Advanced Algorithms – COMS31900

Range Minimum Queries

Raphaël Clifford

Slides by Benjamin Sach

Range minimum query

Preprocess an integer array A (length n) to answer range minimum queries...

n

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	23	17	8	73	51	82	19	32	5	67	91	14	46	9	21	54

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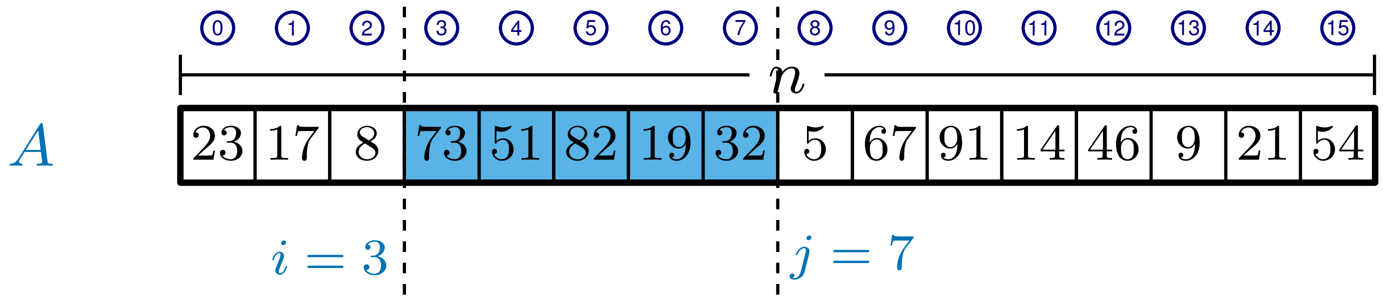

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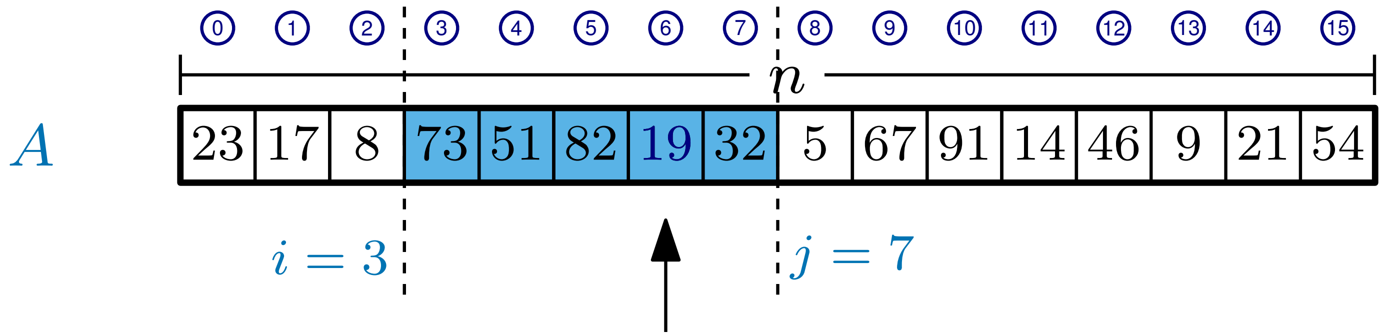

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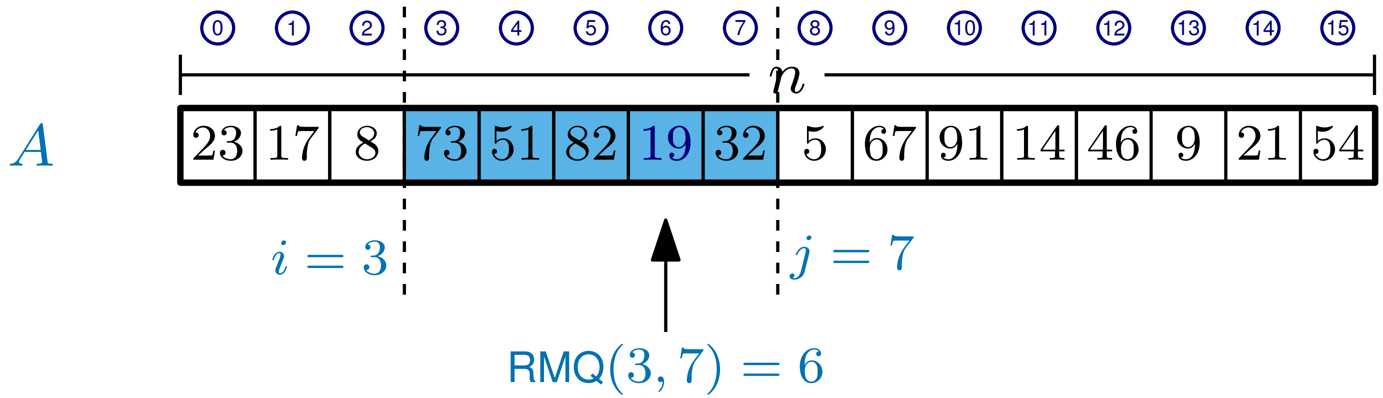

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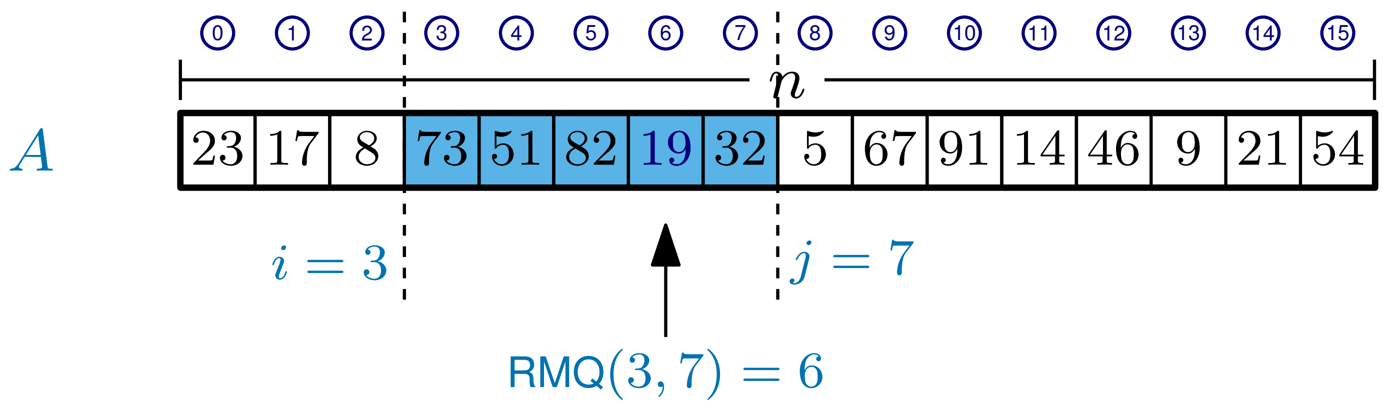

