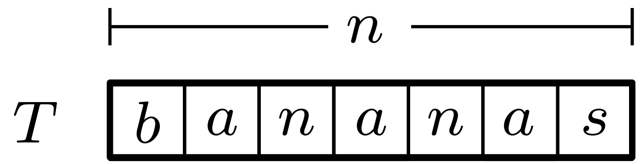
- The symbols themselves must have an order

  throughout we will use alphabetical order

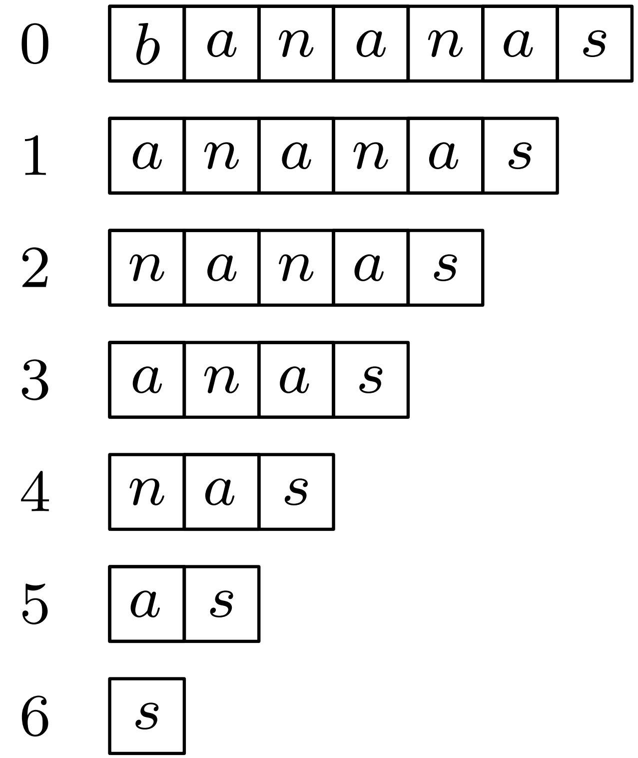
In lexicographical ordering we sort strings based on the first symbol that differs:

$$[a][a] < [b][a]$$

The suffix array



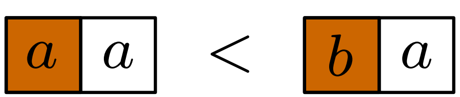
Sort the suffixes lexicographically



- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:



The suffix array

$\longleftarrow n \longrightarrow$

T							
	b	a	n	a	n	a	s

Sort the suffixes lexicographically

0	b	a	n	a	n	a	s
1	a	n	a	n	a	s	
2	n	a	n	a	s		
3	a	n	a	s			
4	n	a	s				
5	a	s					
6	s						

- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$$\boxed{a}\boxed{a} < \boxed{b}\boxed{a}$$

The suffix array

n

T | b | a | n | a | n | a | s |

Sort the suffixes lexicographically

0	$b\ a\ n\ a\ n\ a\ s$
1	$a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
4	$n\ a\ s$
5	$a\ s$
6	s

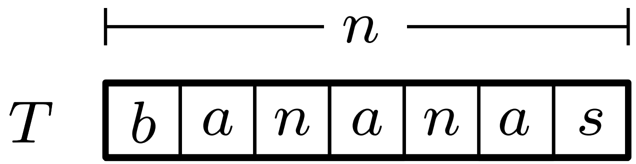
- The symbols themselves must have an order

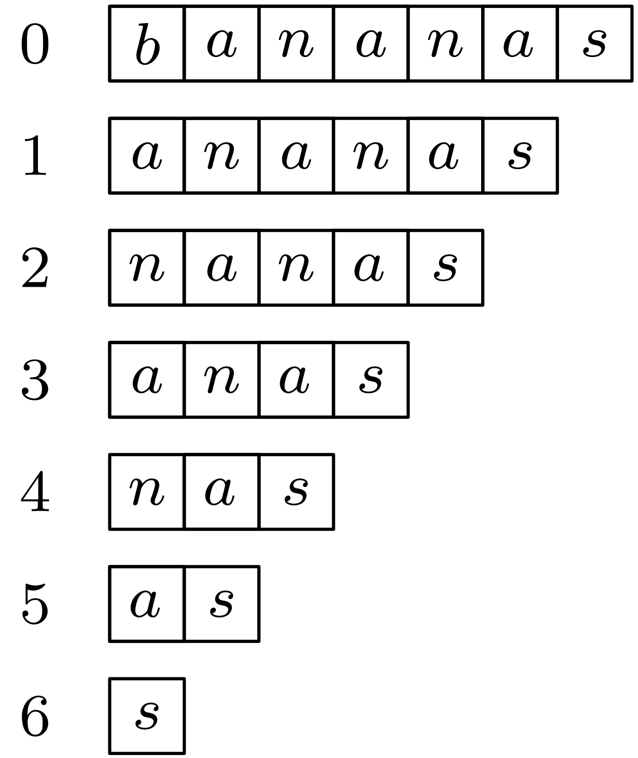
  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$$a\,a \;<\; b\,a \;<\; b\,c$$

The suffix array





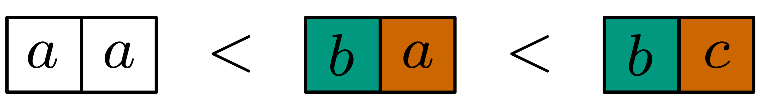
Sort the suffixes

lexicographically

- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:



The suffix array

n

T = $b\ a\ n\ a\ n\ a\ s$

Sort the suffixes lexicographically

0	$b\ a\ n\ a\ n\ a\ s$
1	$a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
4	$n\ a\ s$
5	$a\ s$
6	s

- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$$aa < ba < bc$$

The suffix array

T = | b | a | n | a | n | a | s | (length n)

0	$b\,a\,n\,a\,n\,a\,s$
1	$a\,n\,a\,n\,a\,s$
2	$n\,a\,n\,a\,s$
3	$a\,n\,a\,s$
4	$n\,a\,s$
5	$a\,s$
6	s

Sort the suffixes lexicographically

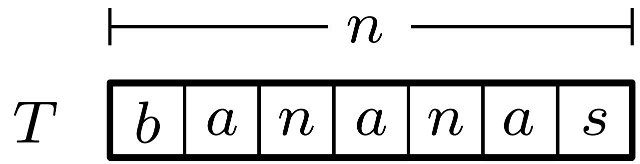
- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$$aa < ba < bc < bca$$

The suffix array

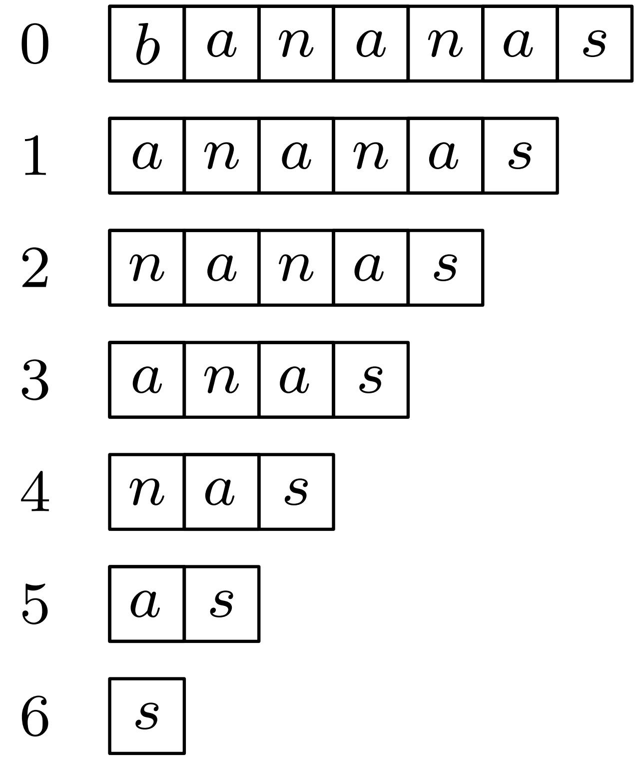


Sort the suffixes

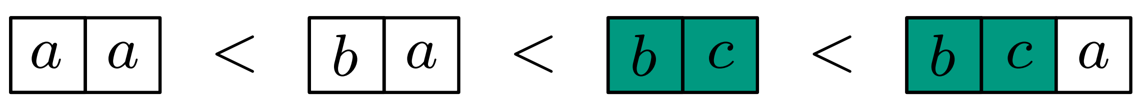
lexicographically

- The symbols themselves must have an order

  throughout we will use alphabetical order



In lexicographical ordering we sort strings based on the first symbol that differs:



The suffix array

n

T							
	b	a	n	a	n	a	s

0	b	a	n	a	n	a	s
1	a	n	a	n	a	s	
2	n	a	n	a	s		
3	a	n	a	s			
4	n	a	s				
5	a	s					
6	s						

Sort the suffixes lexicographically

- The symbols themselves must have an order

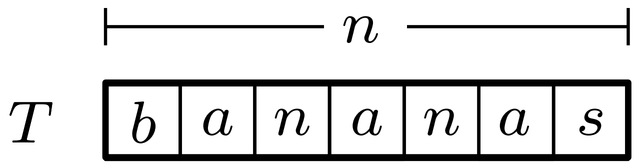
  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

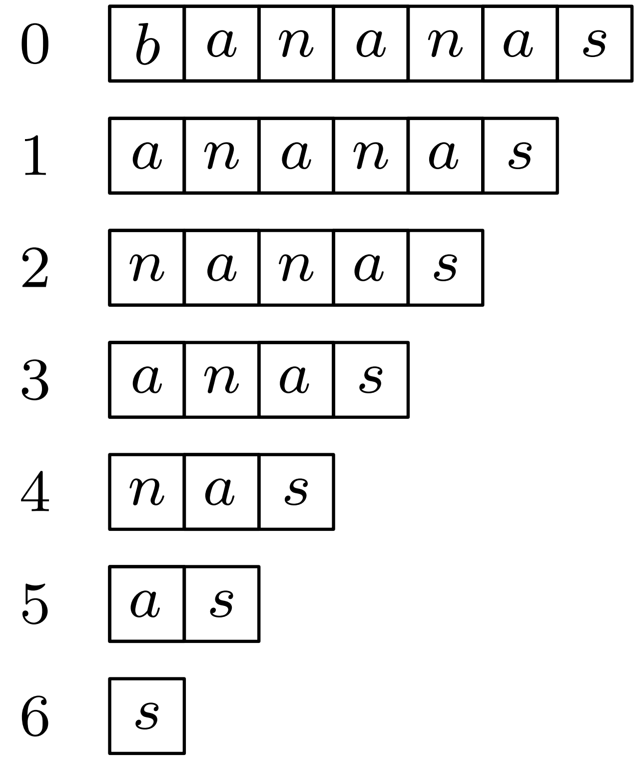
$aa < ba < bc < bca$

(in a ‘tie’, the shorter string is smaller)

The suffix array



Sort the suffixes lexicographically



- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$aa < ba < bc < bca$

(in a 'tie', the shorter string is smaller)

The suffix array

T = | b | a | n | a | n | a | s | (length n)

Sort the suffixes lexicographically

1	a n a n a s
3	a n a s
5	a s
0	b a n a n a s
2	n a n a s
4	n a s
6	s

- The symbols themselves must have an order

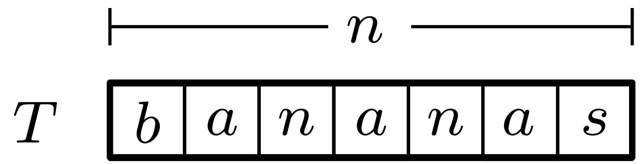
  throughout we will use alphabetical order

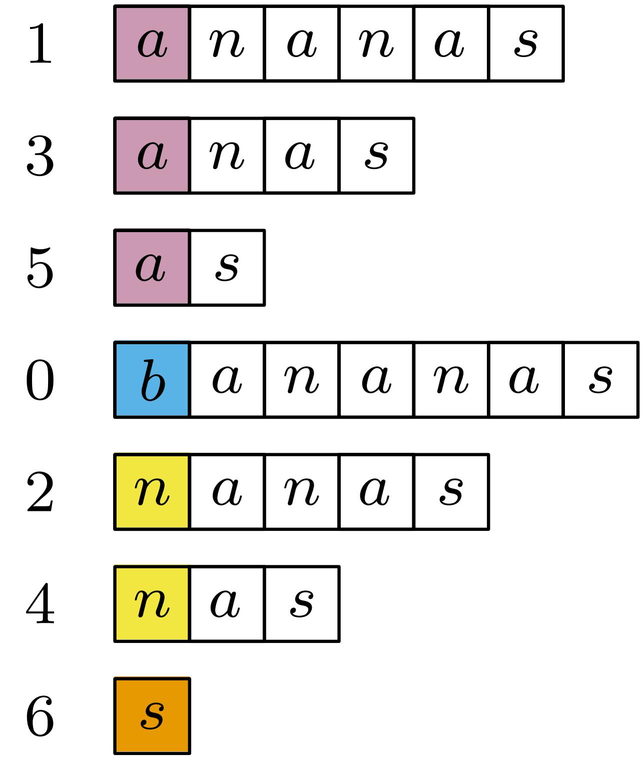
In lexicographical ordering we sort strings based on the first symbol that differs:

$aa < ba < bc < bca$

(in a 'tie', the shorter string is smaller)

The suffix array





Sort the suffixes lexicographically

- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$$aa < ba < bc < bca$$

(in a ‘tie’, the shorter string is smaller)

The suffix array

n

T	b	a	n	a	n	a	s

Sort the suffixes lexicographically

1	a	n	a	n	a	s	
3	a	n	a	s			
5	a	s					
0	b	a	n	a	n	a	s
2	n	a	n	a	s		
4	n	a	s				
6	s						

- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$$aa < ba < bc < bca$$

(in a 'tie', the shorter string is smaller)

The suffix array

T: | b | a | n | a | n | a | s | (length n)

1	$a\ n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
5	$a\ s$
0	$b\ a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
4	$n\ a\ s$
6	s

Sort the suffixes lexicographically

- The symbols themselves must have an order

  throughout we will use alphabetical order

In lexicographical ordering we sort strings based on the first symbol that differs:

$$aa < ba < bc < bca$$

(in a 'tie', the shorter string is smaller)

The suffix array

n

T = | b | a | n | a | n | a | s |

Sort the suffixes lexicographically

1	$a\ n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
5	$a\ s$
0	$b\ a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
4	$n\ a\ s$
6	s

- The symbols themselves must have an order

  throughout we will use alphabetical order

just a fancy name for the order the strings would appear in the dictionary

In lexicographical ordering we sort strings based on the first symbol that differs:

$$aa < ba < bc < bca$$

(in a 'tie', the shorter string is smaller)

The suffix array

n

T | b | a | n | a | n | a | s |

Sort the suffixes lexicographically

1	$a\ n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
5	$a\ s$
0	$b\ a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
4	$n\ a\ s$
6	s

- The symbols themselves must have an order

  throughout we will use alphabetical order

just a fancy name for the order the strings would appear in the dictionary

In lexicographical ordering we sort strings based on the first symbol that differs:

$$aa < ba < bc < bca$$

(in a 'tie', the shorter string is smaller)

If the symbols don't have a natural order, we use their binary representation in memory

The suffix array

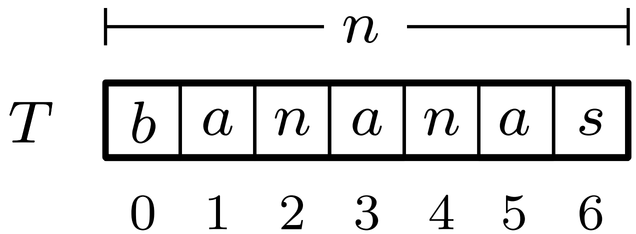
n

T	b	a	n	a	n	a	s

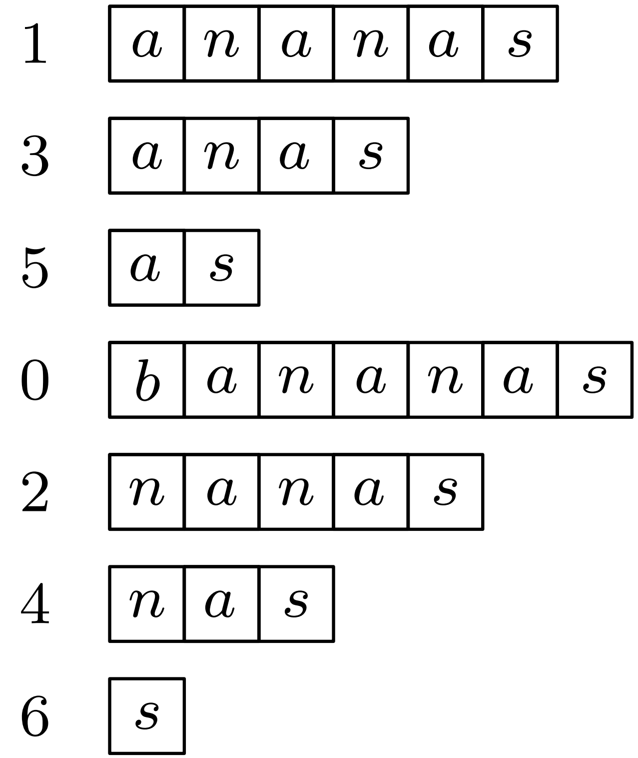
Sort the suffixes lexicographically

1	$a\ n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
5	$a\ s$
0	$b\ a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
4	$n\ a\ s$
6	s

The suffix array



Sort the suffixes lexicographically



The suffix array

T (length n):

b	a	n	a	n	a	s
0	1	2	3	4	5	6

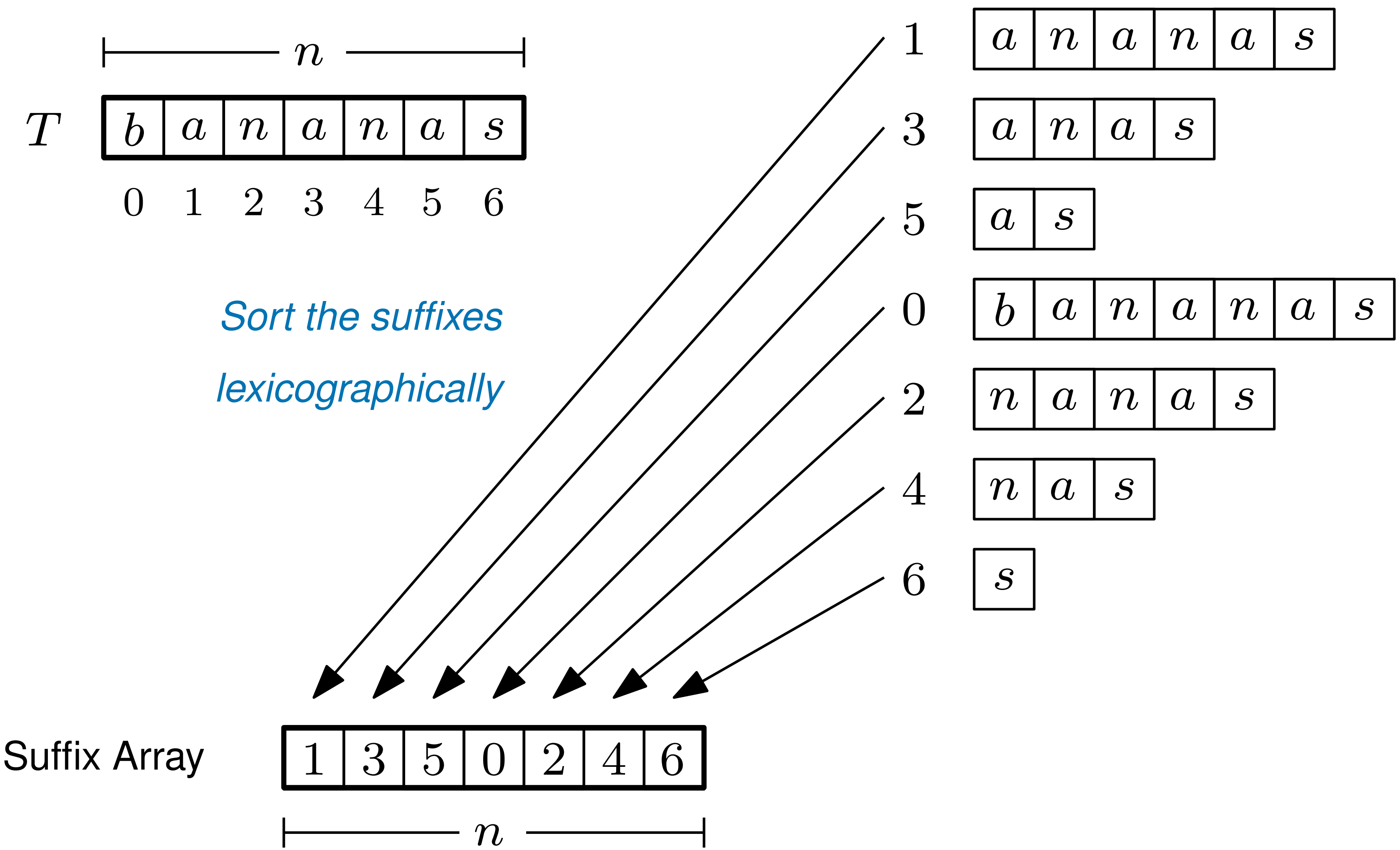
Sort the suffixes lexicographically

1	a	n	a	n	a	s	
3	a	n	a	s			
5	a	s					
0	b	a	n	a	n	a	s
2	n	a	n	a	s		
4	n	a	s				
6	s						

Suffix Array (length n):

1	3	5	0	2	4	6

The suffix array



The suffix array

T (length n):

0	1	2	3	4	5	6
b	a	n	a	n	a	s

Sort the suffixes lexicographically

1	$a\ n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
5	$a\ s$
0	$b\ a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
4	$n\ a\ s$
6	s

Suffix Array (length n):

1	3	5	0	2	4	6

The suffix array

T (length n):

	0	1	2	3	4	5	6
T	b	a	n	a	n	a	s

Sort the suffixes lexicographically

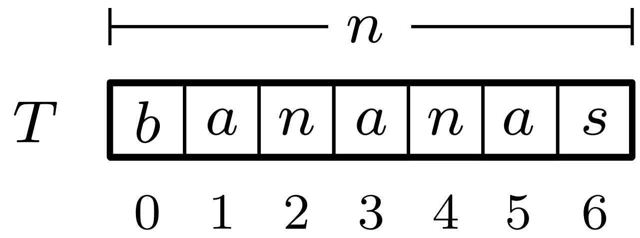
1	$a\ n\ a\ n\ a\ s$
3	$a\ n\ a\ s$
5	$a\ s$
0	$b\ a\ n\ a\ n\ a\ s$
2	$n\ a\ n\ a\ s$
4	$n\ a\ s$
6	s

Suffix Array (length n):

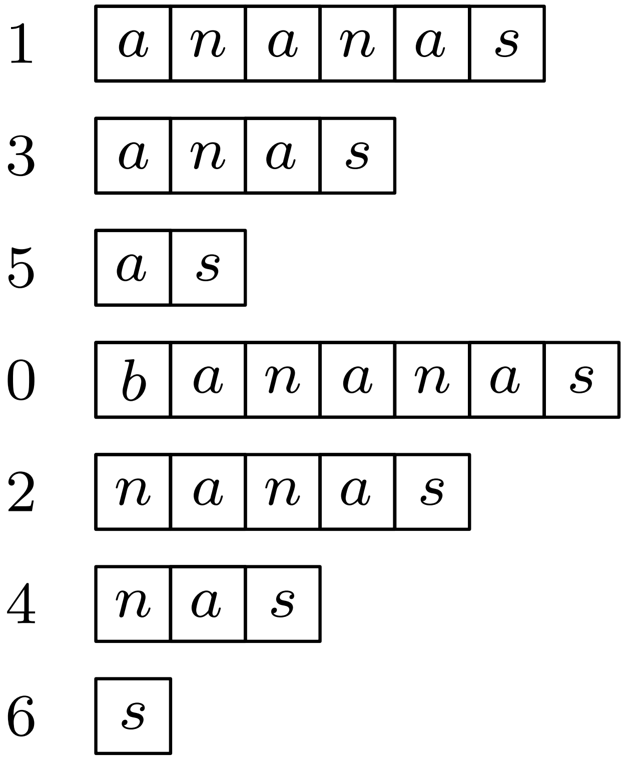
Suffix Array	1	3	5	0	2	4	6

The suffix array is much smaller than the suffix tree (in terms of constants)

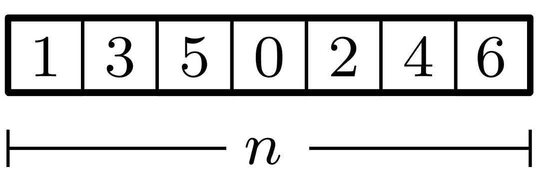
The suffix array



Sort the suffixes lexicographically

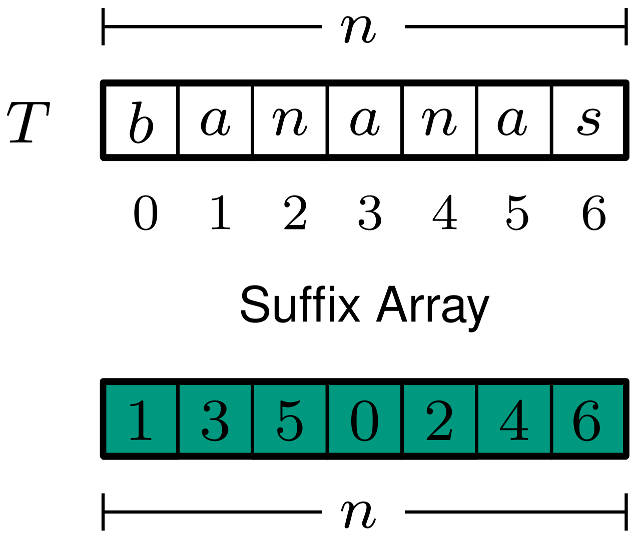


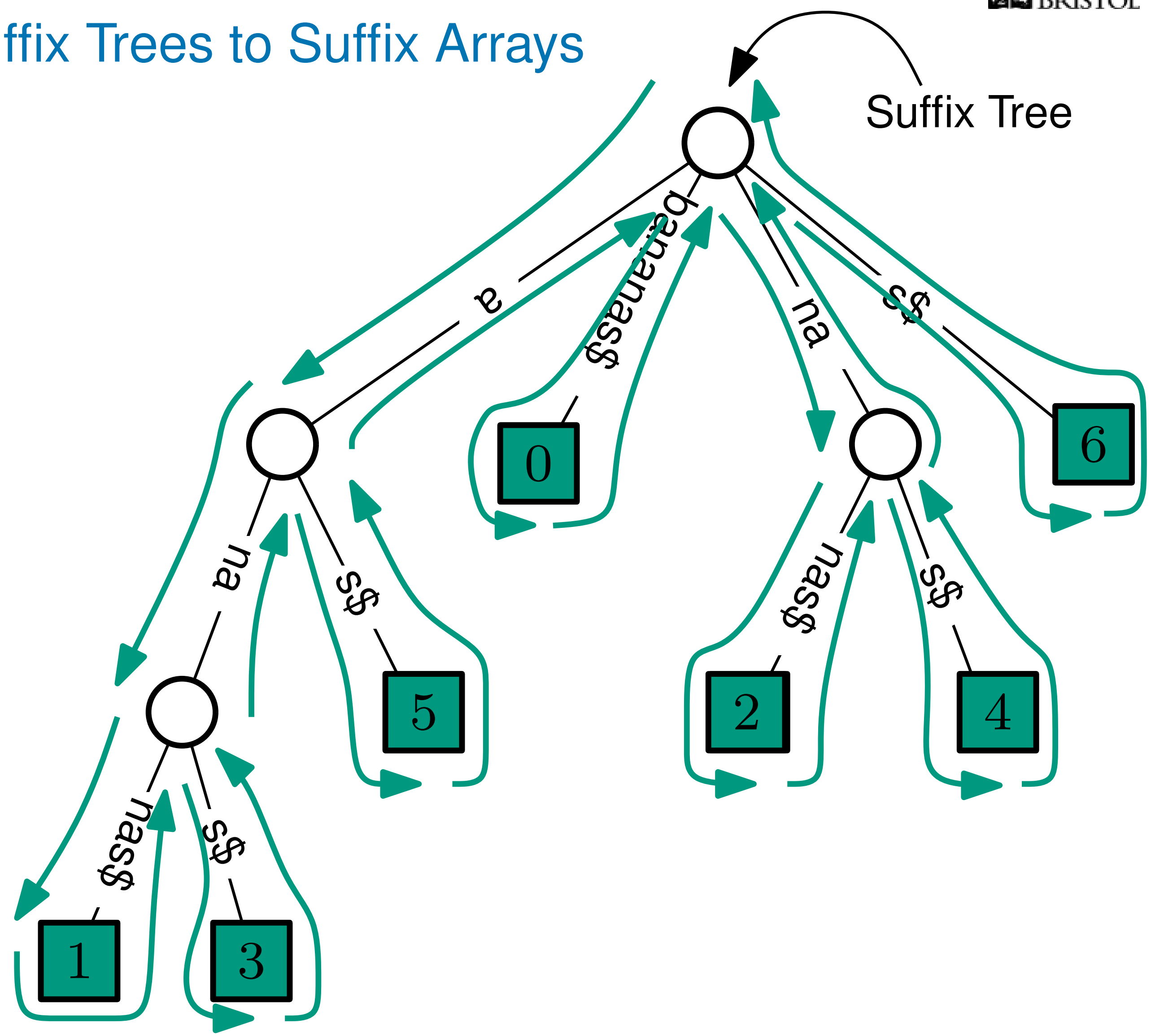
Suffix Array



The suffix array is much smaller than the suffix tree (in terms of constants)

From Suffix Trees to Suffix Arrays





Recall that we can get get the Suffix Array from the Suffix Tree

using depth-first search in $O(n)$ *time*

Searching in the Suffix Array

15	a
1	$a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
7	$a\ c\ c\ b\ b\ d\ c\ d\ a$
0	$b\ a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
3	$b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
10	$b\ b\ d\ c\ d\ a$
4	$b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
11	$b\ d\ c\ d\ a$
6	$c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
2	$c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
9	$c\ b\ b\ d\ c\ d\ a$
8	$c\ c\ b\ b\ d\ c\ d\ a$
13	$c\ d\ a$
14	$d\ a$
5	$d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
12	$d\ c\ d\ a$

n

T	b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Suffix Array	15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

15	a
1	a c b b d c a c c b b d c d a
7	a c c b b d c d a
0	b a c b b d c a c c b b d c d a
3	b b d c a c c b b d c d a
10	b b d c d a
4	b d c a c c b b d c d a
11	b d c d a
6	c a c c b b d c d a
2	c b b d c a c c b b d c d a
9	c b b d c d a
8	c c b b d c d a
13	c d a
14	d a
5	d c a c c b b d c d a
12	d c d a

find P = c b b d c (length m)

T (length n):

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

15	a
1	$a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
7	$a\ c\ c\ b\ b\ d\ c\ d\ a$
0	$b\ a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
3	$b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
10	$b\ b\ d\ c\ d\ a$
4	$b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
11	$b\ d\ c\ d\ a$
6	$c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
2	$c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
9	$c\ b\ b\ d\ c\ d\ a$
8	$c\ c\ b\ b\ d\ c\ d\ a$
13	$c\ d\ a$
14	$d\ a$
5	$d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
12	$d\ c\ d\ a$

find P = $c\ b\ b\ d\ c$ (length m)

Key Idea:

Find an occurence of P

using binary search

T (length n):

b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

occurences could start anywhere

15	a
1	a c b b d c a c c b b d c d a
7	a c c b b d c d a
0	b a c b b d c a c c b b d c d a
3	b b d c a c c b b d c d a
10	b b d c d a
4	b d c a c c b b d c d a
11	b d c d a
6	c a c c b b d c d a
2	c b b d c a c c b b d c d a
9	c b b d c d a
8	c c b b d c d a
13	c d a
14	d a
5	d c a c c b b d c d a
12	d c d a

find P = c b b d c (length m)

Key Idea:

Find an occurence of P

using binary search

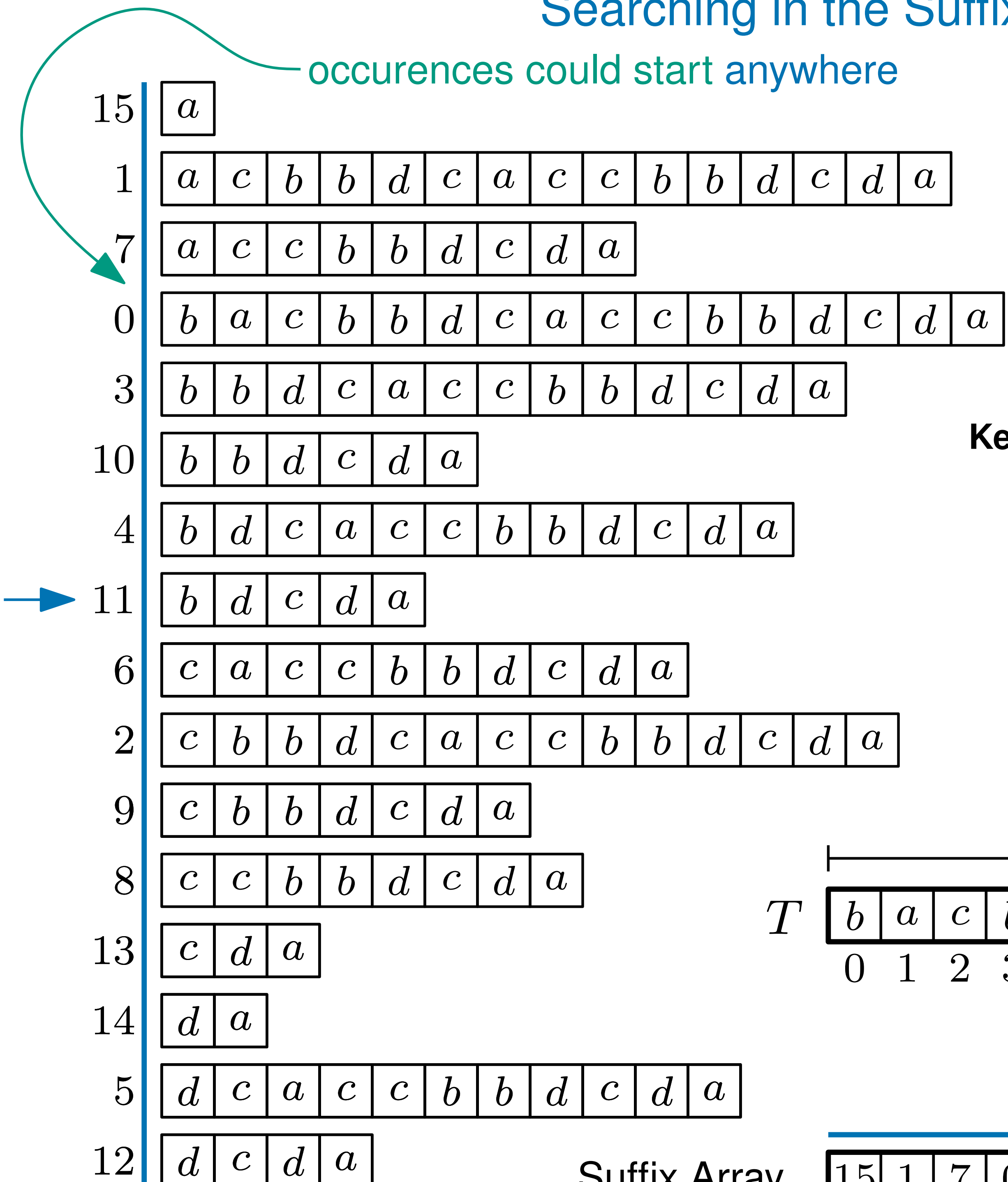
T (length n):

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array



find P (length m):

c	b	b	d	c

Key Idea:

Find an occurence of P

using binary search

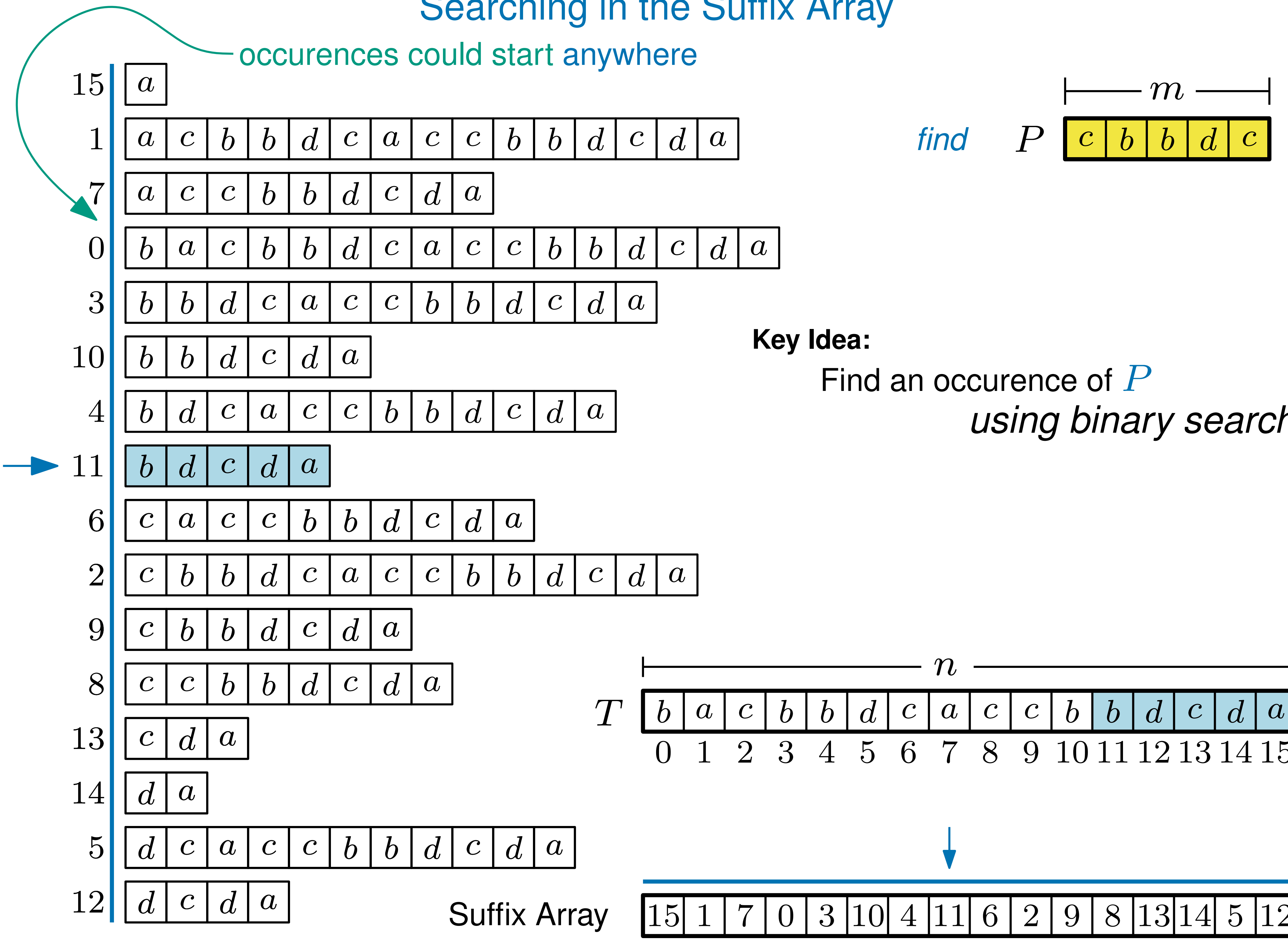
T (length n):

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

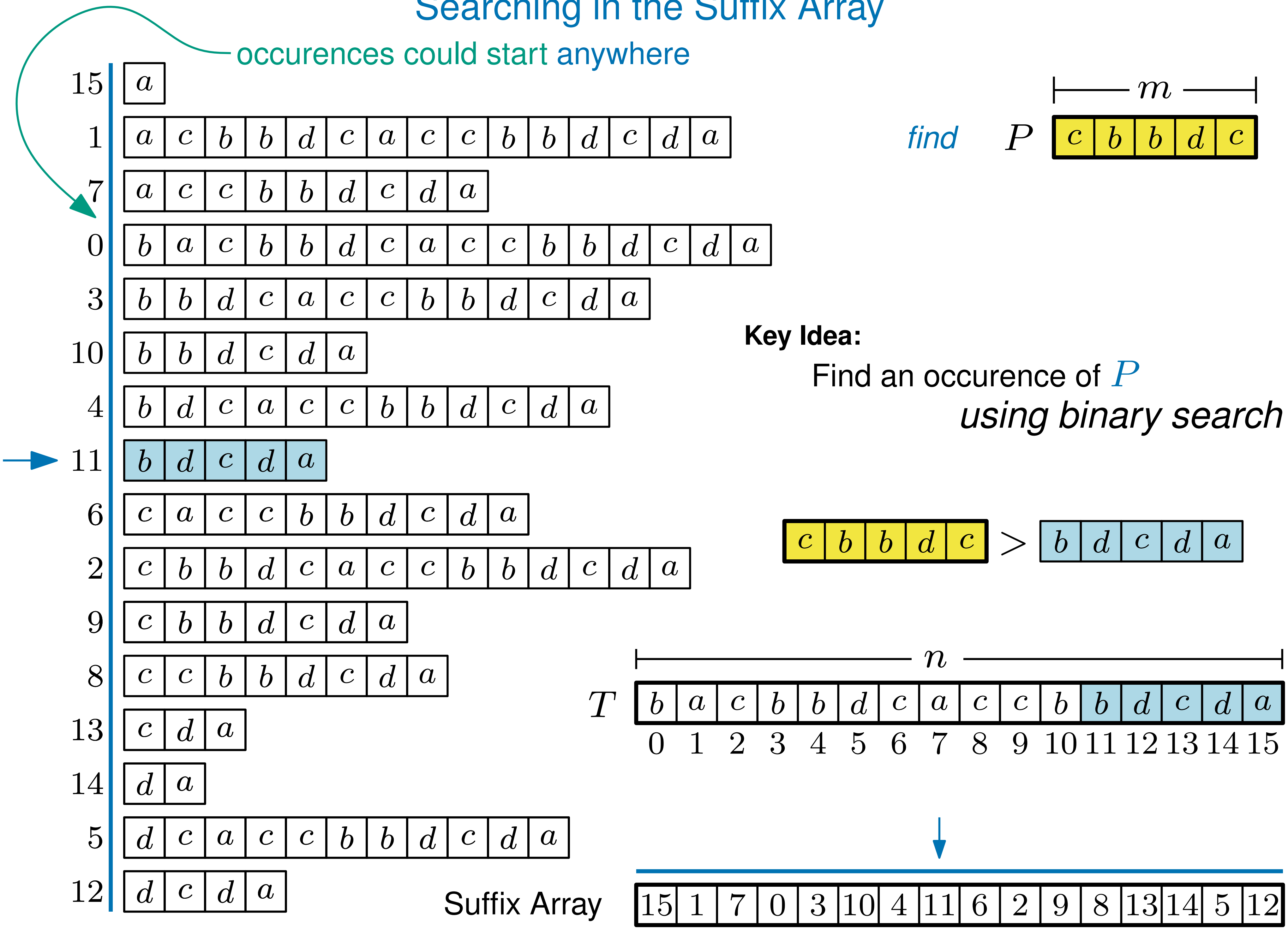


Key Idea:

Find an occurence of P

using binary search

Searching in the Suffix Array

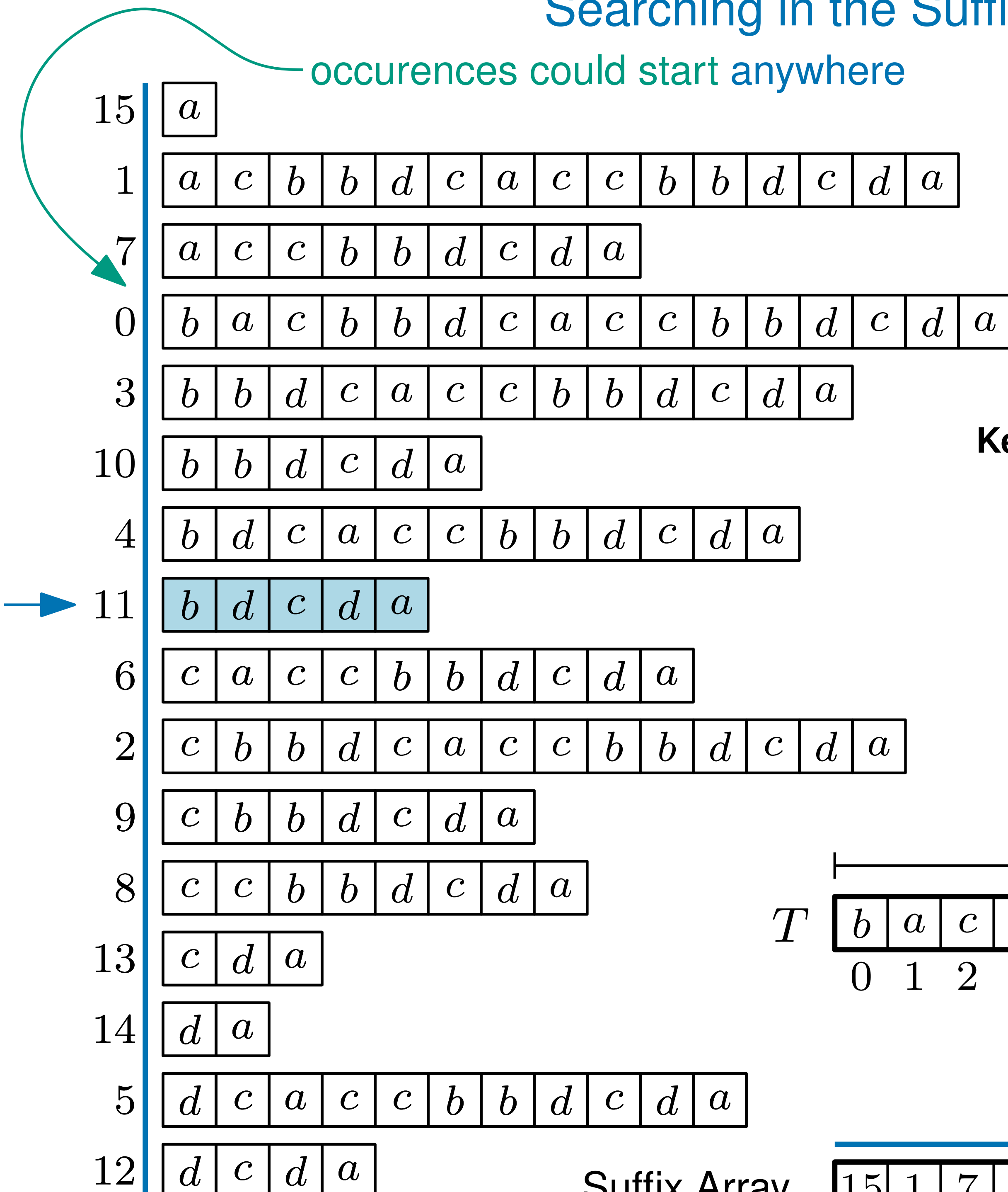


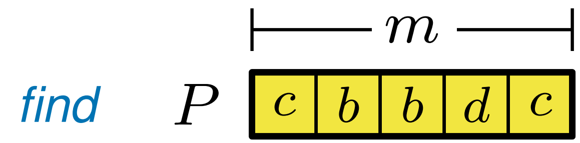
Key Idea:

Find an occurence of P

using binary search

Searching in the Suffix Array

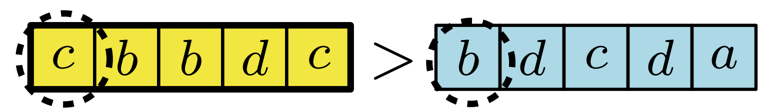


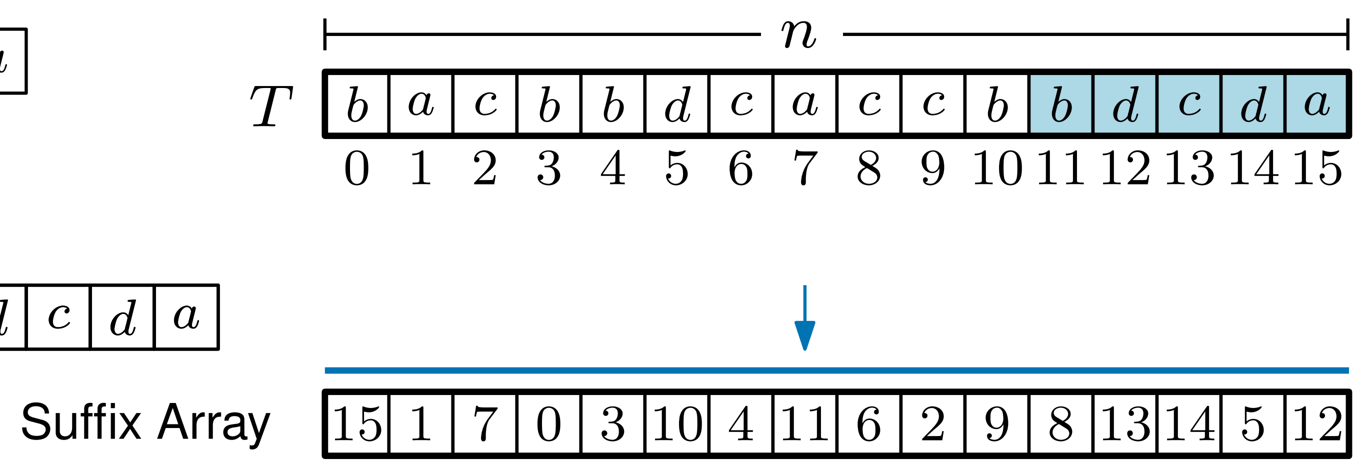


Key Idea:

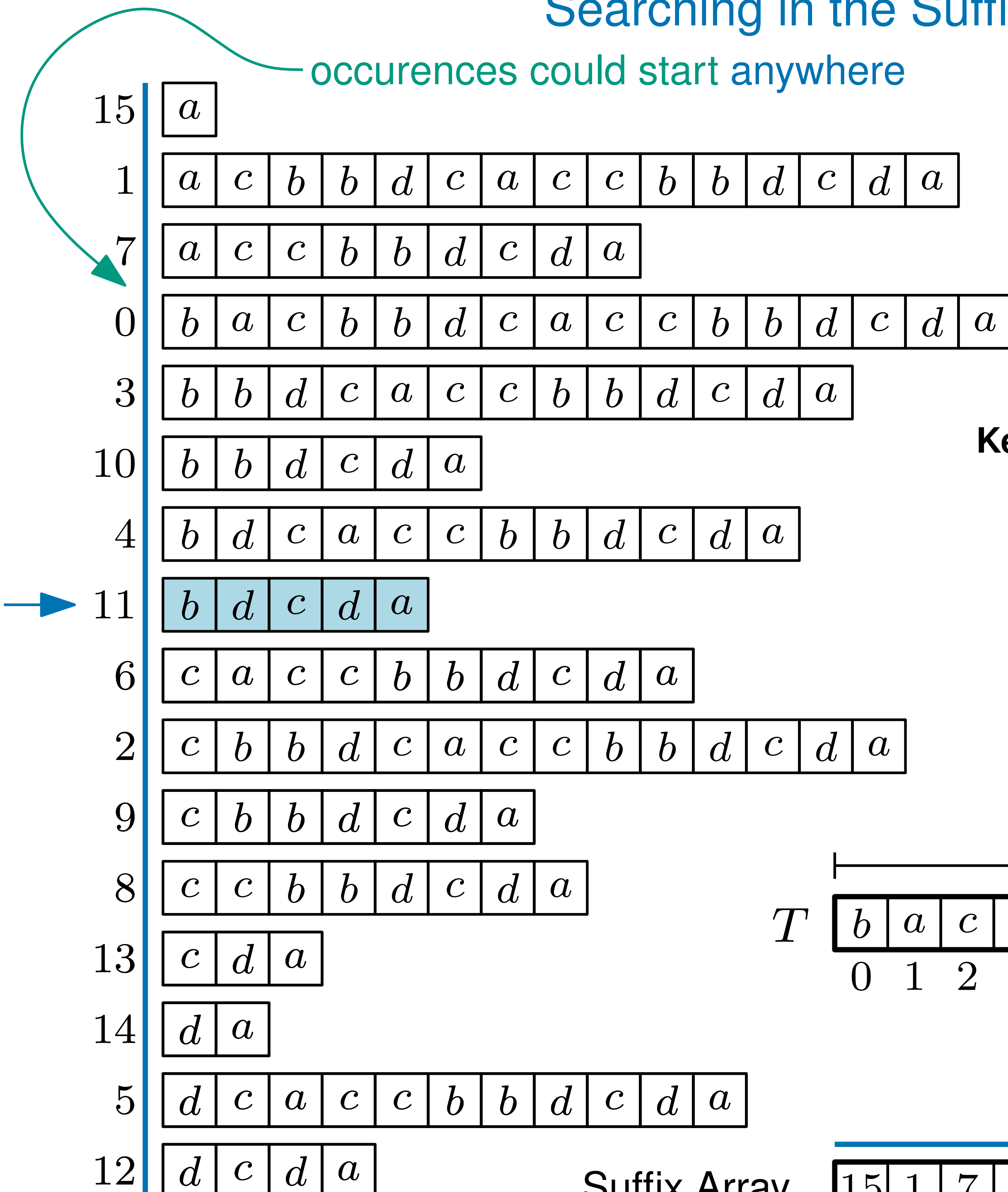
Find an occurence of P

using binary search





Searching in the Suffix Array



find P = c b b d c (length m)

Key Idea:

Find an occurence of P

using binary search

c b b d c $>$ b d c d a

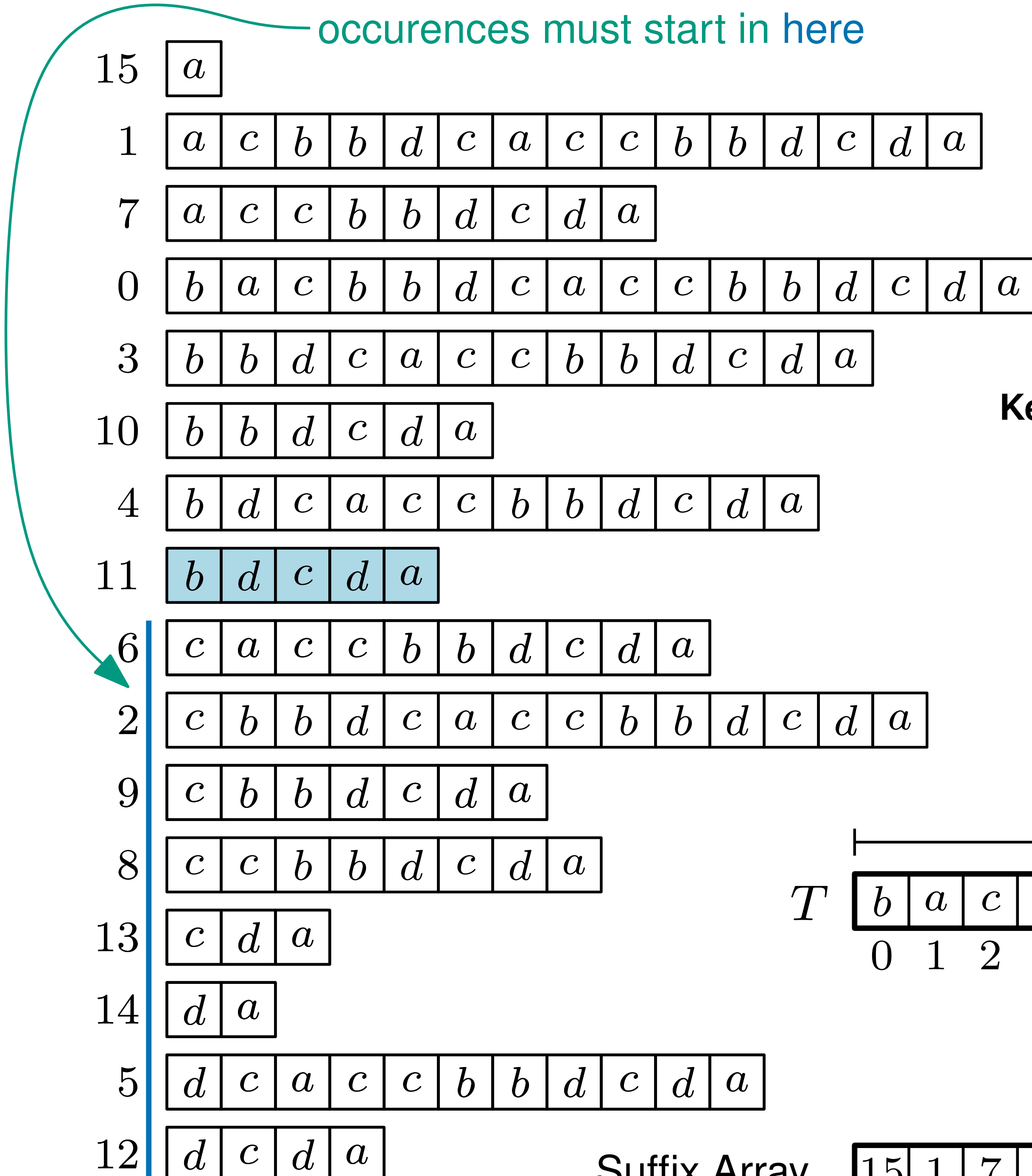
T (length n):

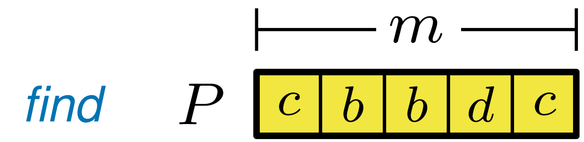
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

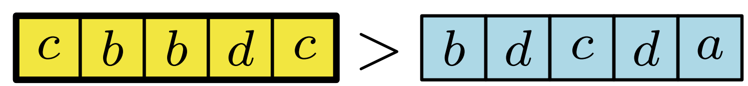


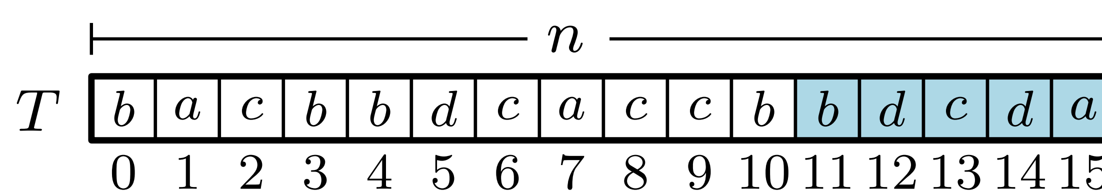


Key Idea:

Find an occurence of P

using binary search

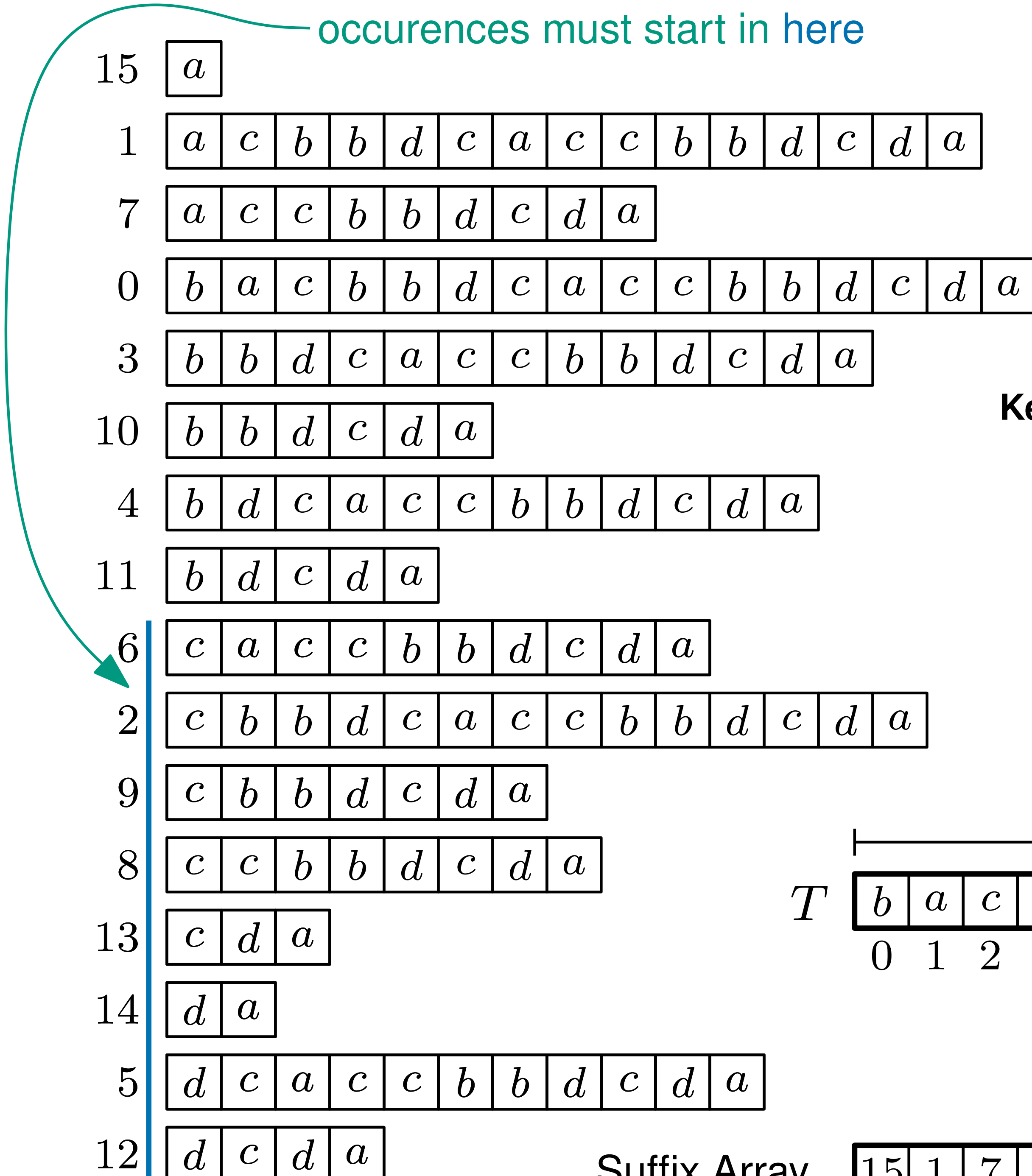




Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array



find P = c b b d c (length m)

Key Idea:

Find an occurence of P

using binary search

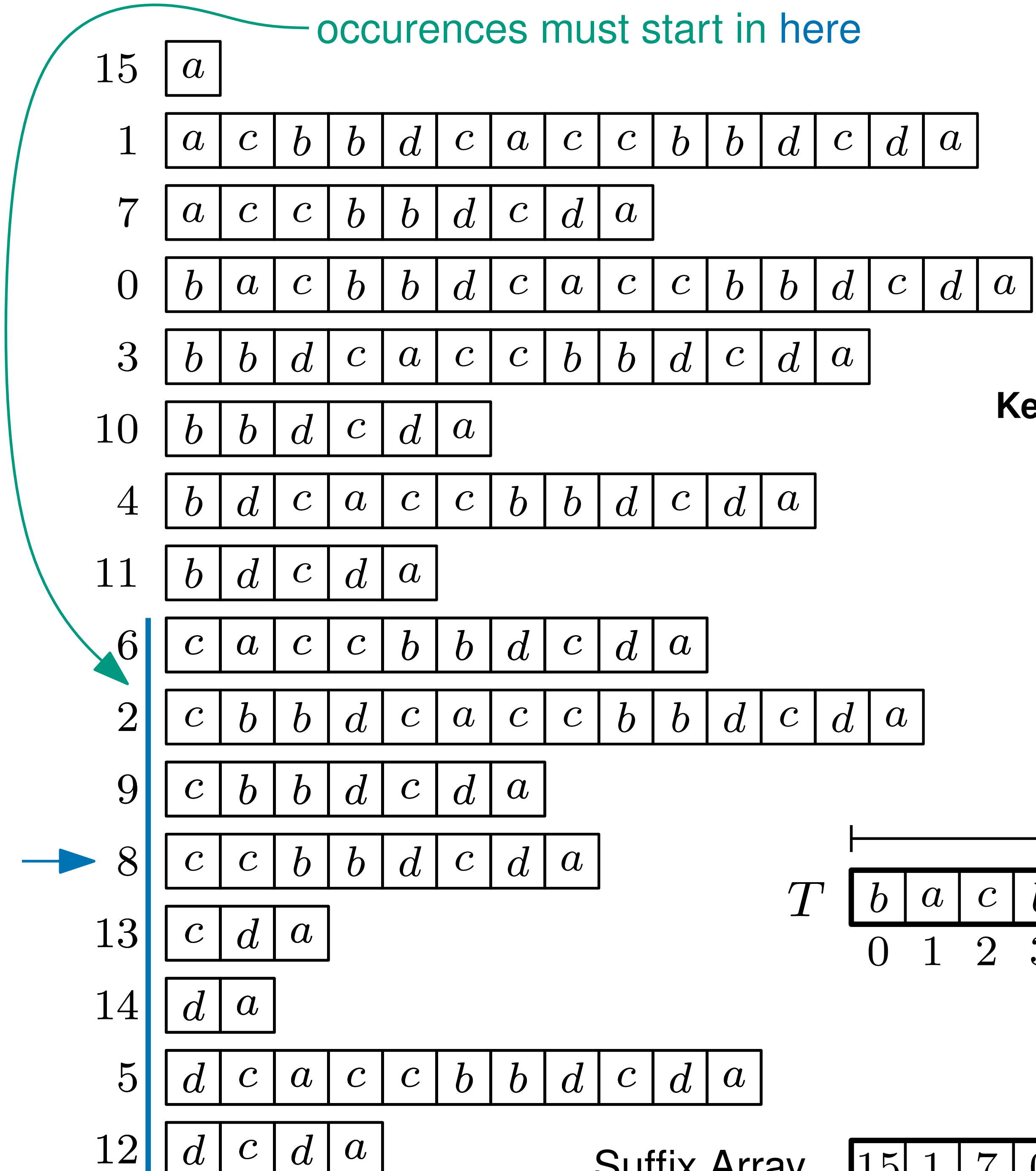
T (length n):

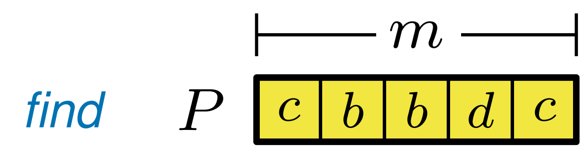
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