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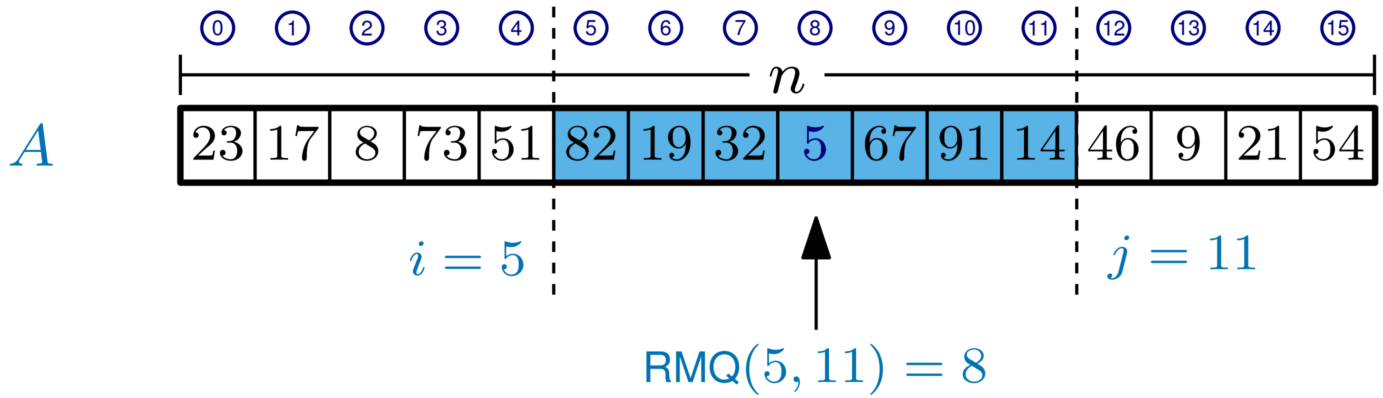

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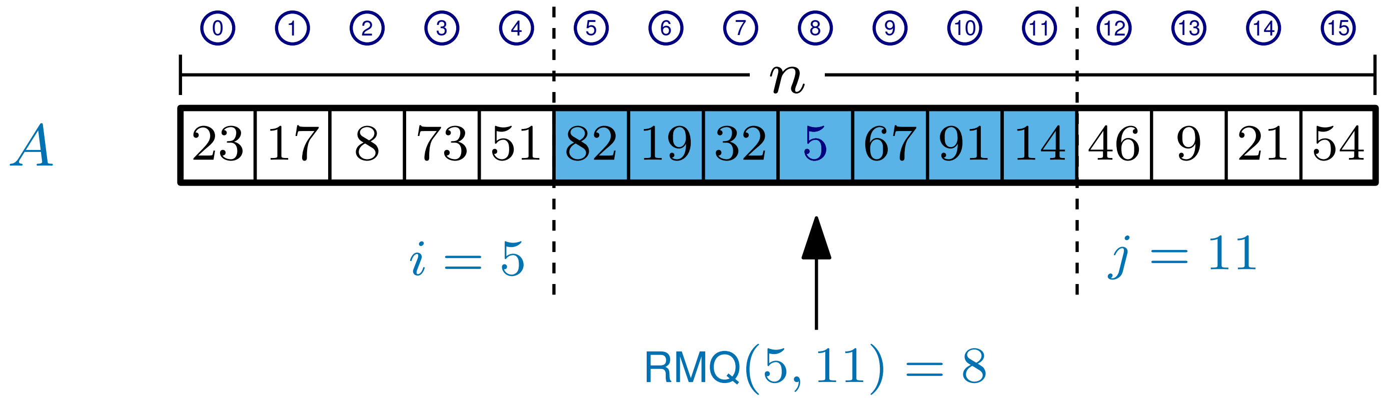

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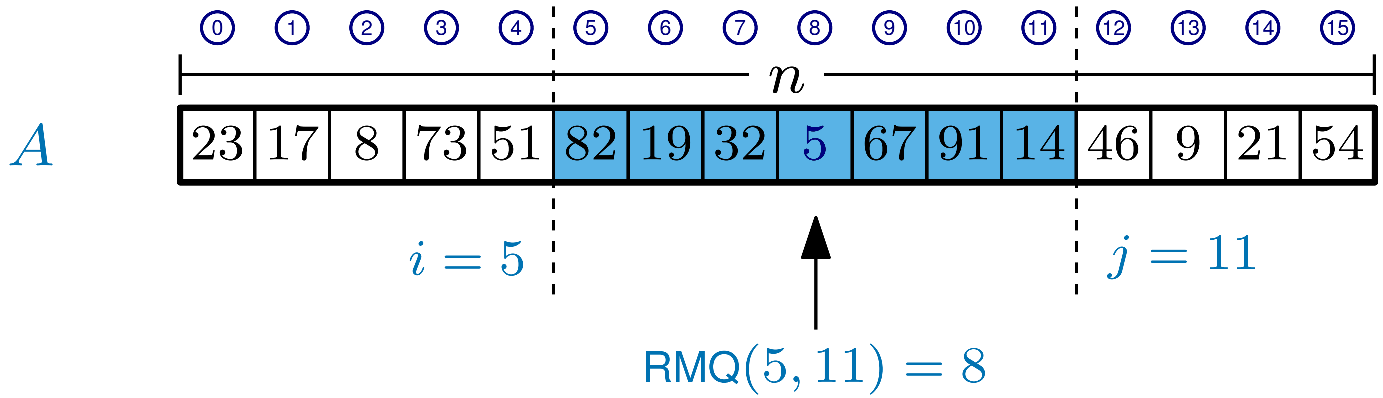

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- Ideally we would like $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time

Block decomposition

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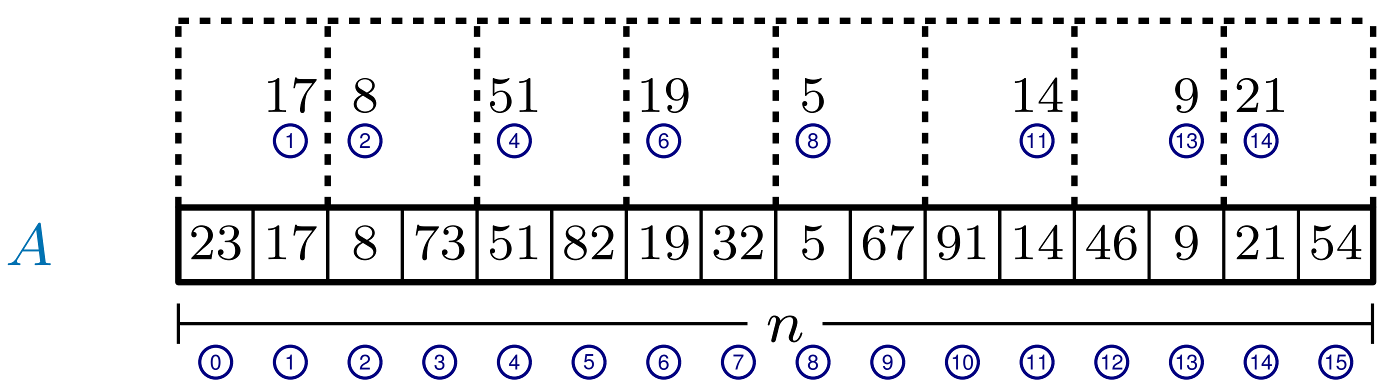

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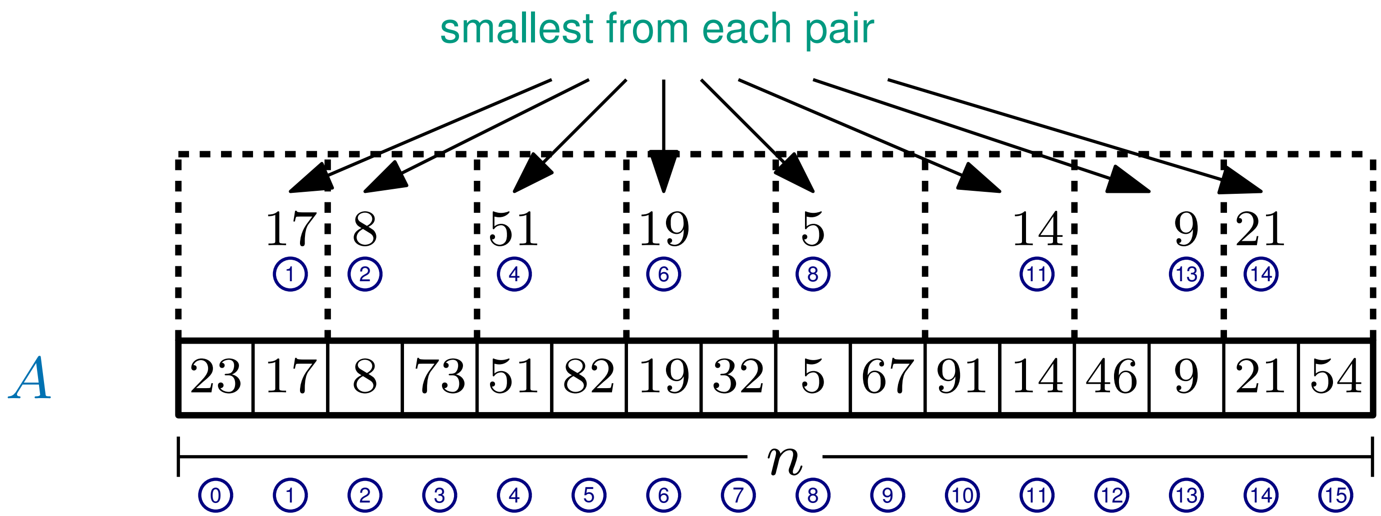

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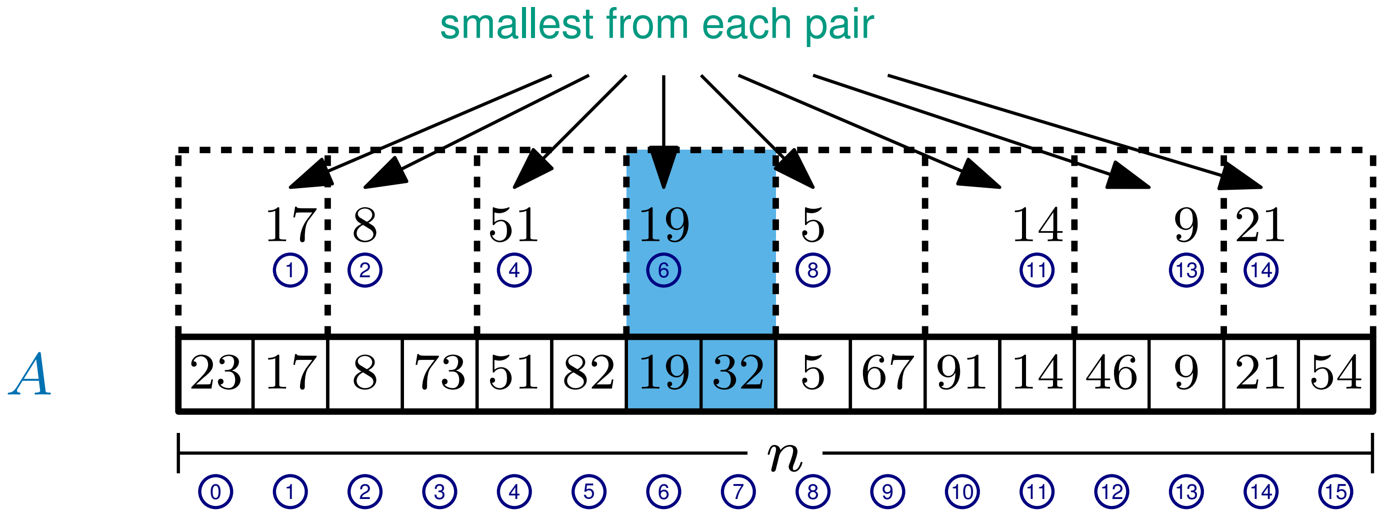

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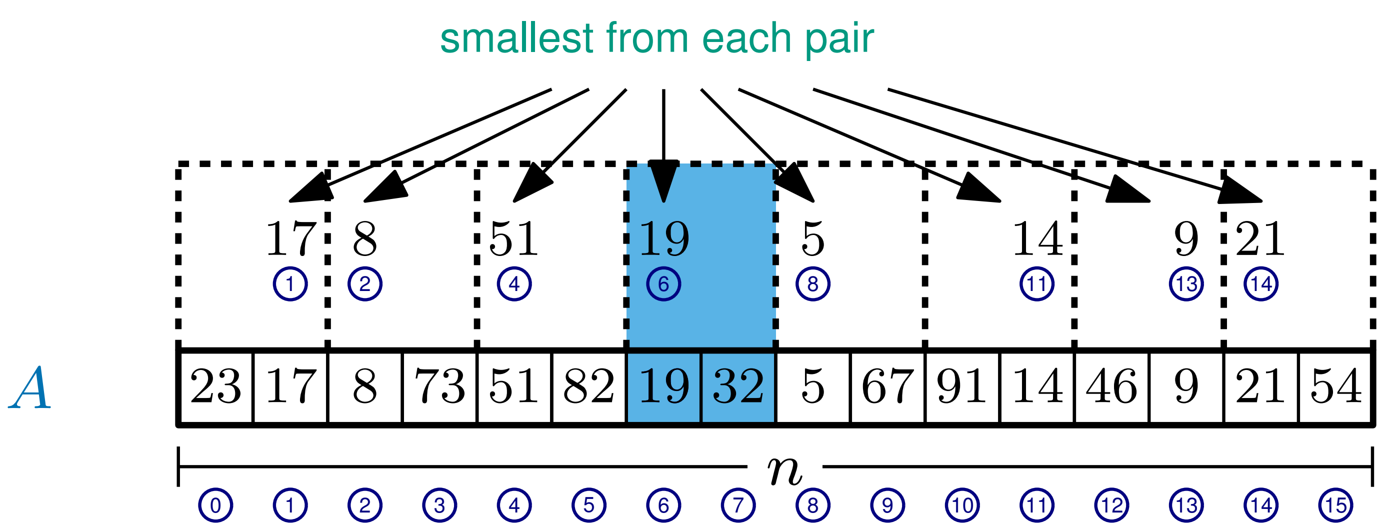

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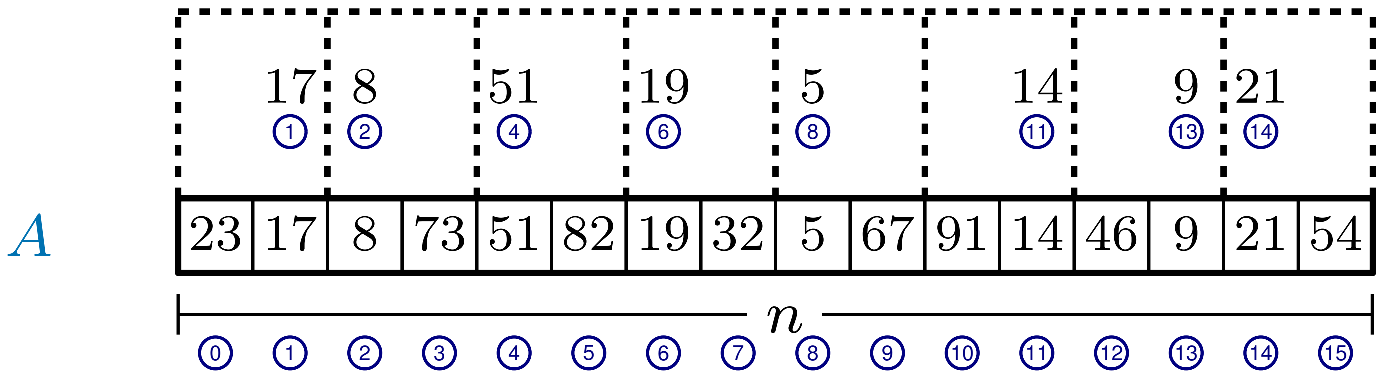