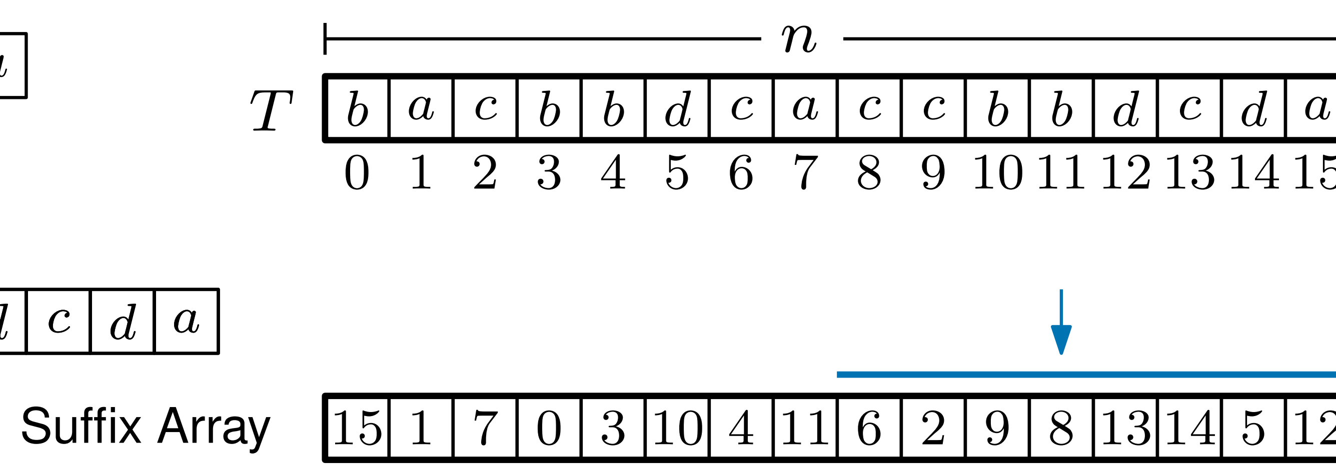




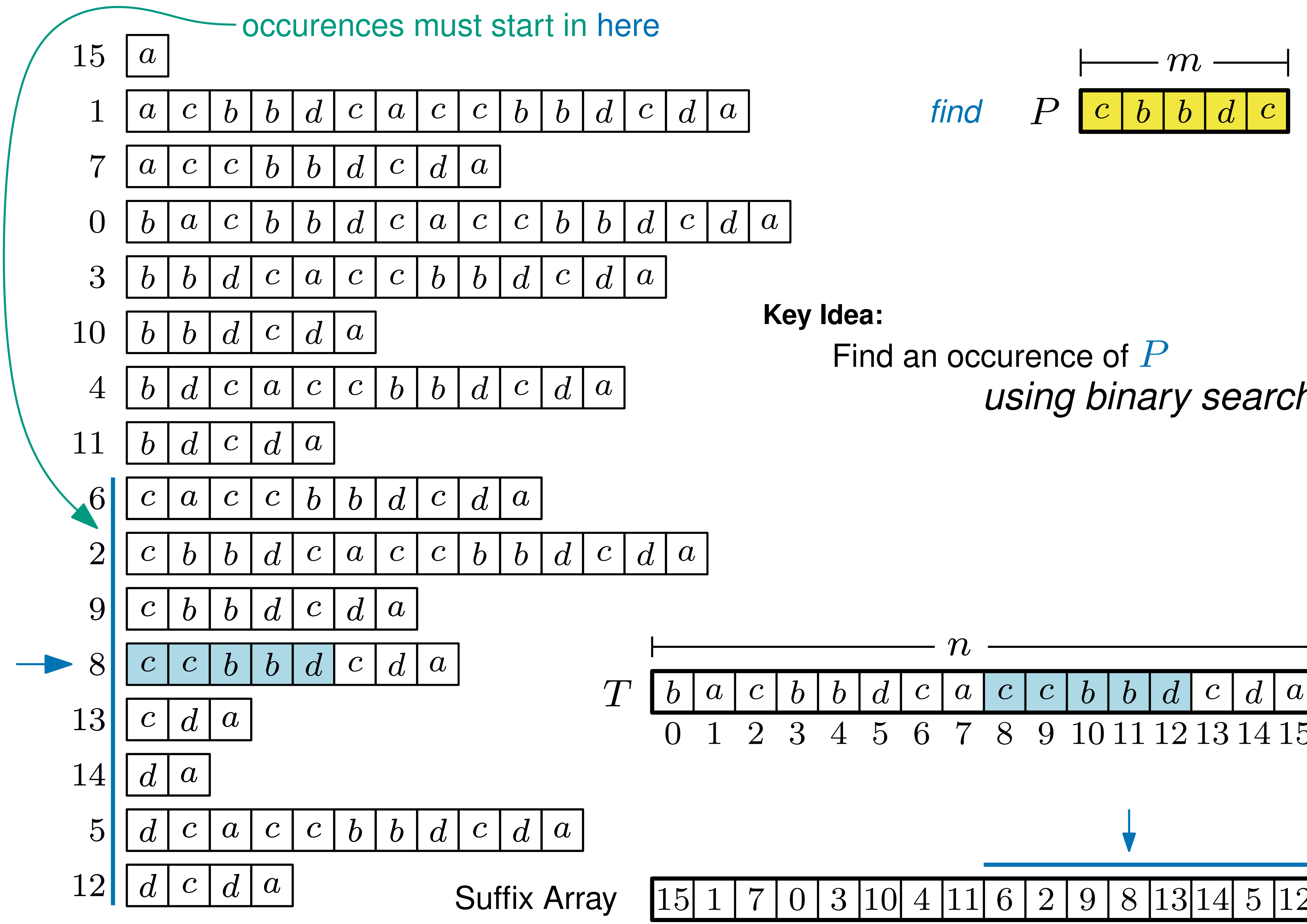
Key Idea:

Find an occurence of P

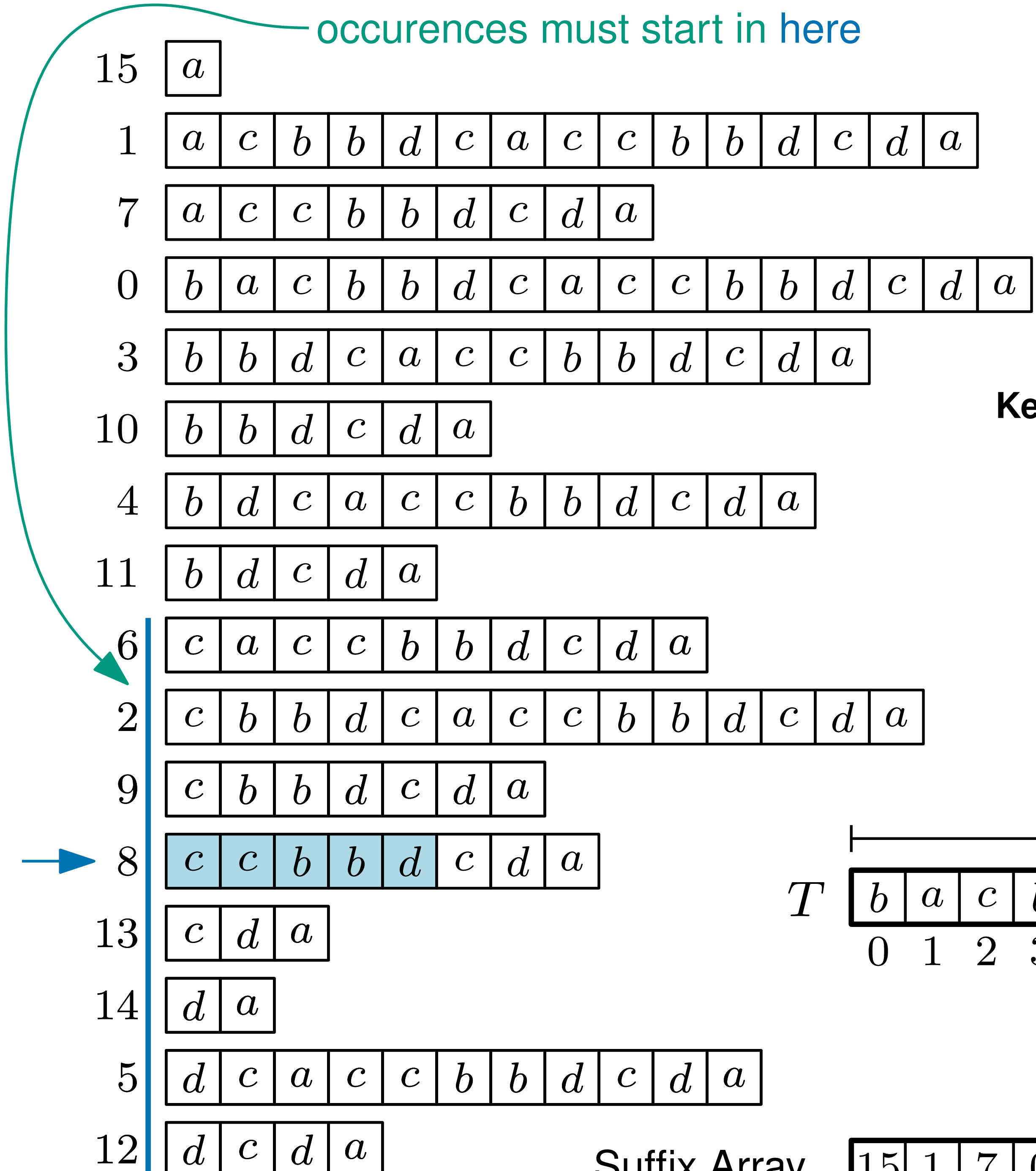
using binary search

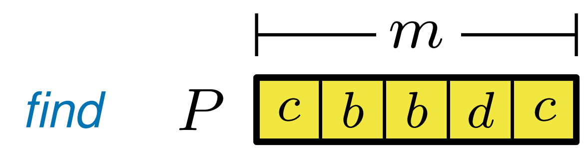


Searching in the Suffix Array



Searching in the Suffix Array

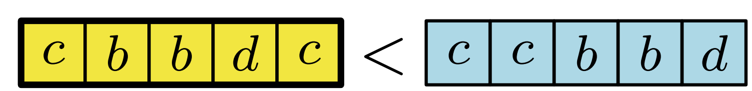


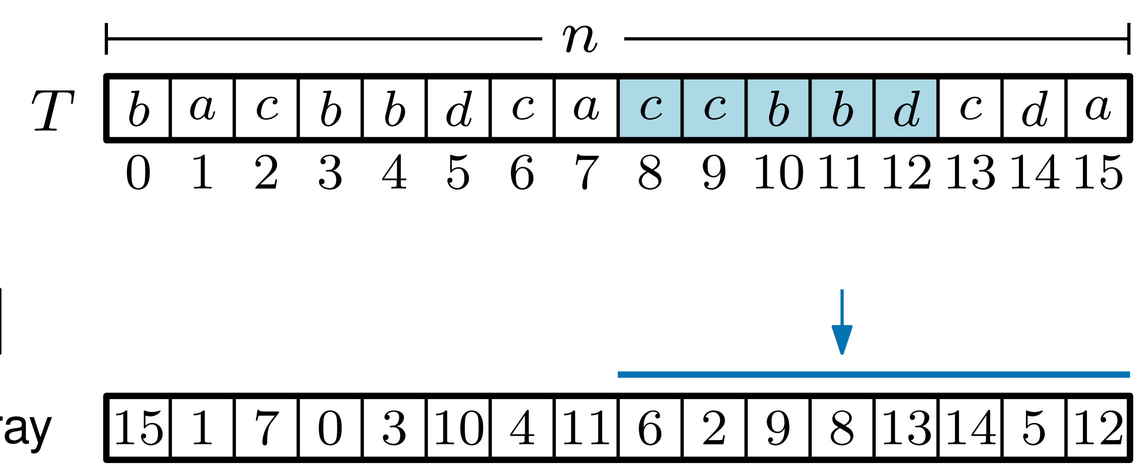


Key Idea:

Find an occurence of P

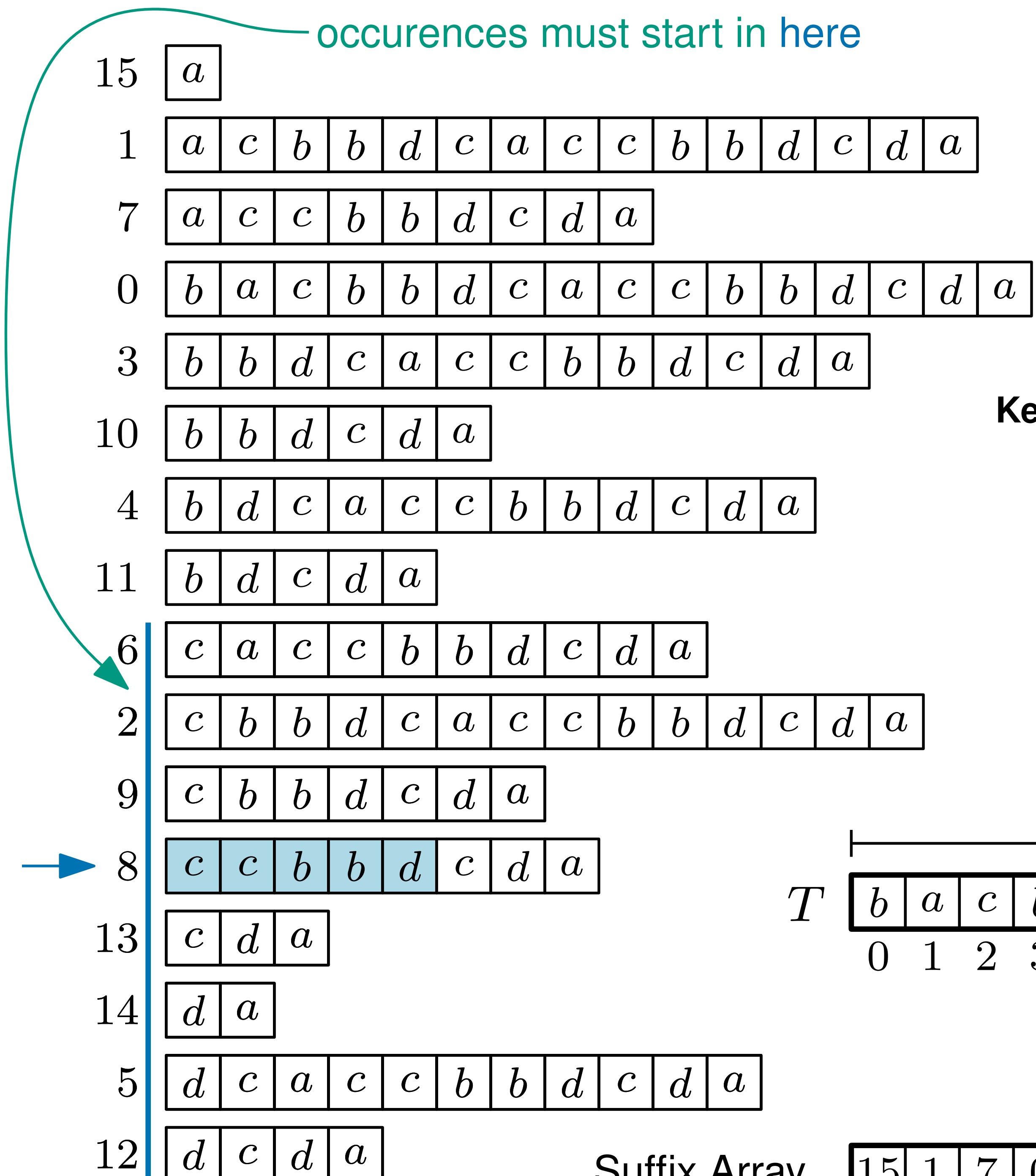
using binary search

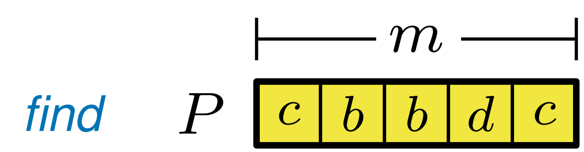




Suffix Array

Searching in the Suffix Array

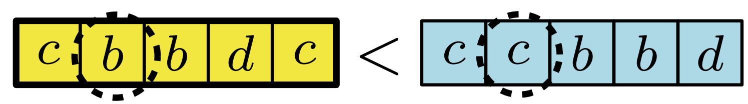


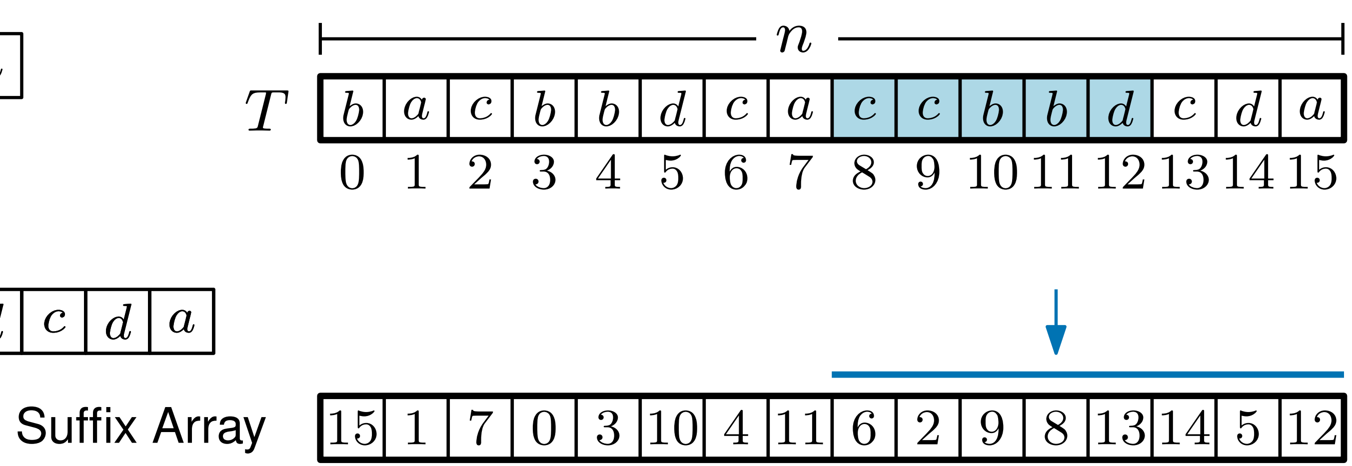


Key Idea:

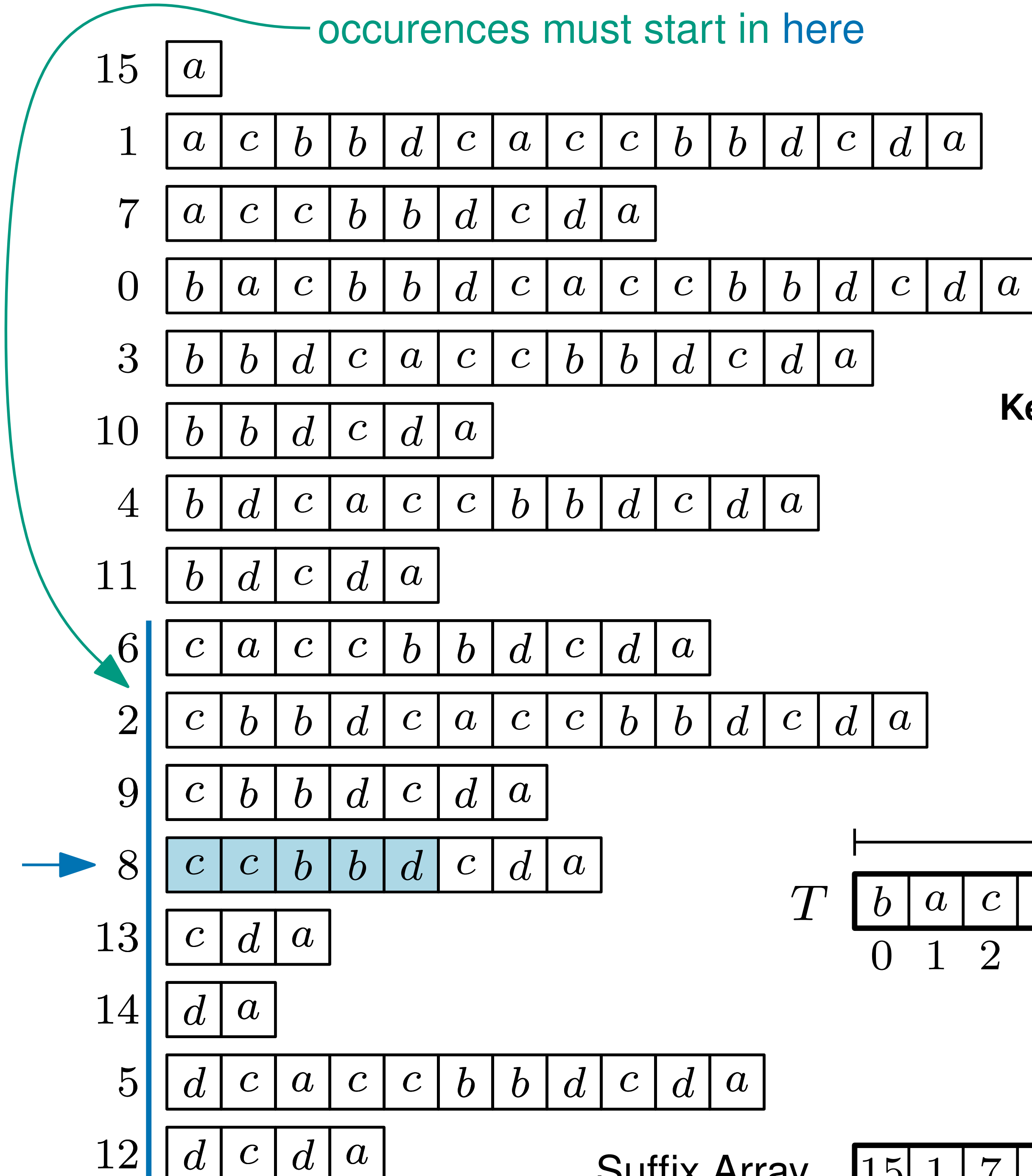
Find an occurence of P

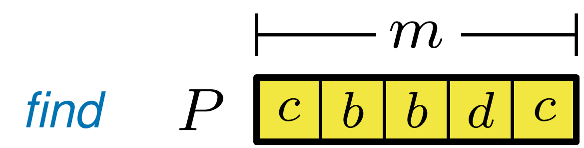
using binary search





Searching in the Suffix Array

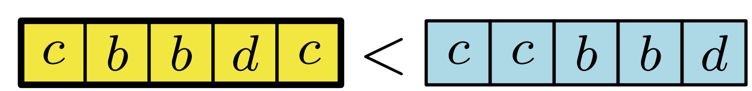


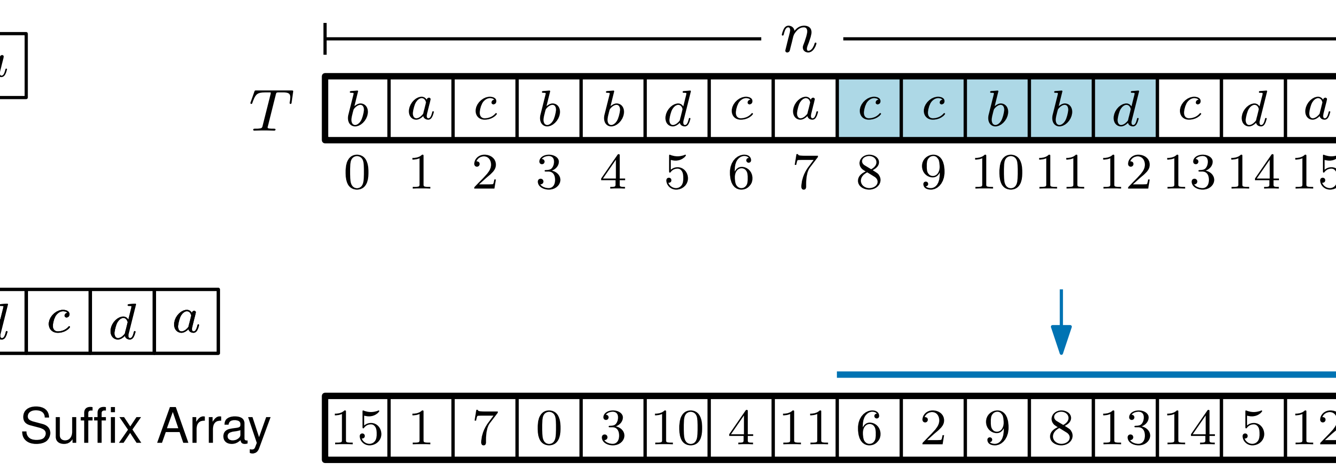


Key Idea:

Find an occurence of P

using binary search





Searching in the Suffix Array

occurences must start in **here**

15	a
1	a c b b d c a c c b b d c d a
7	a c c b b d c d a
0	b a c b b d c a c c b b d c d a
3	b b d c a c c b b d c d a
10	b b d c d a
4	b d c a c c b b d c d a
11	b d c d a
6	c a c c b b d c d a
2	c b b d c a c c b b d c d a
9	c b b d c d a
8	c c b b d c d a
13	c d a
14	d a
5	d c a c c b b d c d a
12	d c d a

find P = c b b d c (length m)

Key Idea:

Find an occurence of P *using binary search*

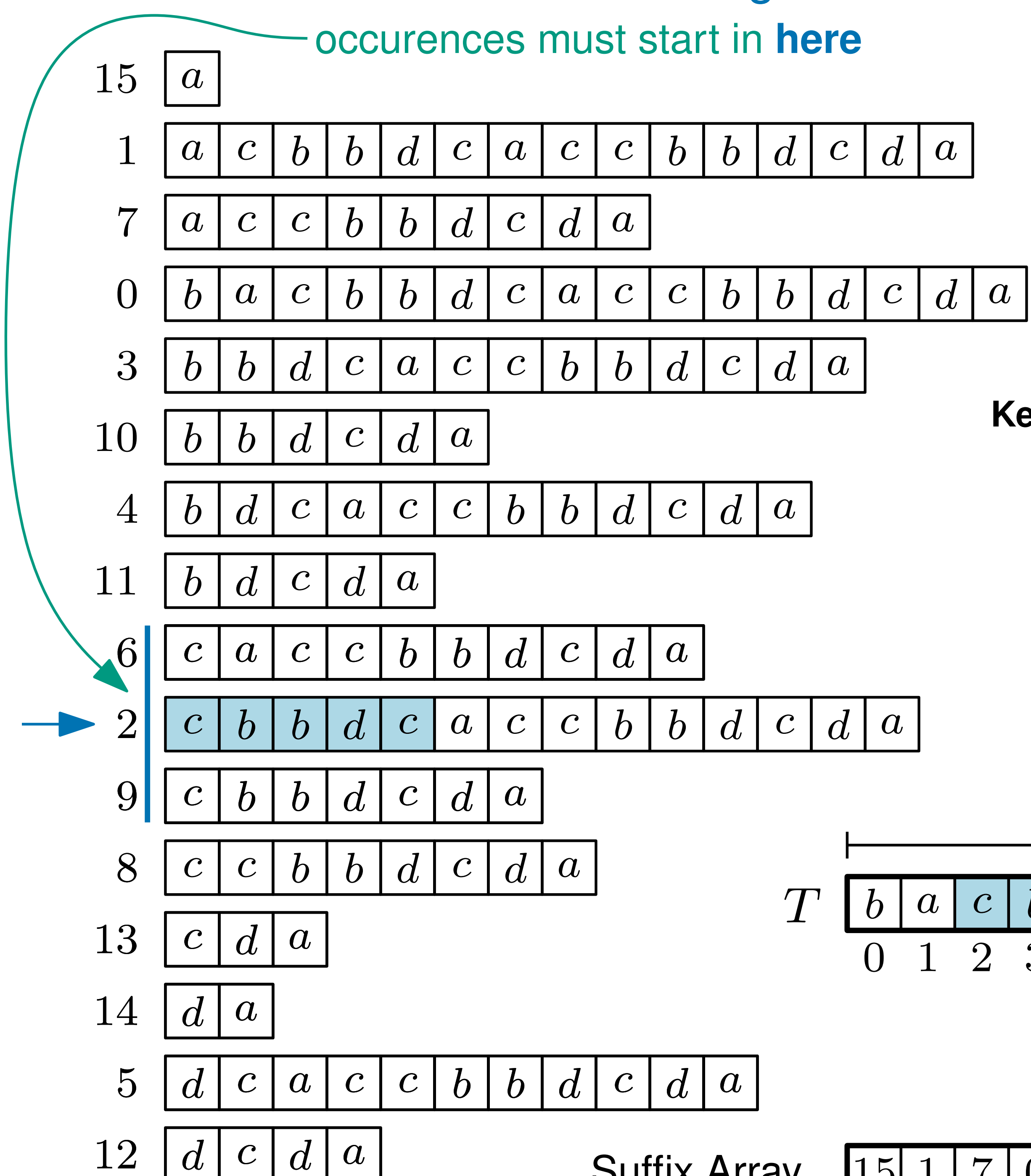
T (length n):

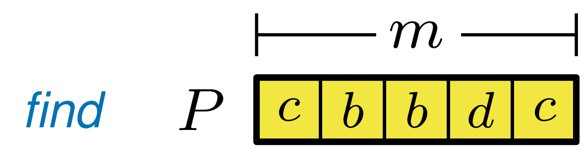
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

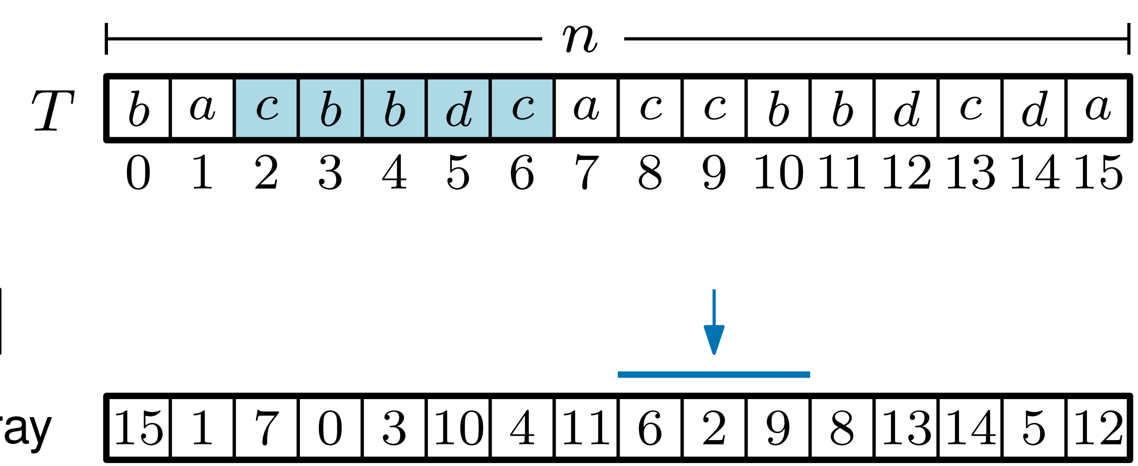




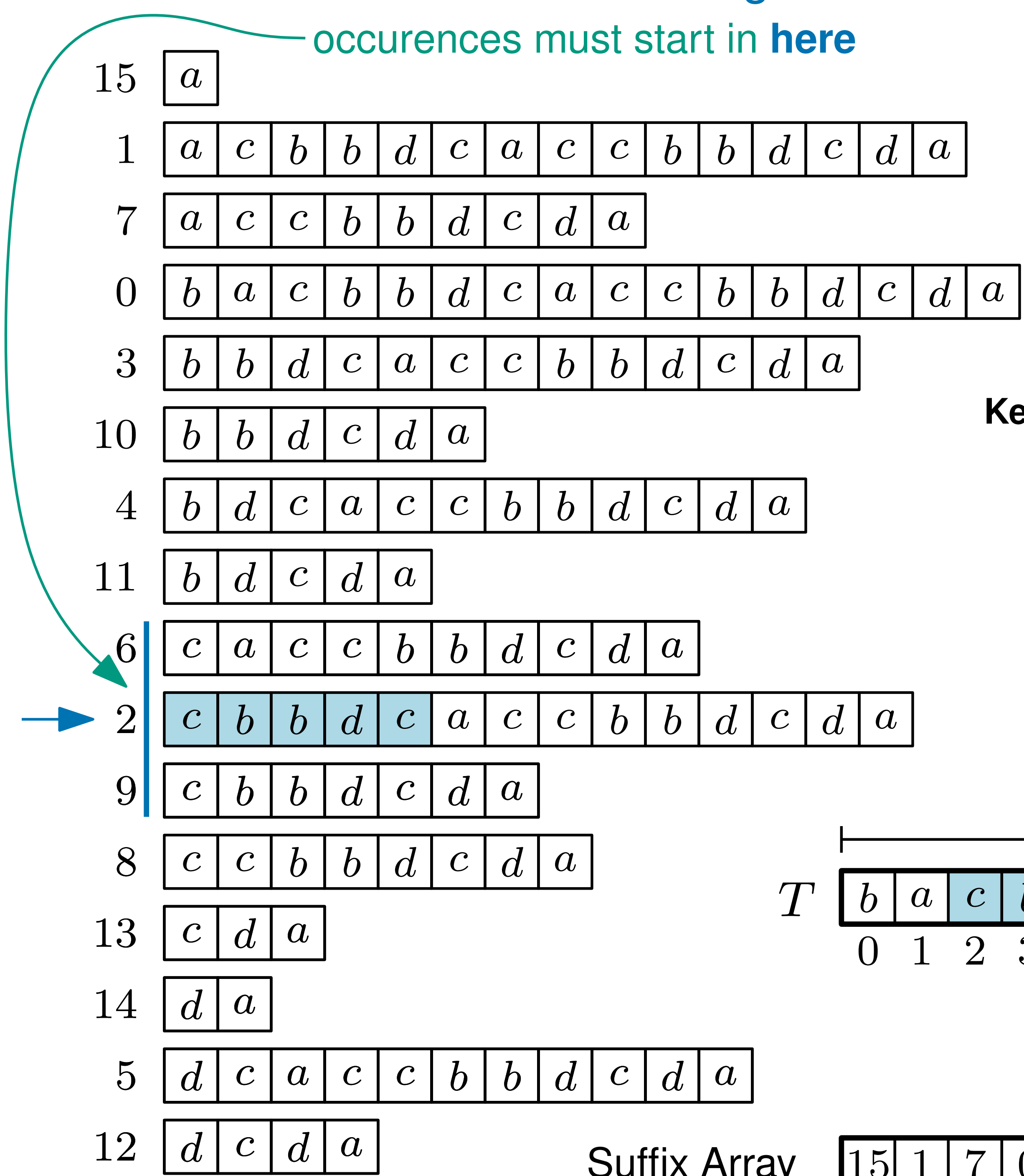
Key Idea:

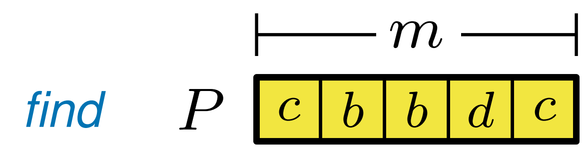
Find an occurence of P

using binary search



Searching in the Suffix Array

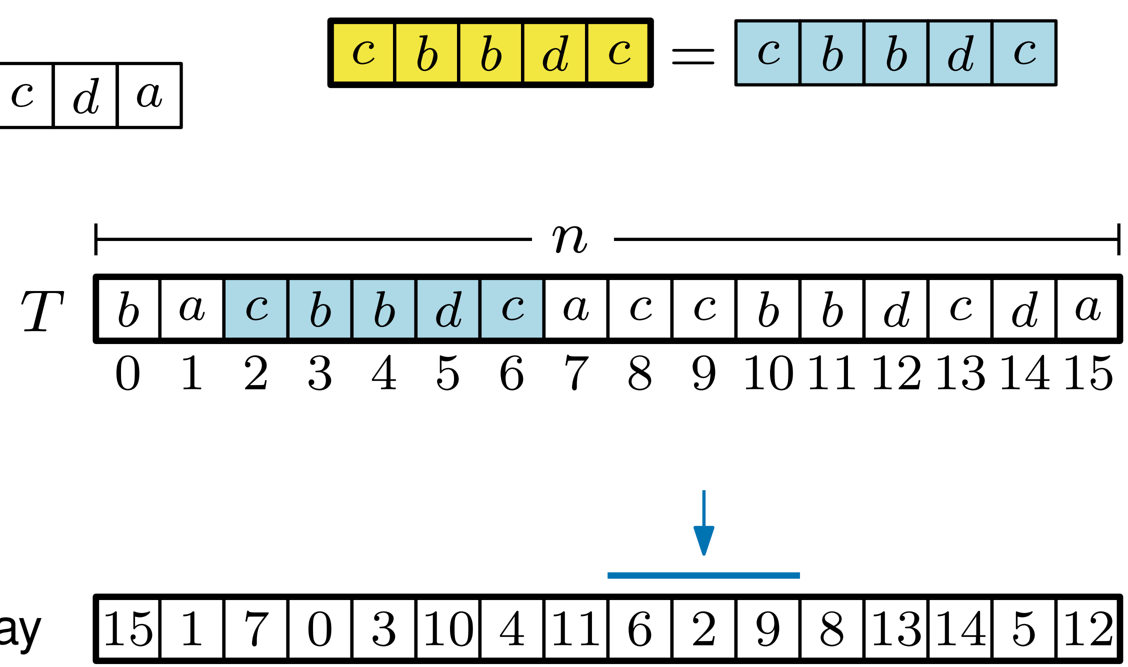




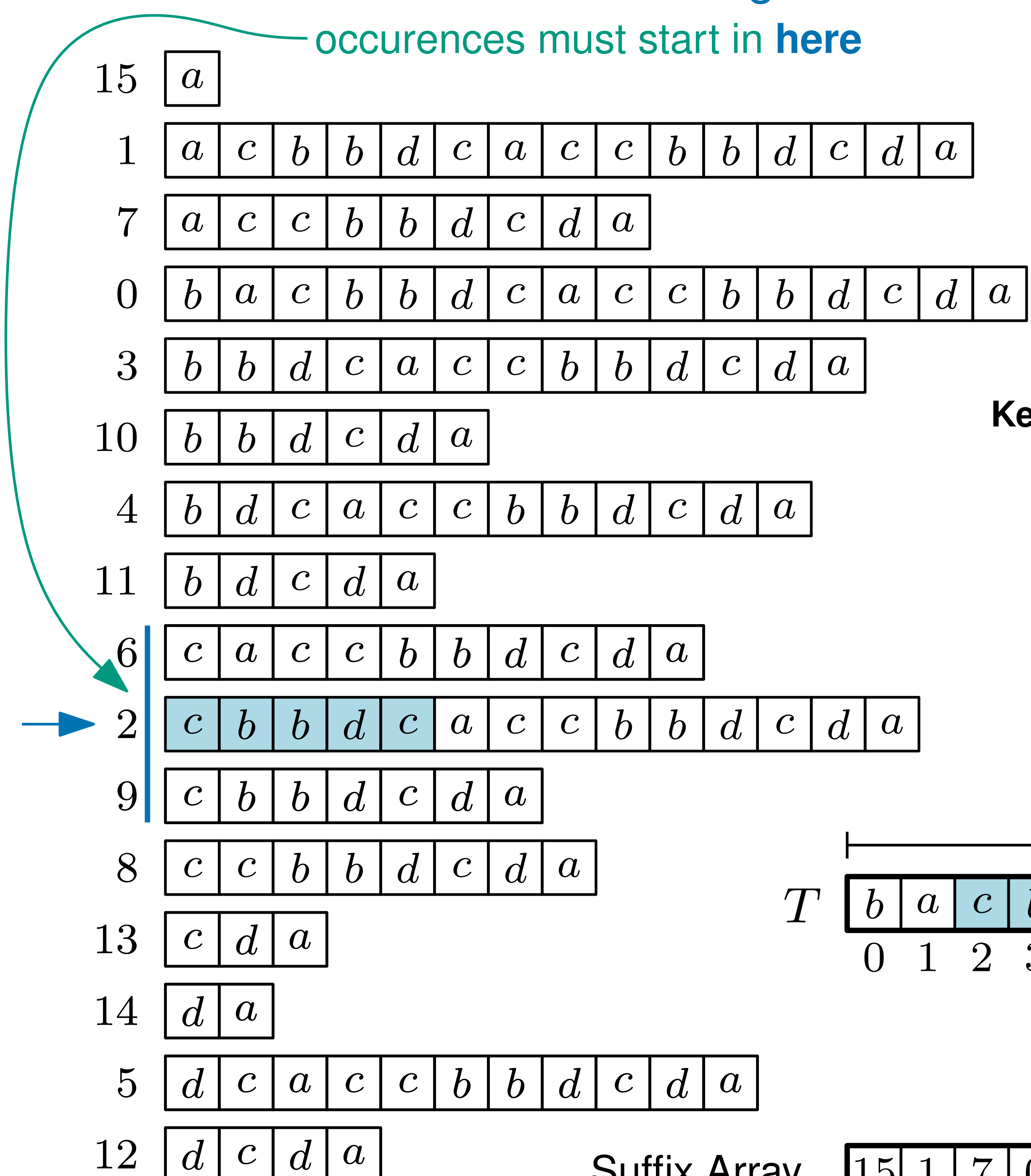
Key Idea:

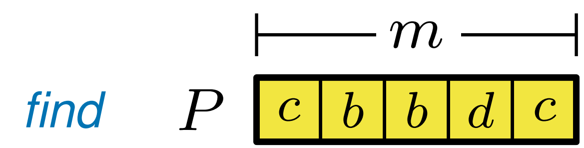
Find an occurence of P

using binary search



Searching in the Suffix Array

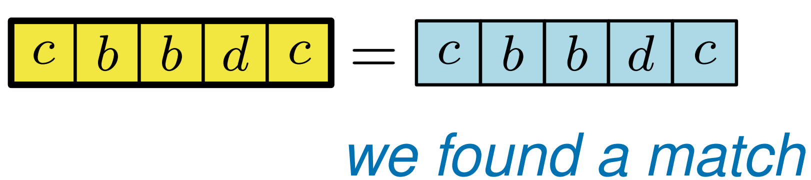


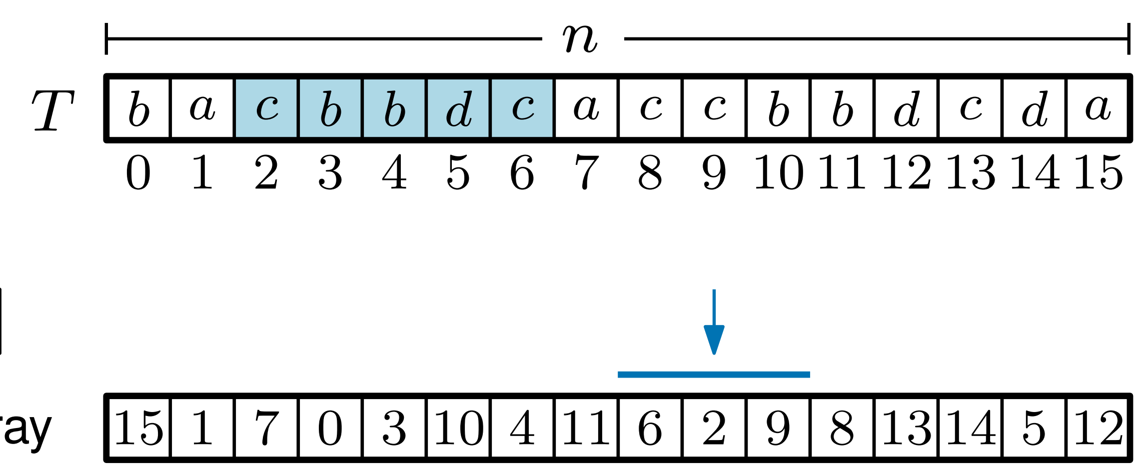


Key Idea:

Find an occurence of P

using binary search





Searching in the Suffix Array

15	a
1	$a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
7	$a\ c\ c\ b\ b\ d\ c\ d\ a$
0	$b\ a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
3	$b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
10	$b\ b\ d\ c\ d\ a$
4	$b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
11	$b\ d\ c\ d\ a$
6	$c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
2	$c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
9	$c\ b\ b\ d\ c\ d\ a$
8	$c\ c\ b\ b\ d\ c\ d\ a$
13	$c\ d\ a$
14	$d\ a$
5	$d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
12	$d\ c\ d\ a$

find P (length m):

c	b	b	d	c

Key Idea:

Find an occurence of P

using binary search

T (length n):

b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

15	a
1	a c b b d c a c c b b d c d a
7	a c c b b d c d a
0	b a c b b d c a c c b b d c d a
3	b b d c a c c b b d c d a
10	b b d c d a
4	b d c a c c b b d c d a
11	b d c d a
6	c a c c b b d c d a
2	c b b d c a c c b b d c d a
9	c b b d c d a
8	c c b b d c d a
13	c d a
14	d a
5	d c a c c b b d c d a
12	d c d a

find P = c b b d c (length m)

Key Idea:

Find an occurence of P

using binary search

How long does this take?

T (length n):

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

15	a
1	$a\,c\,b\,b\,d\,c\,a\,c\,c\,b\,b\,d\,c\,d\,a$
7	$a\,c\,c\,b\,b\,d\,c\,d\,a$
0	$b\,a\,c\,b\,b\,d\,c\,a\,c\,c\,b\,b\,d\,c\,d\,a$
3	$b\,b\,d\,c\,a\,c\,c\,b\,b\,d\,c\,d\,a$
10	$b\,b\,d\,c\,d\,a$
4	$b\,d\,c\,a\,c\,c\,b\,b\,d\,c\,d\,a$
11	$b\,d\,c\,d\,a$
6	$c\,a\,c\,c\,b\,b\,d\,c\,d\,a$
2	$c\,b\,b\,d\,c\,a\,c\,c\,b\,b\,d\,c\,d\,a$
9	$c\,b\,b\,d\,c\,d\,a$
8	$c\,c\,b\,b\,d\,c\,d\,a$
13	$c\,d\,a$
14	$d\,a$
5	$d\,c\,a\,c\,c\,b\,b\,d\,c\,d\,a$
12	$d\,c\,d\,a$

find P = $c\,b\,b\,d\,c$ (length m)

Key Idea:

Find an occurence of P

using binary search

How long does this take?

$O(m)$ time to compare two strings

T (length n):

b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array

15	a
1	$a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
7	$a\ c\ c\ b\ b\ d\ c\ d\ a$
0	$b\ a\ c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
3	$b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
10	$b\ b\ d\ c\ d\ a$
4	$b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
11	$b\ d\ c\ d\ a$
6	$c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
2	$c\ b\ b\ d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
9	$c\ b\ b\ d\ c\ d\ a$
8	$c\ c\ b\ b\ d\ c\ d\ a$
13	$c\ d\ a$
14	$d\ a$
5	$d\ c\ a\ c\ c\ b\ b\ d\ c\ d\ a$
12	$d\ c\ d\ a$

find P = $c\ b\ b\ d\ c$ (length m)

Key Idea:

Find an occurence of P

using binary search

How long does this take?

$O(m)$ time to compare two strings

so $O(m \log n)$ time in total

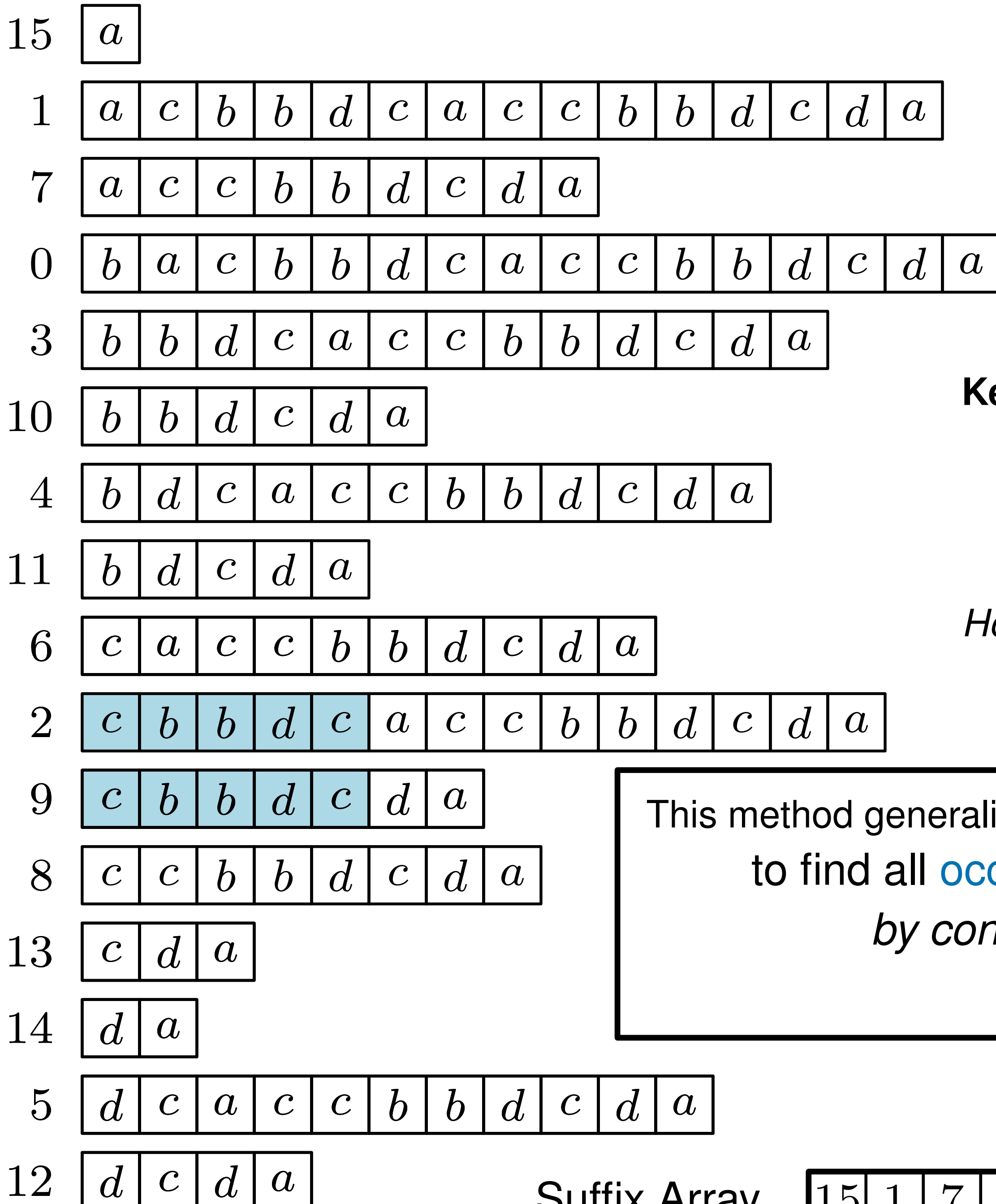
T (length n):

b	a	c	b	b	d	c	a	c	c	b	b	d	c	d	a
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

Searching in the Suffix Array



find P = c b b d c (length m)

Key Idea:

Find an occurence of P *using binary search*

How long does this take?

$O(m)$ time to compare two strings

so $O(m \log n)$ time in total

This method generalises to $O(m \log n + \text{occ})$ time

to find all occ occurences.

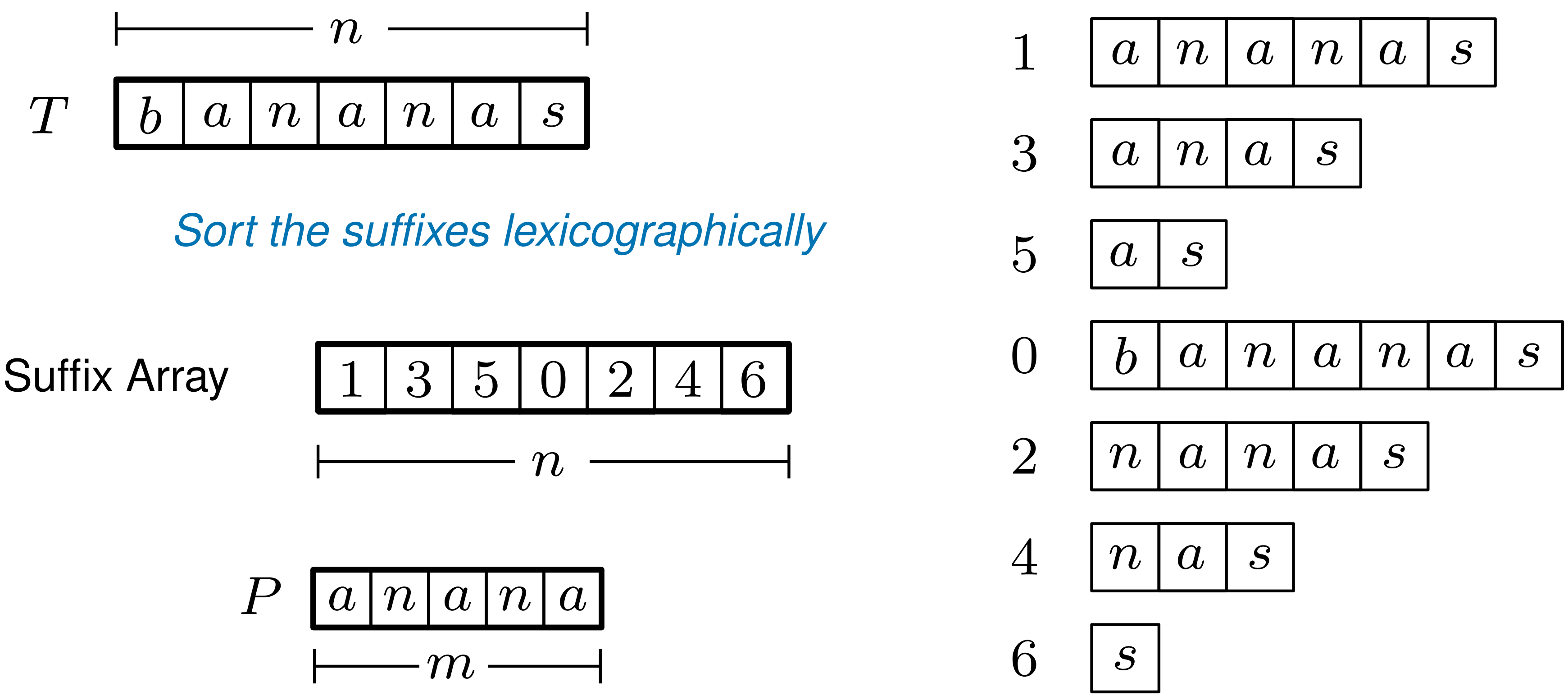
by continuing the binary search

(we will skip the details)

Suffix Array

15	1	7	0	3	10	4	11	6	2	9	8	13	14	5	12

The suffix array

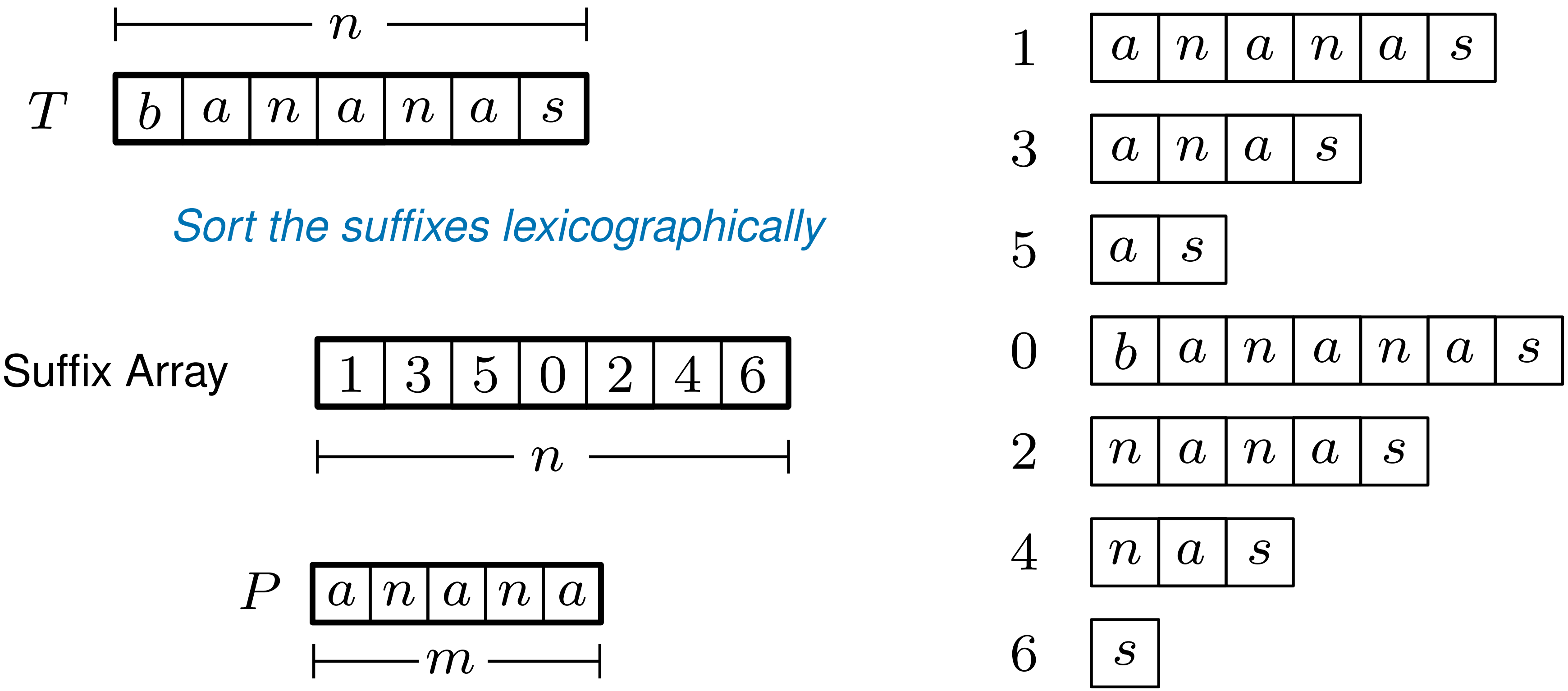


Finding an occurrence of a pattern (length m) takes $O(m \log n)$ time

Finding all occurrences takes $O(m \log n + \text{occ})$ time

where occ is the number of occurences

The suffix array



Finding an occurrence of a pattern (length m) takes $O(m \log n)$ time

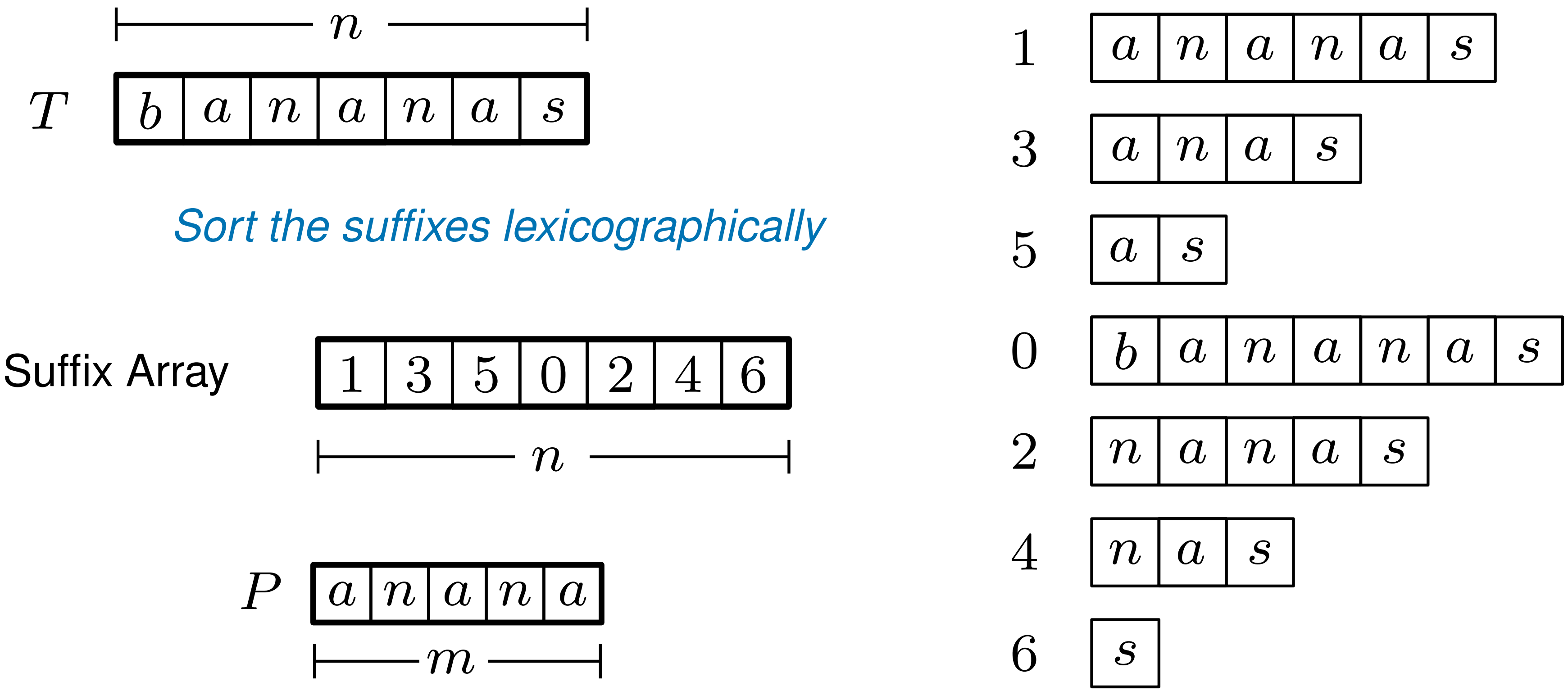
Finding all occurrences takes $O(m \log n + \text{occ})$ time

where occ is the number of occurences

This can be further improved to $O(m + \log n + \text{occ})$ time

(using LCP queries which we will see in a future lecture)

The suffix array



Finding an occurrence of a pattern (length m) takes $O(m \log n)$ time

Finding all occurrences takes $O(m \log n + \text{occ})$ time

where occ is the number of occurences

This can be further improved to $O(m + \log n + \text{occ})$ time

(using LCP queries which we will see in a future lecture)

Do we really need to build the suffix tree to construct the suffix array?

The DC3 method

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

0	**1**	2	3	**4**	5	6	**7**	8	9	**10**	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

The DC3 method

B_1 contains indices with

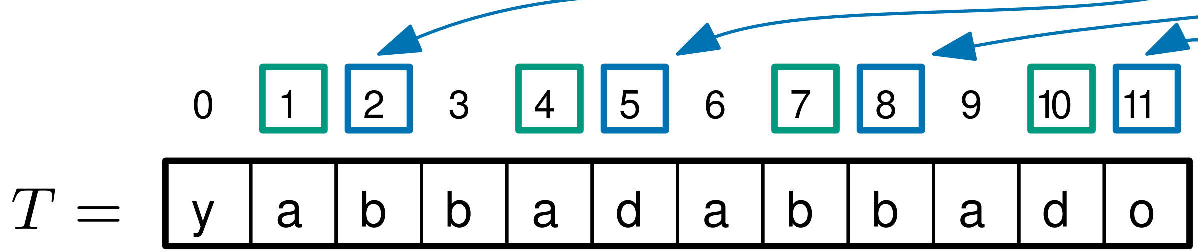
$i \bmod 3 = 1$

	0	**1**	2	3	**4**	5	6	**7**	8	9	**10**	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

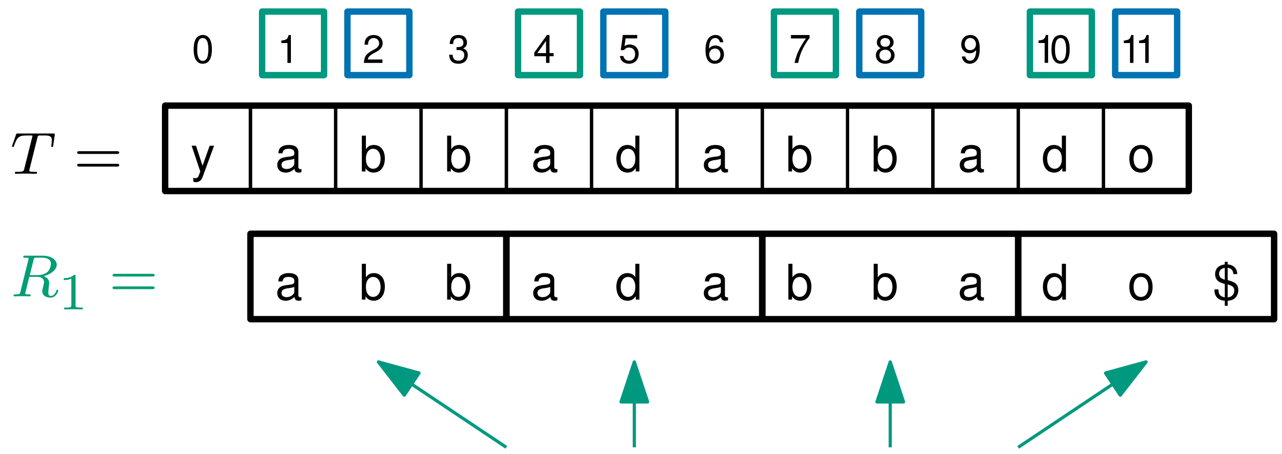
$R_1 =$	a b b	a d a	b b a	d o $

Introduce a new "filler symbol" $.

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



R_1 is split into *blocks* of length 3

Introduce a new "filler symbol" $.

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$

a	b	b	a	d	a	b	b	a	d	o	$

Introduce a new "filler symbol" $.

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$ | a b b | a d a | b b a | d o $ |

$R_2 =$ | b b a | d a b | b a d | o $ $ |

Introduce a new "filler symbol" $.

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$ | a b b | a d a | b b a | d o $ |

$R_2 =$ | b b a | d a b | b a d | o $ $ |

Introduce a new "filler symbol" $.

R_2 is also split into *blocks* of length 3

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$ [a b b][a d a][b b a][d o $]

$R_2 =$ [b b a][d a b][b a d][o $ $]

Introduce a new "filler symbol" $.

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]

$T =$

y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$

a b b	a d a	b b a	d o $

$R_2 =$

b b a	d a b	b a d	o $ $

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$	a b b	a d a	b b a	d o $
$R_2 =$	b b a	d a b	b a d	o $ $

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Number the blocks in lexicographical order

($ is the smallest symbol)

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$ abb ada bba do$

$R_2 =$ bba dab bad o$$

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

abb	ada	bba	do$	bba	dab	bad	o$$
1							

Number the blocks in lexicographical order

($ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$

a b b	a d a	b b a	d o $

$R_2 =$

b b a	d a b	b a d	o $ $

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2						

Number the blocks in lexicographical order

($ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$	a	b	b	a	d	a	b	b	a	d	o	$
$R_2 =$	b	b	a	d	a	b	b	a	d	o	$	$

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2					3	

Number the blocks in lexicographical order

($ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

$R_1 =$

a b b	a d a	b b a	d o $

$R_2 =$

b b a	d a b	b a d	o $ $

Introduce a new “filler symbol” $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4		4		3	

Number the blocks in lexicographical order

($ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$ [a b b | a d a | b b a | d o $]

$R_2 =$ [b b a | d a b | b a d | o $ $]

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4		4	5	3	

Number the blocks in lexicographical order

($ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$	a b b	a d a	b b a	d o $
$R_2 =$	b b a	d a b	b a d	o $ $

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	

Number the blocks in lexicographical order

($ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]	
$T =$	y	a	b	b	a	d	a	b	b	a	d	o	
$R_1 =$		a	b	b	a	d	a	b	b	a	d	o	\$
$R_2 =$			b	b	a	d	a	b	b	a	d	o	\$ \$

Introduce a new "filler symbol" \$.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

(\$ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$

a	b	b	a	d	a	b	b	a	d	o	$

$R_2 =$

b	b	a	d	a	b	b	a	d	o	$	$

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

($ is the smallest symbol)

This can be done by sorting the blocks in $O(n)$ time using radix sort

we assume that the bit representation of each symbol uses $O(\log n)$ bits.