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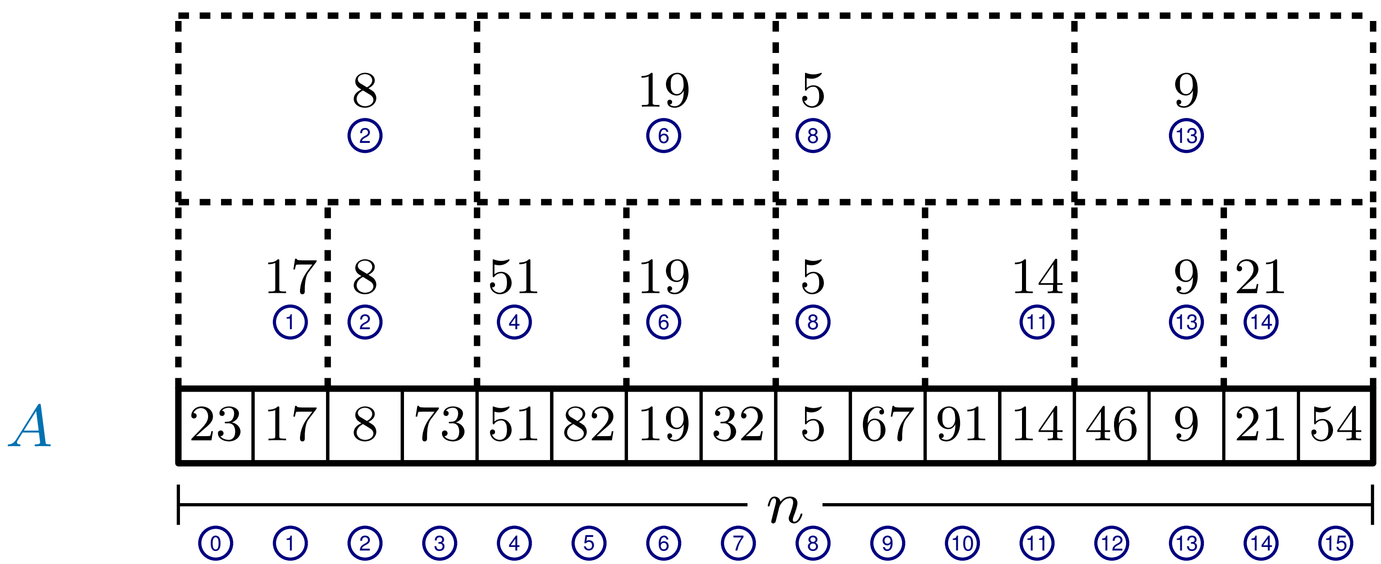

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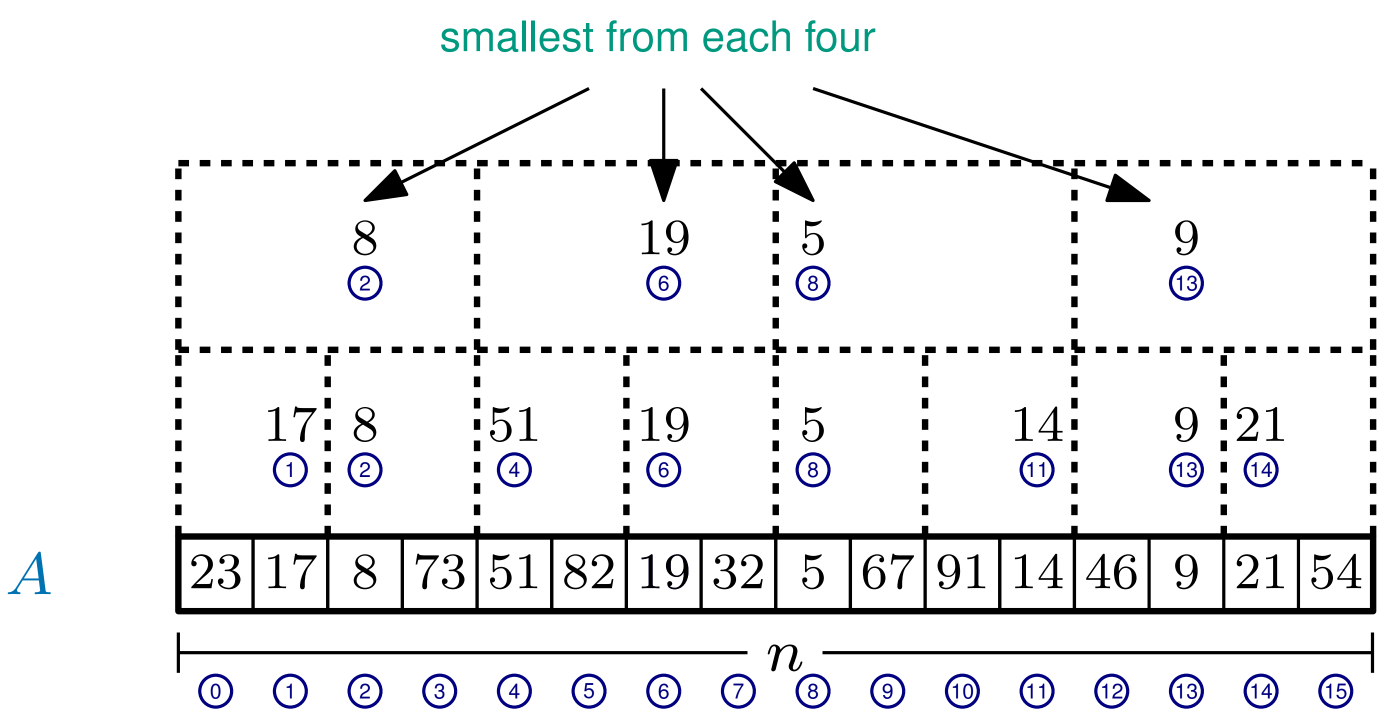

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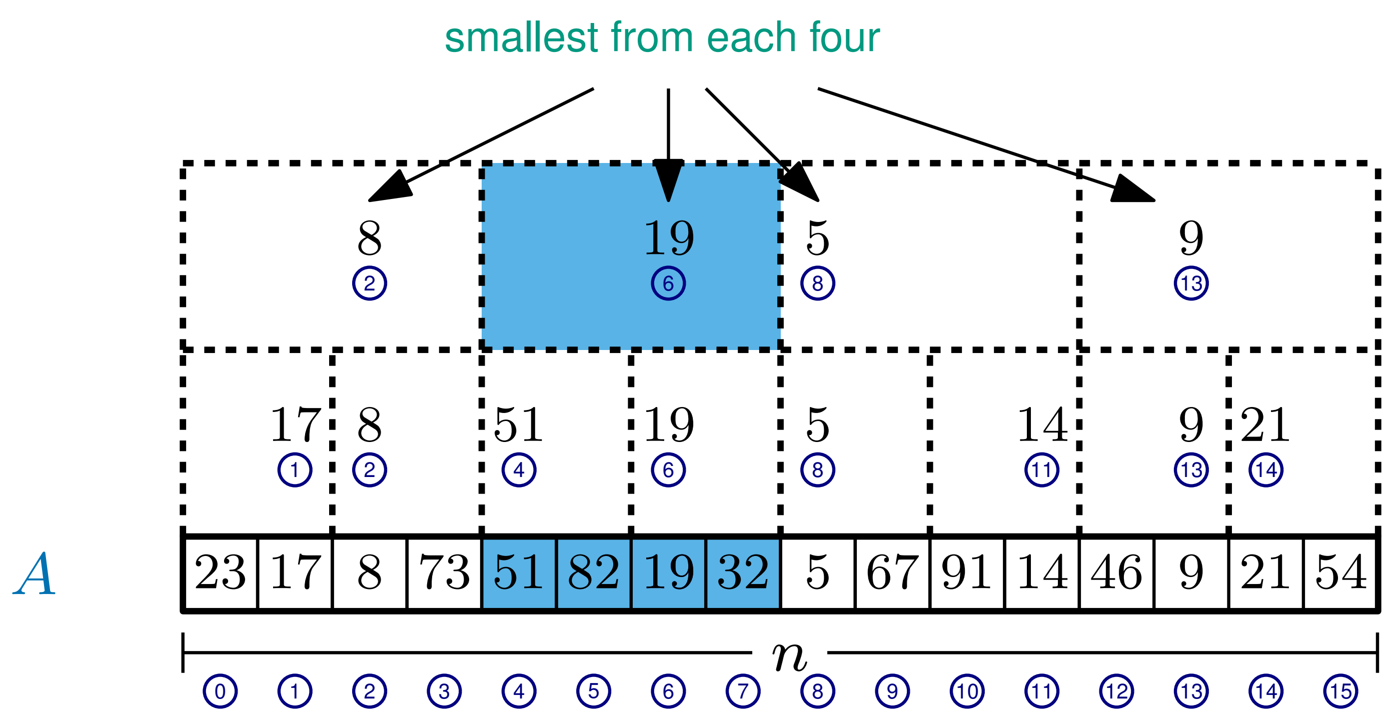

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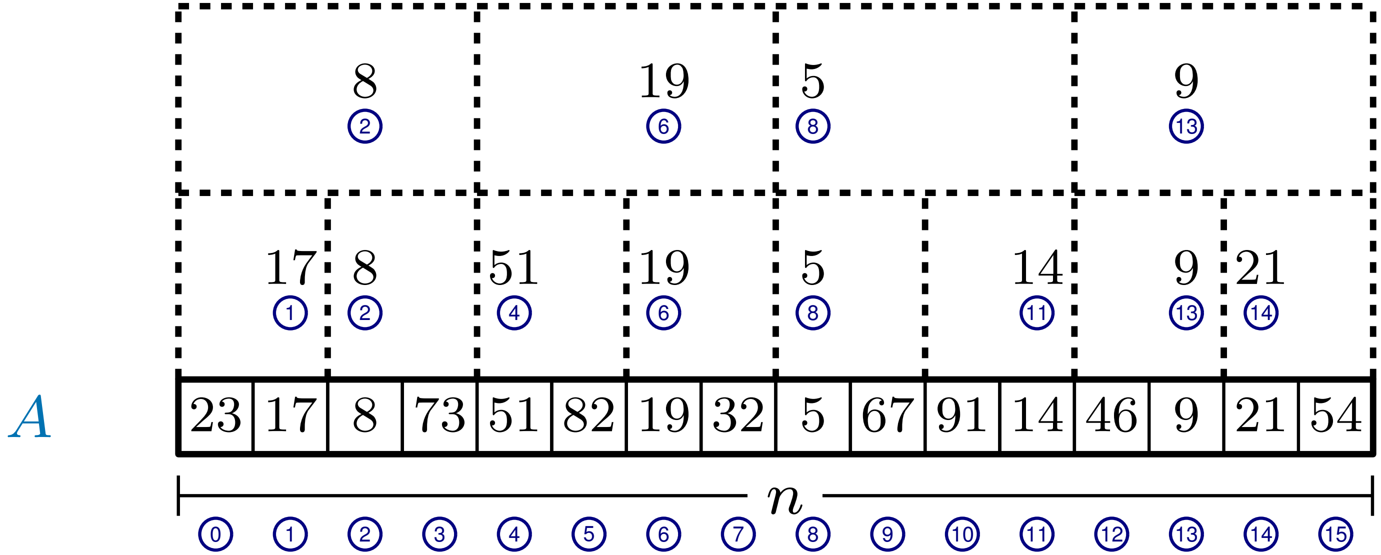

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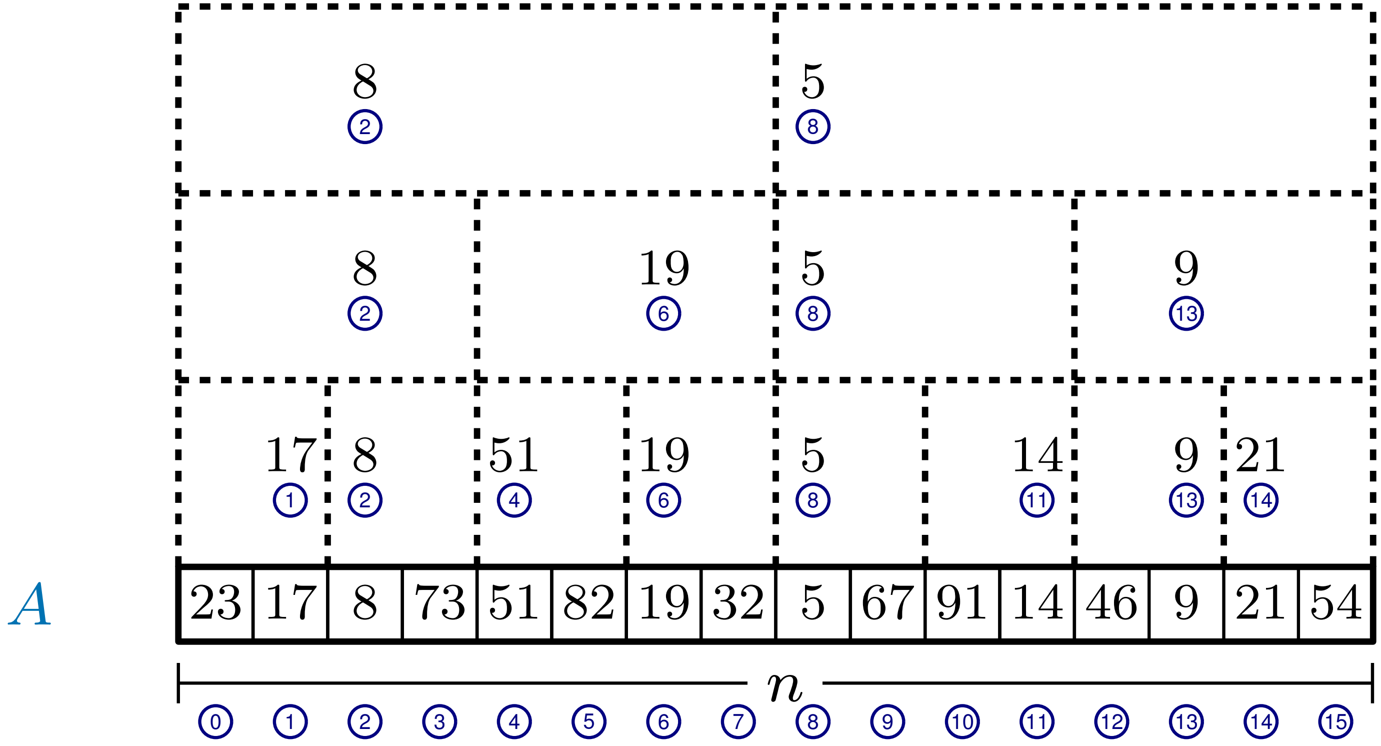