(which is a common and realistic assumption)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$

a	b	b	a	d	a	b	b	a	d	o	$

$R_2 =$

b	b	a	d	a	b	b	a	d	o	$	$

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

($ is the smallest symbol)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11		
$T =$	y	a	b	b	a	d	a	b	b	a	d	o		
$R_1 =$		a	b	b	a	d	a	b	b	a	d	o	\$	
$R_2 =$			b	b	a	d	a	b	b	a	d	o	\$	\$

Introduce a new "filler symbol" \$.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

(\$ is the smallest symbol)

let $R' =$

1	2	4	6	4	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

$R_1 =$ [a b b] [a d a] [b b a] [d o \$]

$R_2 =$ [b b a] [d a b] [b a d] [o \$ \$]

Introduce a new "filler symbol" \$.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

(\$ is the smallest symbol)

let $R' =$

1	2	4	6	4	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

$R_1 =$ [a b b] [a d a] [b b a] [d o \$]

$R_2 =$ [b b a] [d a b] [b a d] [o \$ \$]

Introduce a new "filler symbol" \$.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

(\$ is the smallest symbol)

let $R' =$

1	2	4	6	4	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	**1**	**2**	3	**4**	**5**	6	**7**	**8**	9	**10**	**11**
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

$R_1 =$ [a b b] [a d a] [b b a] [d o $]

$R_2 =$ [b b a] [d a b] [b a d] [o $ $]

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

($ is the smallest symbol)

let $R' =$

0	1	2	3	4	5	6	7
1	2	4	6	4	5	3	7

compute the suffix array of R':

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

$R_1 =$ [a b b] [a d a] [b b a] [d o $]

$R_2 =$ [b b a] [d a b] [b a d] [o $ $]

Introduce a new “filler symbol” $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

($ is the smallest symbol)

let $R' =$

1	2	4	6	4	5	3	7
0	1	2	3	4	5	6	7

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

$R_1 =$ [a b b] [a d a] [b b a] [d o $]

$R_2 =$ [b b a] [d a b] [b a d] [o $ $]

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	7

N

How do we compute the suffix array for R'?

Recursion! *(Notice that R' has length $2n/3$)*

($ is the smallest symbol)

			6	4	5	3	7
0	1	2	3	4	5	6	7

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

$R_1 =$ | a b b | a d a | b b a | d o $ |

$R_2 =$ | b b a | d a b | b a d | o $ $ |

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

($ is the smallest symbol)

let $R' =$

0	1	2	3	4	5	6	7
1	2	4	6	4	5	3	7

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

University of BRISTOL

0	1	2	3	4	5	6	7	8	9	10	11

$T =$

y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$

a b b	a d a	b b a	d o \$

$R_2 =$

b b a	d a b	b a d	o \$ \$

Introduce a new "filler symbol" \$.

Concatenate R_1 and R_2 to obtain R:

a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

(\$ is the smallest symbol)

let $R' =$

1	2	4	6	4	5	3	7
0	1	2	3	4	5	6	7

compute the suffix array of R':

0	1	6	4	2	5	3	7

what use is this?!

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

$R_1 =$ | a b b | a d a | b b a | d o $ |

$R_2 =$ | b b a | d a b | b a d | o $ $ |

Introduce a new "filler symbol" $.

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
1	2	4	6	4	5	3	7

Number the blocks in lexicographical order

($ is the smallest symbol)

let $R' =$

1	2	4	6	4	5	3	7
0	1	2	3	4	5	6	7

compute the suffix array of R':

0	1	6	4	2	5	3	7

what use is this?!

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

University of BRISTOL

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

[1] a b b a d a b b a d o

[5] d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$

Take any two suffixes in $B_1 \cup B_2$

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

1: a b b a d a b b a d o

5: d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

what use

is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

1: a b b a d a b b a d o

5: d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

University of BRISTOL

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o
1		a	b	b	a	d	a	b	b	a	d	o
5						d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

1: a b b a d a b b a d o

5: d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o
1		a	b	b	a	d	a	b	b	a	d	o
5						d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
					d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

1: a b b a d a b b a d o

5: d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

a b b | a d a | b b a | d o $ | b b a | d a b | b a d | o $ $

d a b | b a d | o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

1: a b b a d a b b a d o

5: d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

a b b a d a b b a d o $ b b a d a b b a d o $ $

d a b b a d o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

University of BRISTOL

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Suffix											
1	a	b	b	a	d	a	b	b	a	d	o
5	d	a	b	b	a	d	o				

Suffix 1 is smaller than suffix 5

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
					d a b	b a d	o \$ \$

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

7: b b a d o

5: d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

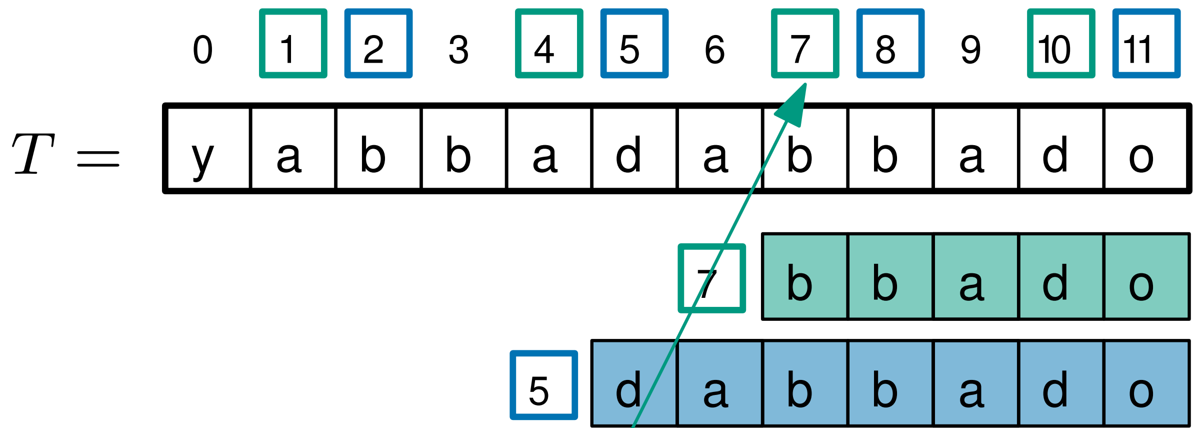
compute the suffix array of R':

0	1	6	4	2	5	3	7

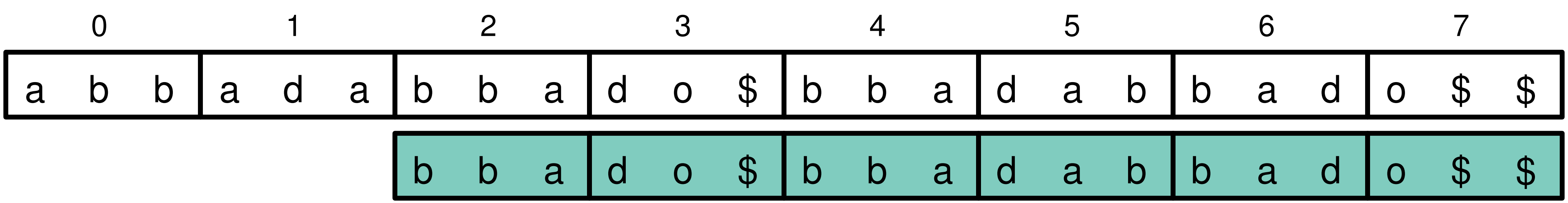
The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



Concatenate R_1 and R_2 to obtain R:



Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use

is this?!

---

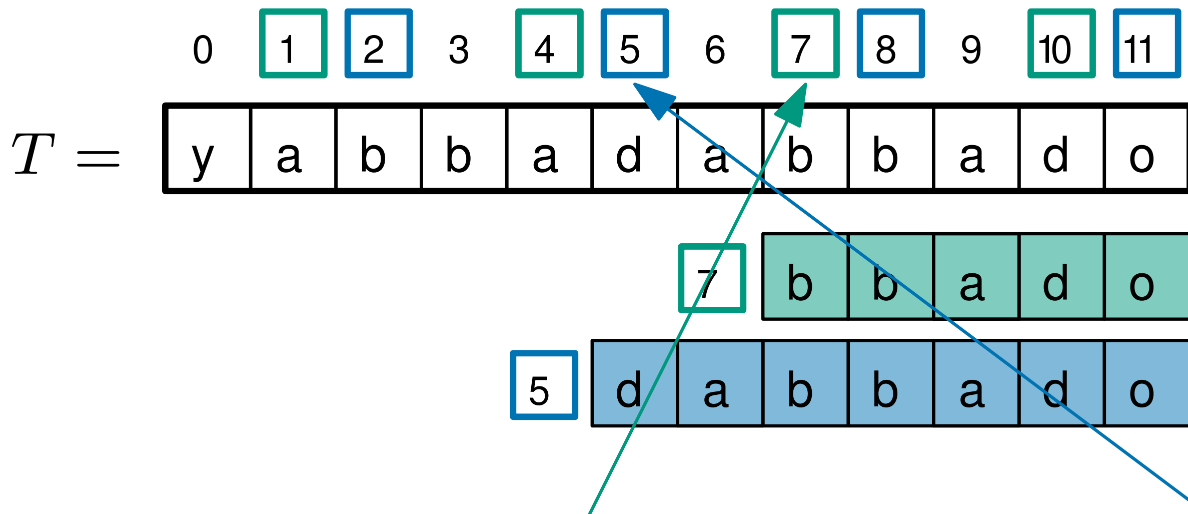
compute the suffix array of R':

0	1	6	4	2	5	3	7

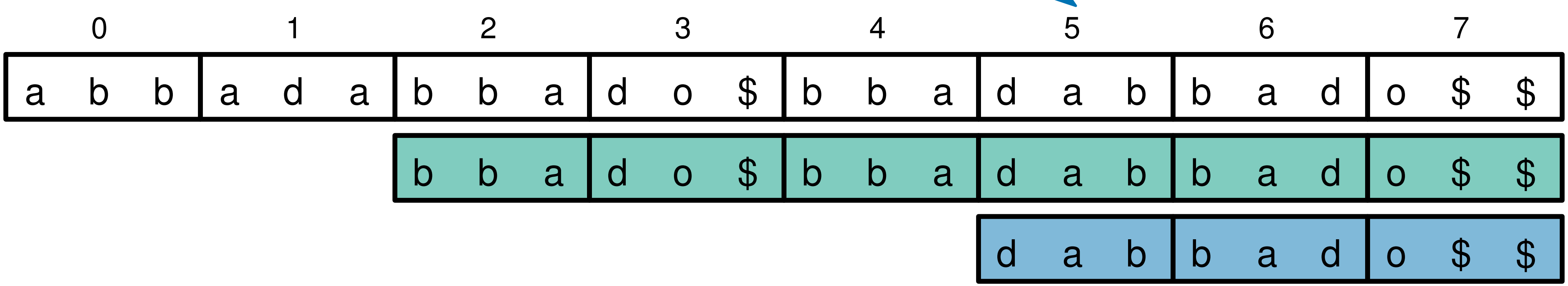
The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



Concatenate R_1 and R_2 to obtain R:



Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

7	b	b	a	d	o	
5	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
		b b a	d o $	b b a	d a b	b a d	o $ $
					d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

$T =$

0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
y	a	b	b	a	d	a	b	b	a	d	o

7	b	b	a	d	o

5	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	(2)	3	4	(5)	6	7
a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
		b b a	d o \$	b b a	d a b	b a d	o \$ \$
					d a b	b a d	o \$ \$

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use

is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

7: b b a d o

5: d a b b a d o

Suffix 7 is smaller than suffix 5

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

b b a | d o $ | b b a | d a b | b a d | o $ $

d a b | b a d | o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

$T =$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o
2			b	b	a	d	a	b	b	a	d	o
5						d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

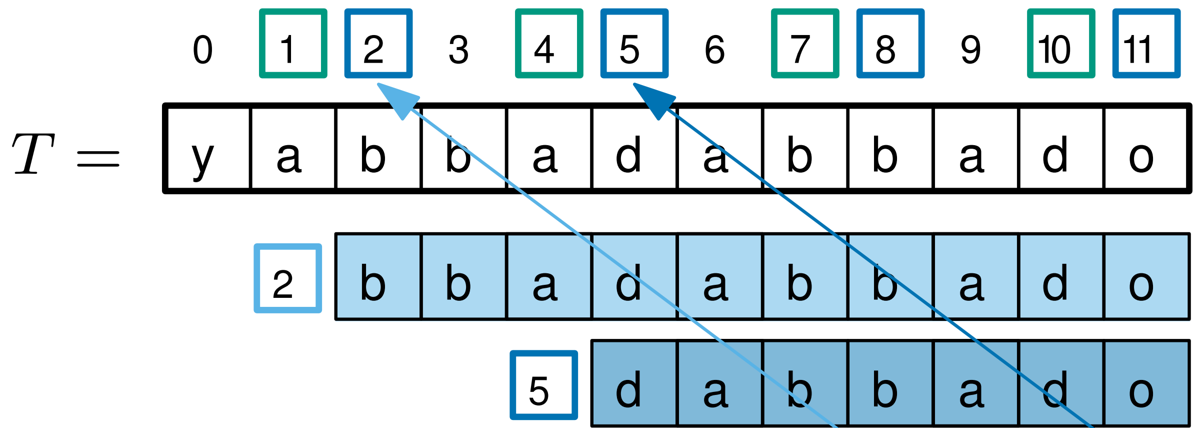
compute the suffix array of R':

0	1	6	4	2	5	3	7

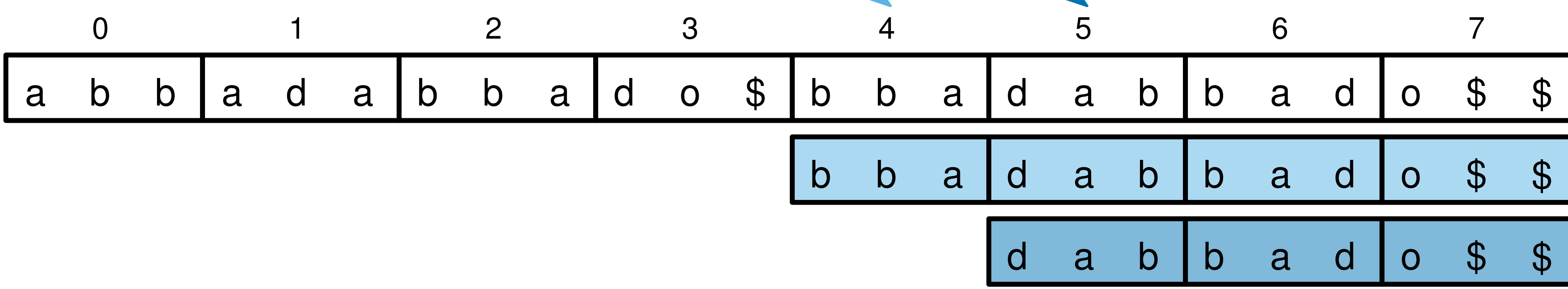
The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



Concatenate R_1 and R_2 to obtain R:



Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	[1]	[2]	3	[4]	[5]	6	[7]	[8]	9	[10]	[11]
$T =$	y	a	b	b	a	d	a	b	b	a	d	o
			[2] b	b	a	d	a	b	b	a	d	o
						[5] d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $
				b b a	d a b	b a d	o $ $
					d a b	b a d	o $ $

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

---

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o
2			b	b	a	d	a	b	b	a	d	o
5						d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
				b b a	d a b	b a d	o \$ \$
					d a b	b a d	o \$ \$

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o
2			b	b	a	d	a	b	b	a	d	o
5						d	a	b	b	a	d	o

Suffix 2 is smaller than suffix 5

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o \$	b b a	d a b	b a d	o \$ \$
				b b a	d a b	b a d	o \$ \$
					d a b	b a d	o \$ \$

Take any two suffixes in $B_1 \cup B_2$ *and find them in* R

their order is given by the suffix array of R':

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

what use is this?!

compute the suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

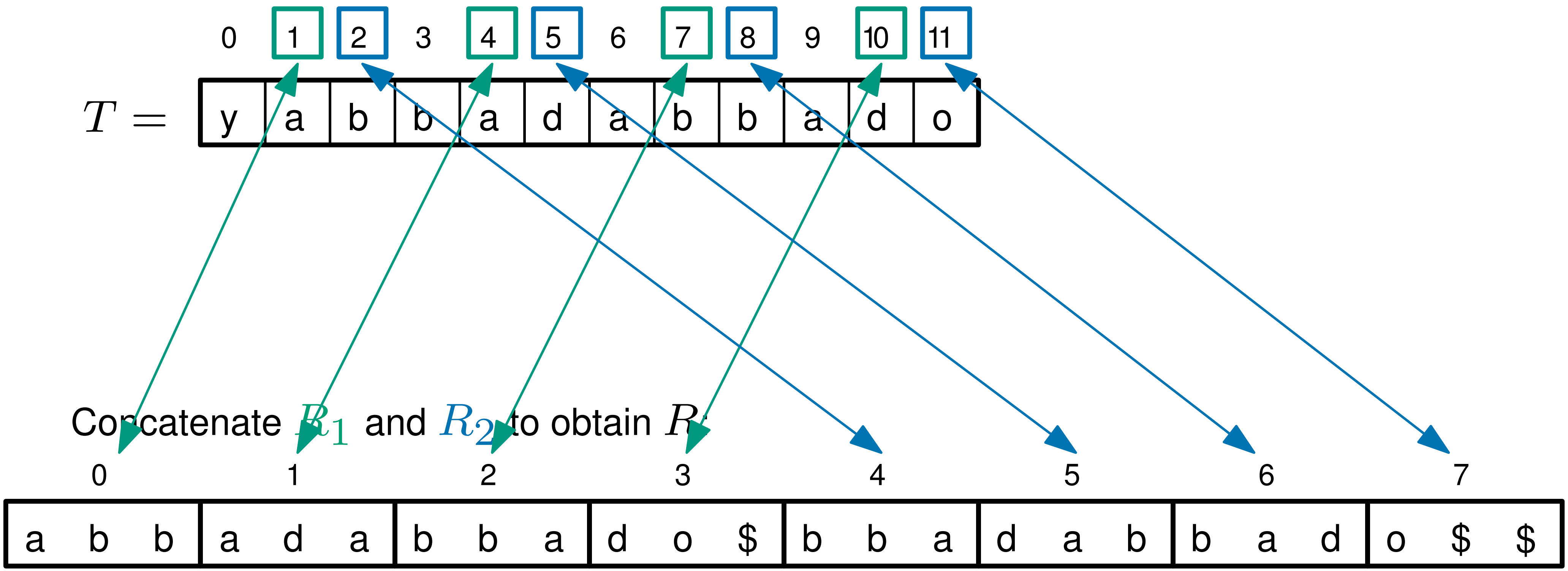
The suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



Concatenate R_1 and R_2 to obtain R'

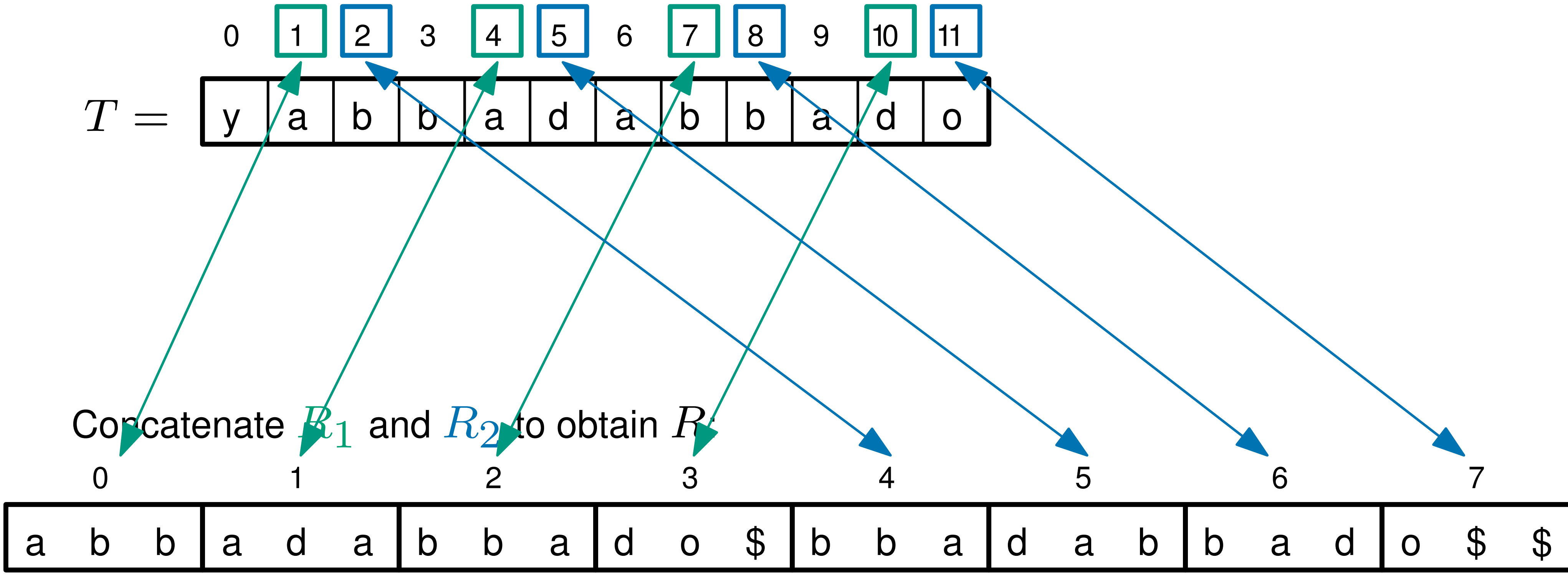
The suffix array of R':

0	1	6	4	2	5	3	7

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



The suffix array of R':