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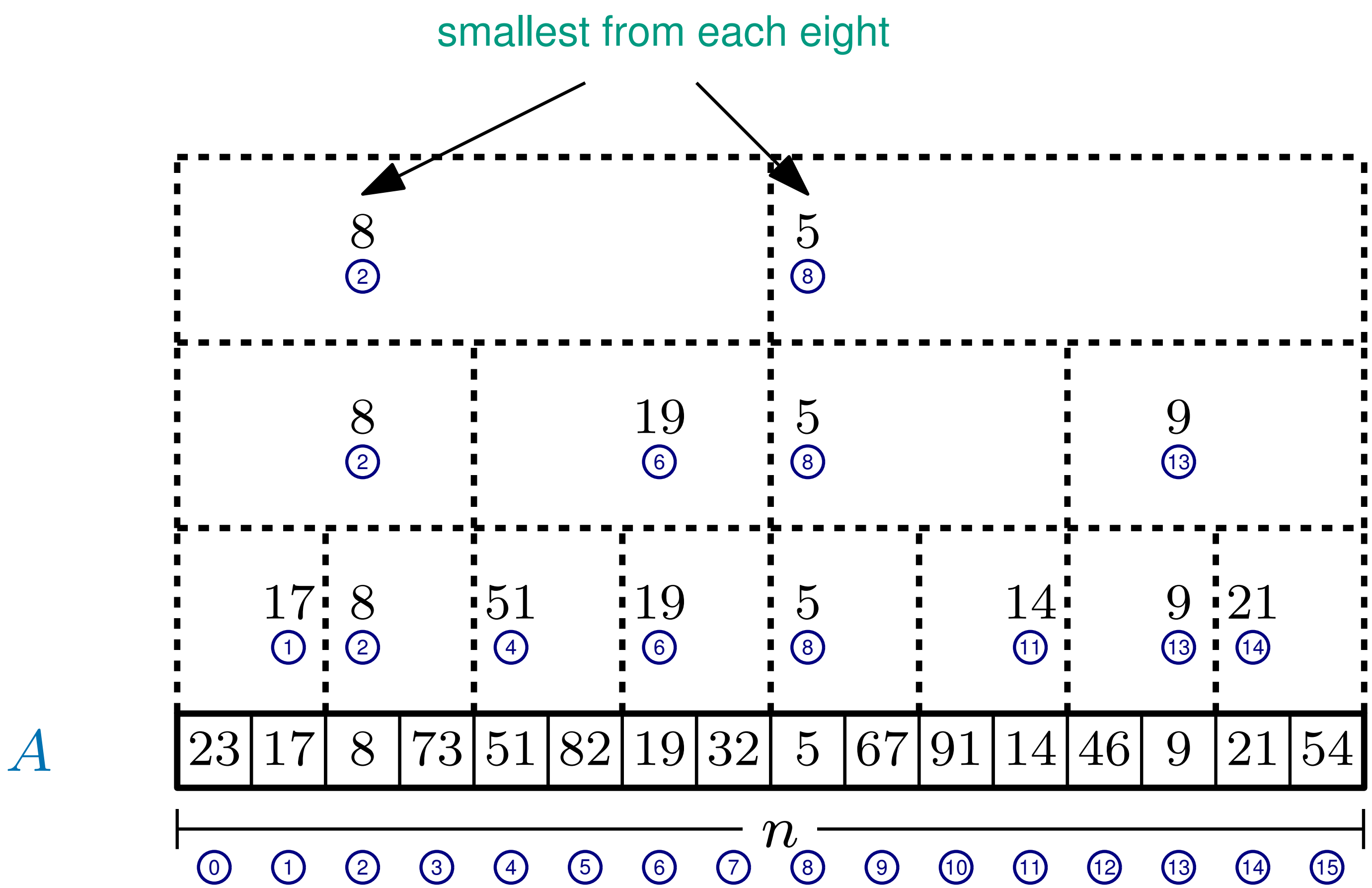

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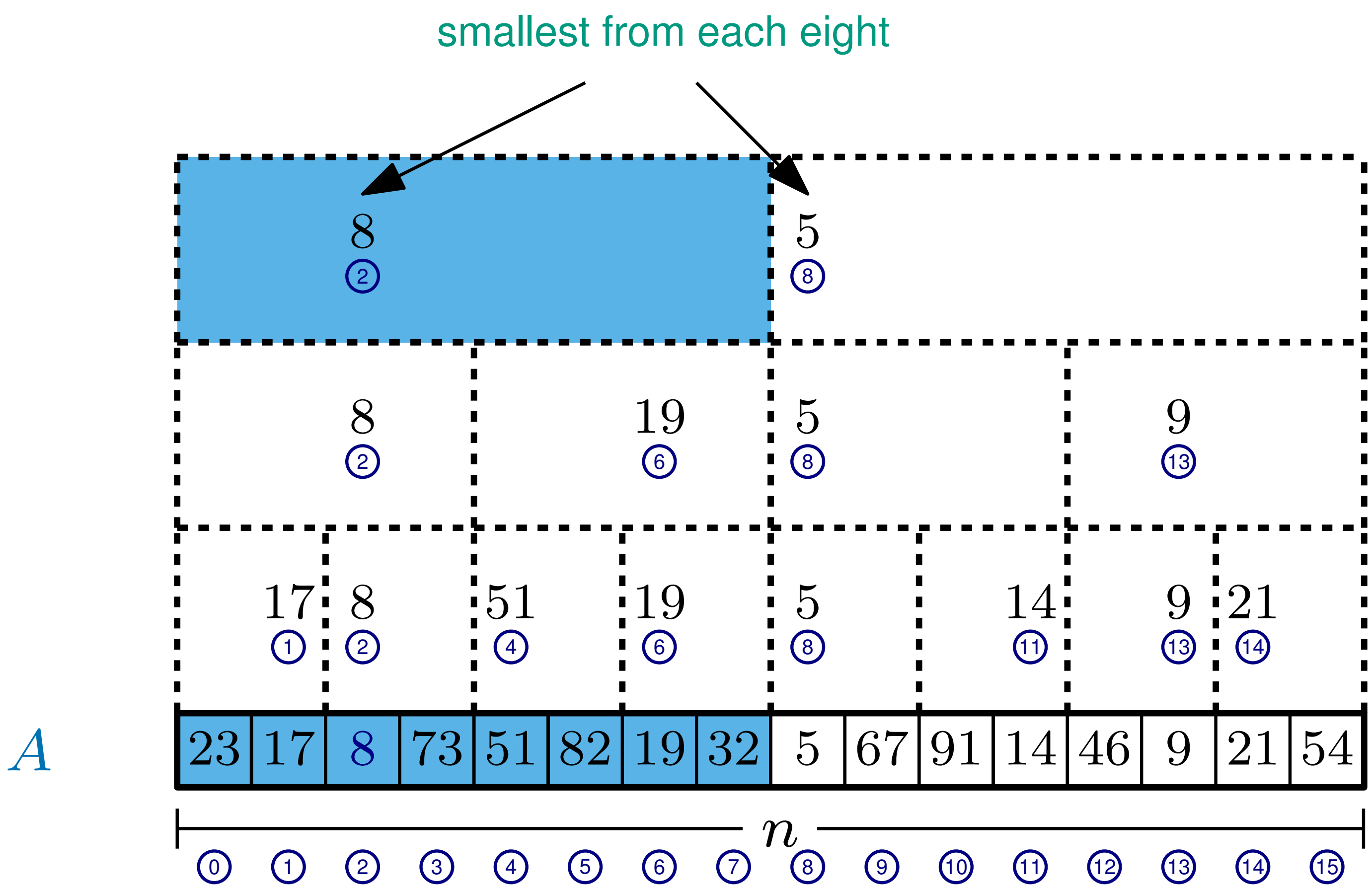

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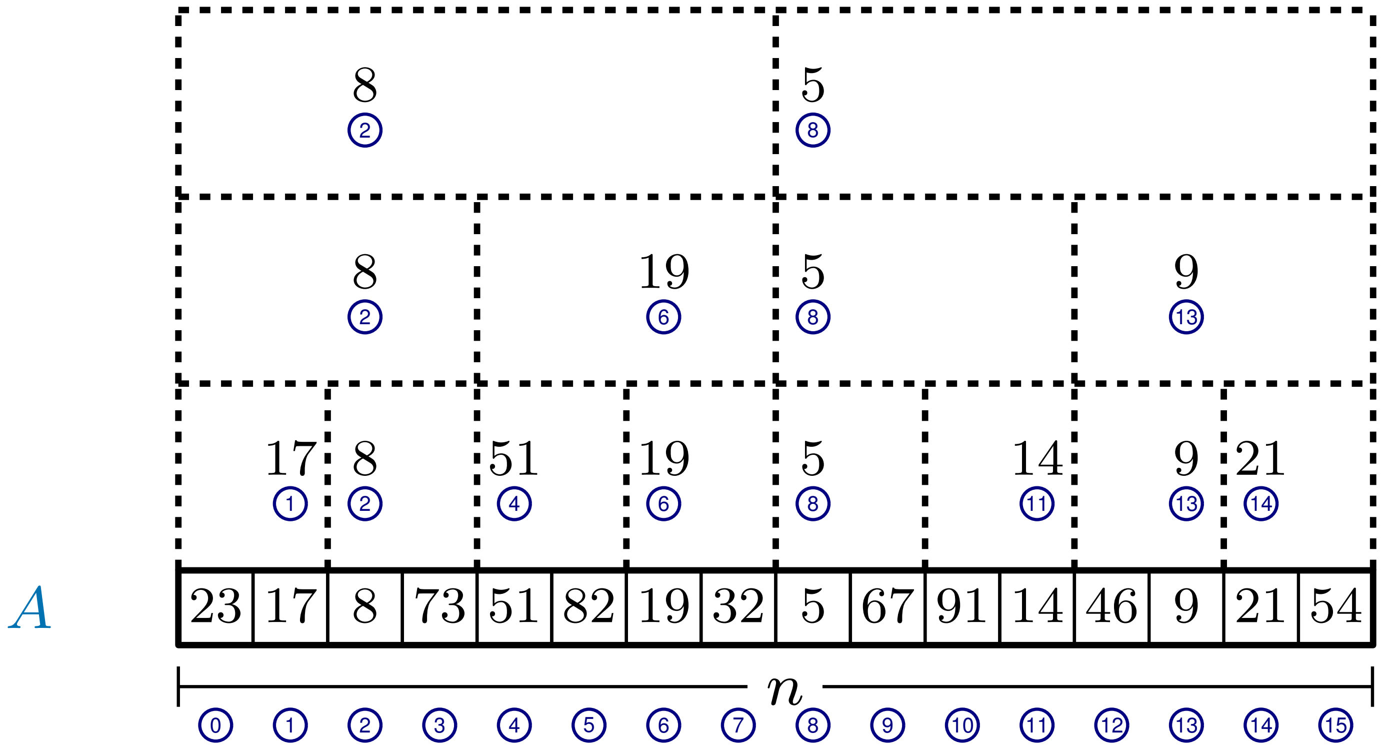

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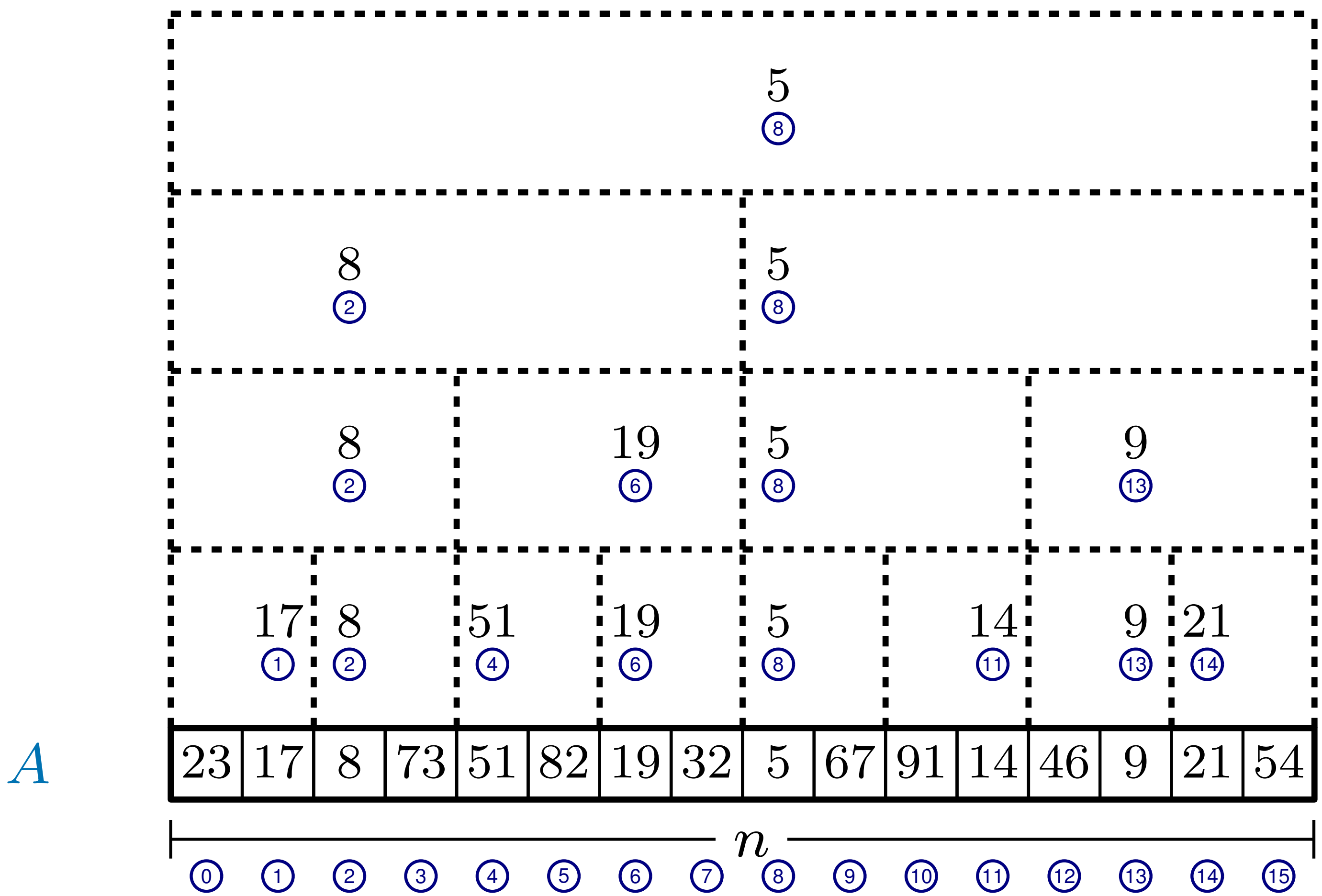