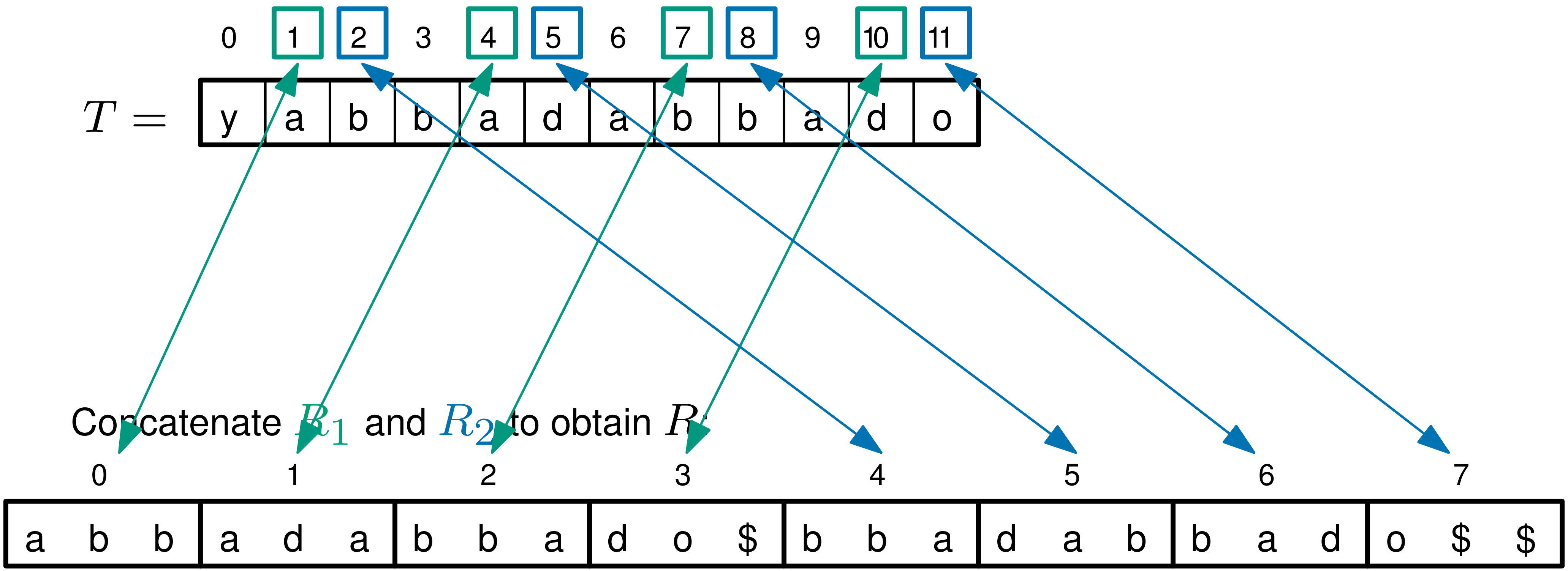
0	1	6	4	2	5	3	7

after relabelling,

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



The suffix array of R':

0	1	6	4	2	5	3	7

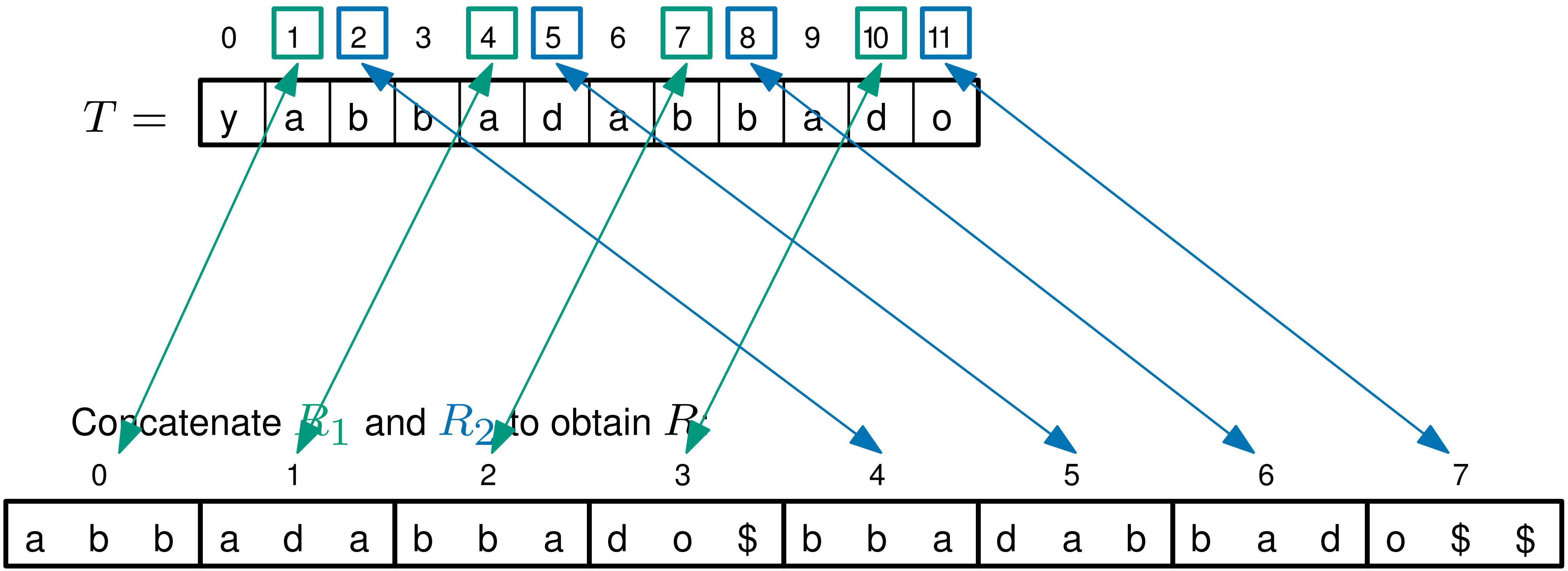
after relabelling,

1	4	8	2	7	5	10	11

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



The suffix array of R':

0	1	6	4	2	5	3	7

after relabelling,

1	4	8	2	7	5	10	11

we have the suffix array of just the suffixes from $B_1 \cup B_2$

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ (above)

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

The suffix array of R':

0	1	6	4	2	5	3	7

after relabelling,

1	4	8	2	7	5	10	11

we have the suffix array of just the suffixes from $B_1 \cup B_2$

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

$T =$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

Concatenate R_1 and R_2 to obtain R:

0	1	2	3	4	5	6	7
a b b	a d a	b b a	d o $	b b a	d a b	b a d	o $ $

The suffix array of R':

0	1	6	4	2	5	3	7

after relabelling,

1	4	8	2	7	5	10	11

we have the suffix array of just the suffixes from $B_1 \cup B_2$

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Suffix array for just $B_1 \cup B_2$

1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ (above)

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

Indices: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

$T =$

y	a	b	b	a	d	a	b	b	a	d	o

0	y	a	b	b	a	d	a	b	b	a	d	o
3	b	a	d	a	b	b	a	d	o			
6	a	b	b	a	d	o						
9	a	d	o									

Suffix array for just $B_1 \cup B_2$

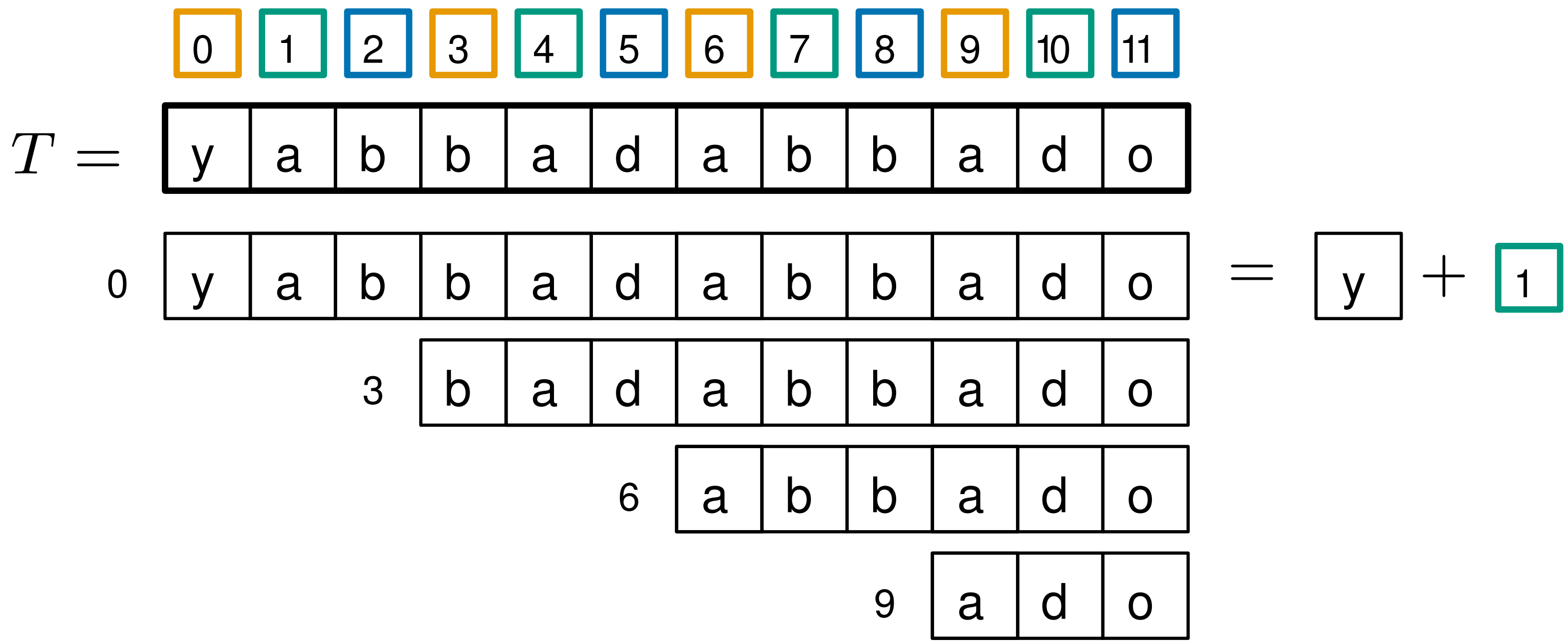
0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$



Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

0	y	a	b	b	a	d	a	b	b	a	d	o
3	b	a	d	a	b	b	a	d	o			
6	a	b	b	a	d	o						
9	a	d	o									

= y + 1

suffix 0 is a y
followed by suffix 1

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

0	y	a	b	b	a	d	a	b	b	a	d	o
3	b	a	d	a	b	b	a	d	o			
6	a	b	b	a	d	o						
9	a	d	o									

0: y a b b a d a b b a d o = y + 1

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

0 y a b b a d a b b a d o = y + 1

3 b a d a b b a d o = b + 4

6 a b b a d o = a + 7

9 a d o = a + 10

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

0	y	a	b	b	a	d	a	b	b	a	d	o	=	y	+	1
3	b	a	d	a	b	b	a	d	o				=	b	+	4
6	a	b	b	a	d	o							=	a	+	7
9	a	d	o										=	a	+	10

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

Suffix array for just $B_1 \cup B_2$

rank:	0	1	2	3	4	5	6	7
	1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

0	y	a	b	b	a	d	a	b	b	a	d	o	$=$	y	$+$ 1	$(y, 0)$
3				b	a	d	a	b	b	a	d	o	$=$	b	$+$ 4	$(b, 1)$
6							a	b	b	a	d	o	$=$	a	$+$ 7	$(a, 4)$
9										a	d	o	$=$	a	$+$ 10	$(a, 6)$

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

Suffix array for just $B_1 \cup B_2$

rank:	0	1	2	3	4	5	6	7
	1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Suffix				
0: y a b b a d a b b a d o	=	y + 1		$(y, 0)$
3: b a d a b b a d o	=	b + 4		$(b, 1)$
6: a b b a d o	=	a + 7		$(a, 4)$
9: a d o	=	a + 10		$(a, 6)$

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

We then sort in $O(n)$ time using radix sort

Suffix array for just $B_1 \cup B_2$

rank:	0	1	2	3	4	5	6	7
	1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Suffix	Suffix text				
6	abbado	=	a	+	7	$(a, 4)$
9	ado	=	a	+	10	$(a, 6)$
3	badabbado	=	b	+	4	$(b, 1)$
0	yabbadabbado	=	y	+	1	$(y, 0)$

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

We then sort in $O(n)$ time using radix sort

Suffix array for just $B_1 \cup B_2$

rank:	0	1	2	3	4	5	6	7
	1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

6 | a | b | b | a | d | o | $=$ a $+$ 7 $\quad (a, 4)$

9 | a | d | o | $=$ a $+$ 10 $\quad (a, 6)$

3 | b | a | d | a | b | b | a | d | o | $=$ b $+$ 4 $\quad (b, 1)$

0 | y | a | b | b | a | d | a | b | b | a | d | o | $=$ y $+$ 1 $\quad (y, 0)$

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

We then sort in $O(n)$ time using radix sort

Suffix array for just $B_1 \cup B_2$

rank:	0	1	2	3	4	5	6	7
	1	4	8	2	7	5	10	11

How do we find the ordering of the suffixes from B_0? (where $i \bmod 3 = 0$)

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Suffix			=			
6	a b b a d o	=	a	+	7	$(a, 4)$
9	a d o	=	a	+	10	$(a, 6)$
3	b a d a b b a d o	=	b	+	4	$(b, 1)$
0	y a b b a d a b b a d o	=	y	+	1	$(y, 0)$

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

We then sort in $O(n)$ time using radix sort

rank:	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	1	4	8	2	7	5	10	11

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

suffix																
6	a	b	b	a	d	o							=	a + 7		$(a, 4)$
9	a	d	o										=	a + 10		$(a, 6)$
3	b	a	d	a	b	b	a	d	o				=	b + 4		$(b, 1)$
0	y	a	b	b	a	d	a	b	b	a	d	o	=	y + 1		$(y, 0)$

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

We then sort in $O(n)$ time using radix sort

rank:	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	1	4	8	2	7	5	10	11
Suffix array for just B_0	6	9	3	0				

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

6: a b b a d o = a + 7 → $(a, 4)$

9: a d o = a + 10 → $(a, 6)$

3: b a d a b b a d o = b + 4 → $(b, 1)$

0: y a b b a d a b b a d o = y + 1 → $(y, 0)$

Each suffix $i \in B_0$ is represented by $(T[i], r)$ where r is the rank of suffix $(i+1)$

(the ranks are given by the array below)

We then sort in $O(n)$ time using radix sort

rank:	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	1	4	8	2	7	5	10	11
Suffix array for just B_0	6	9	3	0				

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Merge them like in mergesort...

	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	1	4	8	2	7	5	10	11
Suffix array for just B_0	6	9	3	0				

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

University of BRISTOL

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort...

which is smaller, suffix 1 or 6 ?

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort...

which is smaller, suffix 1 or 6 ?

6 = a + 7

1 = a + 2

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Merge them like in mergesort...

which is smaller, suffix 1 or 6 ?

$6 = \boxed{a} + 7 \qquad (a, 4)$

$1 = \boxed{a} + 2 \qquad (a, 3)$

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort...

which is smaller, suffix 1 or 6 ?

6 = a + 7 $\quad (a, 4)$

1 = a + 2 $\quad (a, 3)$

It takes $O(1)$ time to decide that 1 is smaller

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
1	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort. . . [1]

which is smaller, suffix [1] or [6] ?

[6] = [a] + [7] $(a, 4)$

[1] = [a] + [2] $(a, 3)$

It takes $O(1)$ time to decide that [1] is smaller

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Merge them like in mergesort. . .

1

which is smaller, suffix 4 or 6 ?

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort. . .   1

which is smaller, suffix 4 or 6 ?

6 = a + 7

4 = a + 5

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort. . .   1

which is smaller, suffix 4 or 6 ?

6 = a + 7 $(a, 4)$

4 = a + 5 $(a, 5)$

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort. . . 1

which is smaller, suffix 4 or 6 ?

6 = a + 7 $(a, 4)$

4 = a + 5 $(a, 5)$

Again, it takes $O(1)$ time to decide that 6 is smaller

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	4	8	2	7	5	10	11

Suffix array for just B_0

6	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Merge them like in mergesort...

1	6

	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	1 (crossed out)	4	8	2	7	5	10	11
Suffix array for just B_0	6 (crossed out)	9	3	0				

How do we merge these?

University of BRISTOL

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Merge them like in mergesort...

1	6

which is smaller, suffix 4 or 9 ?

	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	~~1~~	4	8	2	7	5	10	11
Suffix array for just B_0	~~6~~	9	3	0				

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Merge them like in mergesort...

1	6

which is smaller, suffix 4 or 9 ? (4 is smaller)

	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	~~1~~	4	8	2	7	5	10	11
Suffix array for just B_0	~~6~~	9	3	0				

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort...

1	6	4

which is smaller, suffix 4 or 9 ? (4 is smaller)

	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	~~1~~	~~4~~	8	2	7	5	10	11
Suffix array for just B_0	~~6~~	9	3	0				

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort. . .

1	6	4

which is smaller, suffix 8 or 9 ?

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	~~4~~	8	2	7	5	10	11

Suffix array for just B_0

~~6~~	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort...

1	6	4

which is smaller, suffix 8 or 9 ?

9 = a + 10

8 = b + 9

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	~~4~~	8	2	7	5	10	11

Suffix array for just B_0

~~6~~	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort...

1	6	4

which is smaller, suffix 8 or 9 ?

9 = a + 10

8 = b + 9

Uh oh! how do we compare 9 to 10 ?

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	~~4~~	8	2	7	5	10	11

Suffix array for just B_0

~~6~~	9	3	0

How do we merge these?

B_1 contains indices with $i \bmod 3 = 1$

The DC3 method

B_2 contains indices with $i \bmod 3 = 2$

0	1	2	3	4	5	6	7	8	9	10	11
y	a	b	b	a	d	a	b	b	a	d	o

$T =$ y a b b a d a b b a d o

Merge them like in mergesort. . .

1	6	4

which is smaller, suffix 8 or 9 ?

9 = a + d + 11

8 = b + a + 10

It *still* takes $O(1)$ time to decide that 9 is smaller

Uh oh! how do we compare 9 to 10 ?

Suffix array for just $B_1 \cup B_2$

0	1	2	3	4	5	6	7
~~1~~	~~4~~	8	2	7	5	10	11

Suffix array for just B_0

~~6~~	9	3	0

How do we merge these?

The DC3 method

B_1 contains indices with $i \bmod 3 = 1$

B_2 contains indices with $i \bmod 3 = 2$

	0	1	2	3	4	5	6	7	8	9	10	11
$T =$	y	a	b	b	a	d	a	b	b	a	d	o

Merge them like in mergesort...

1	6	4

Overall this merging phase takes $O(n)$ time

(because processing each suffix takes $O(1)$ time)

	0	1	2	3	4	5	6	7
Suffix array for just $B_1 \cup B_2$	~~1~~	~~4~~	8	2	7	5	10	11
Suffix array for just B_0	~~6~~	9	3	0				

How do we merge these?

The DC3 method

Theorem

The DC3 algorithm constructs a suffix array in $O(n)$ time.

The DC3 method

Theorem

The DC3 algorithm constructs a suffix array in $O(n)$ time.

Proof

Suppose $T(n)$ is the running time. We have

$$T(n) = T(2n/3) + O(n)$$

The DC3 method

Theorem

The DC3 algorithm constructs a suffix array in $O(n)$ time.

Proof

Suppose $T(n)$ is the running time. We have

radix sorting and merging

$$T(n) = T(2n/3) + O(n)$$

recursion to construct
a suffix array of size $2n/3$

The DC3 method

Theorem

The DC3 algorithm constructs a suffix array in $O(n)$ time.

Proof

Suppose $T(n)$ is the running time. We have

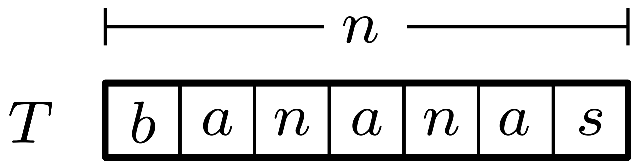
radix sorting and merging

$$T(n) = T(2n/3) + O(n)$$

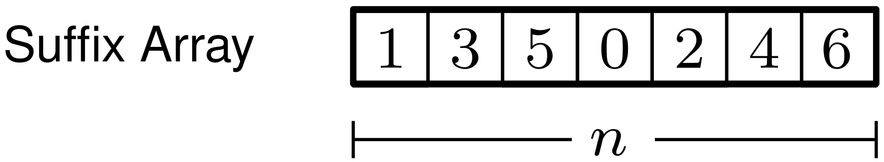
recursion to construct a suffix array of size $2n/3$

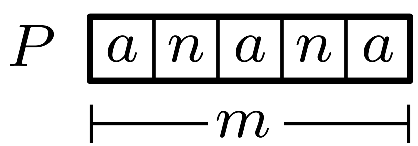
Solving this recurrence gives $T(n) \in O(n)$.

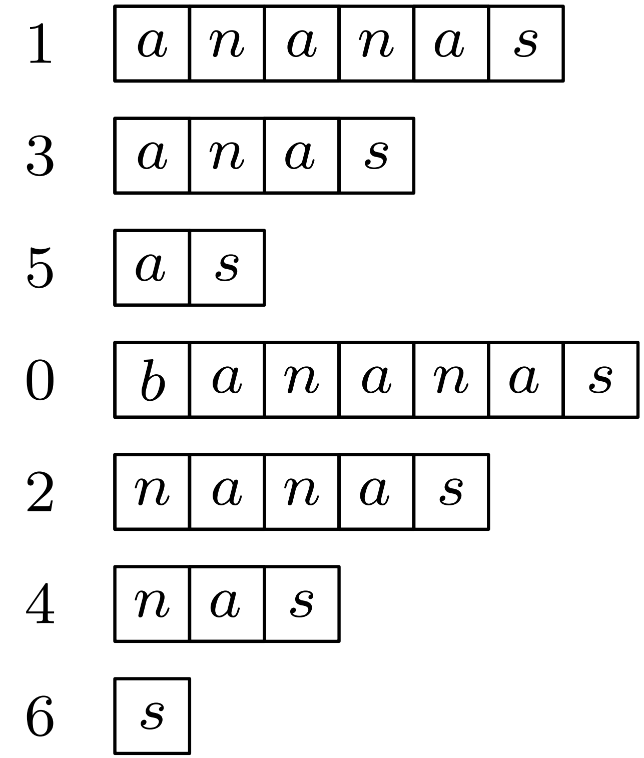
The suffix array



Sort the suffixes lexicographically







Finding an occurrence of a pattern (length m) takes $O(m \log n)$ time

Finding all occurrences takes $O(m \log n + \text{occ})$ time

where occ is the number of occurences

This can be further improved to $O(m + \log n + \text{occ})$ time

(using LCP queries which we will see in a future lecture)

We can construct the suffix array in $O(n)$ time