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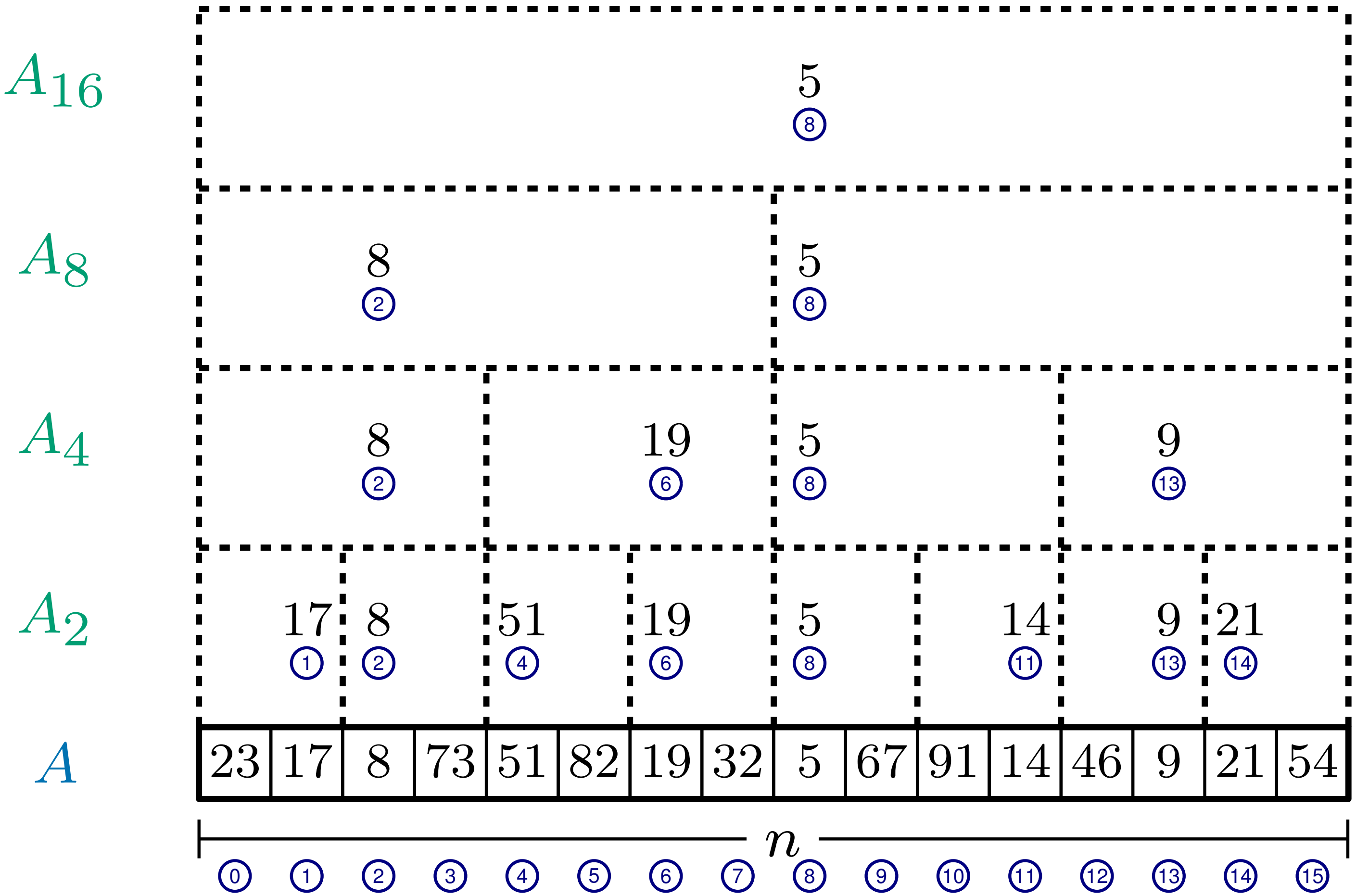

Block decomposition

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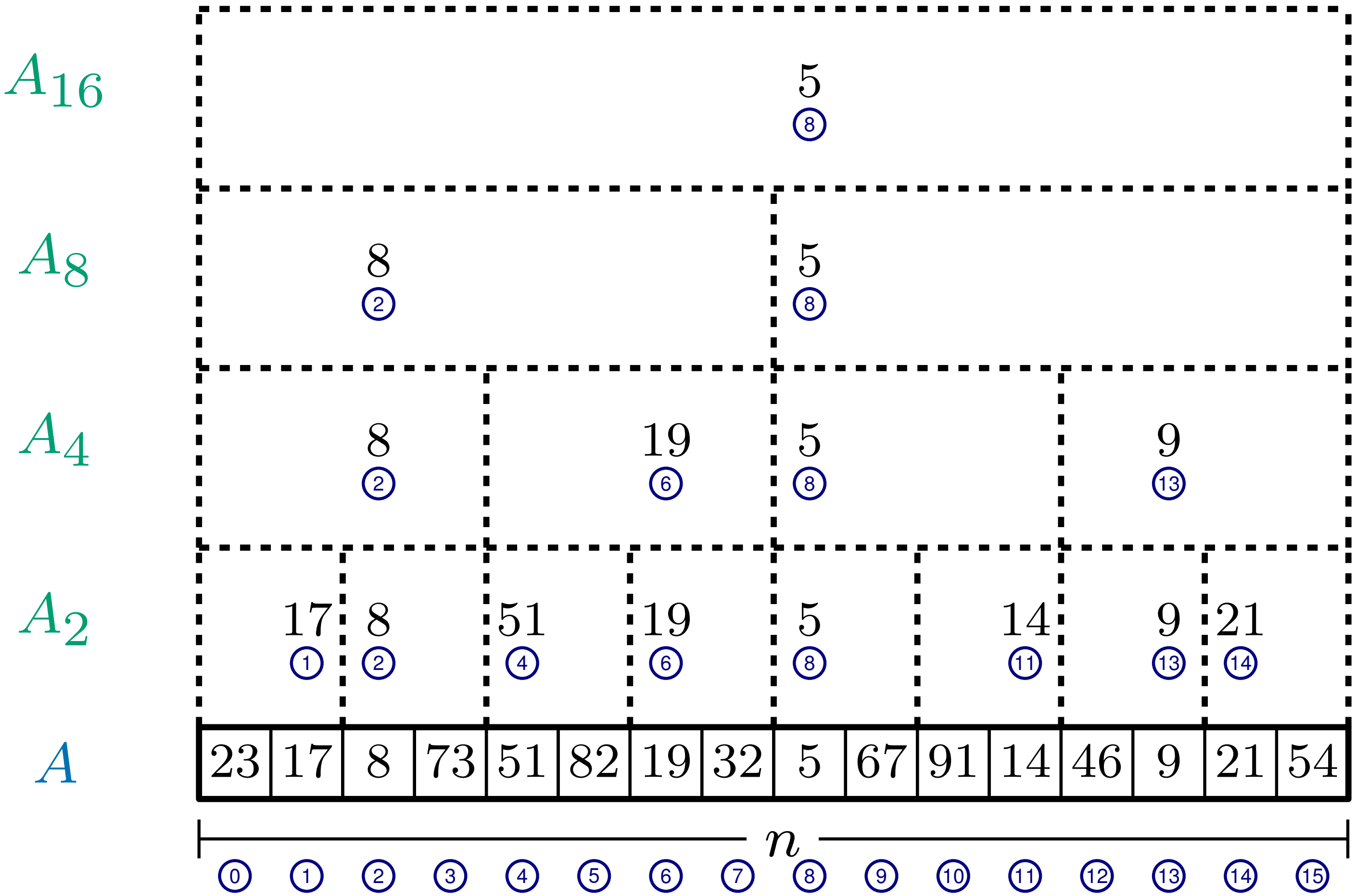
where v is the minimum in $A[ik, (i+1)k]$ and x is its location in A.



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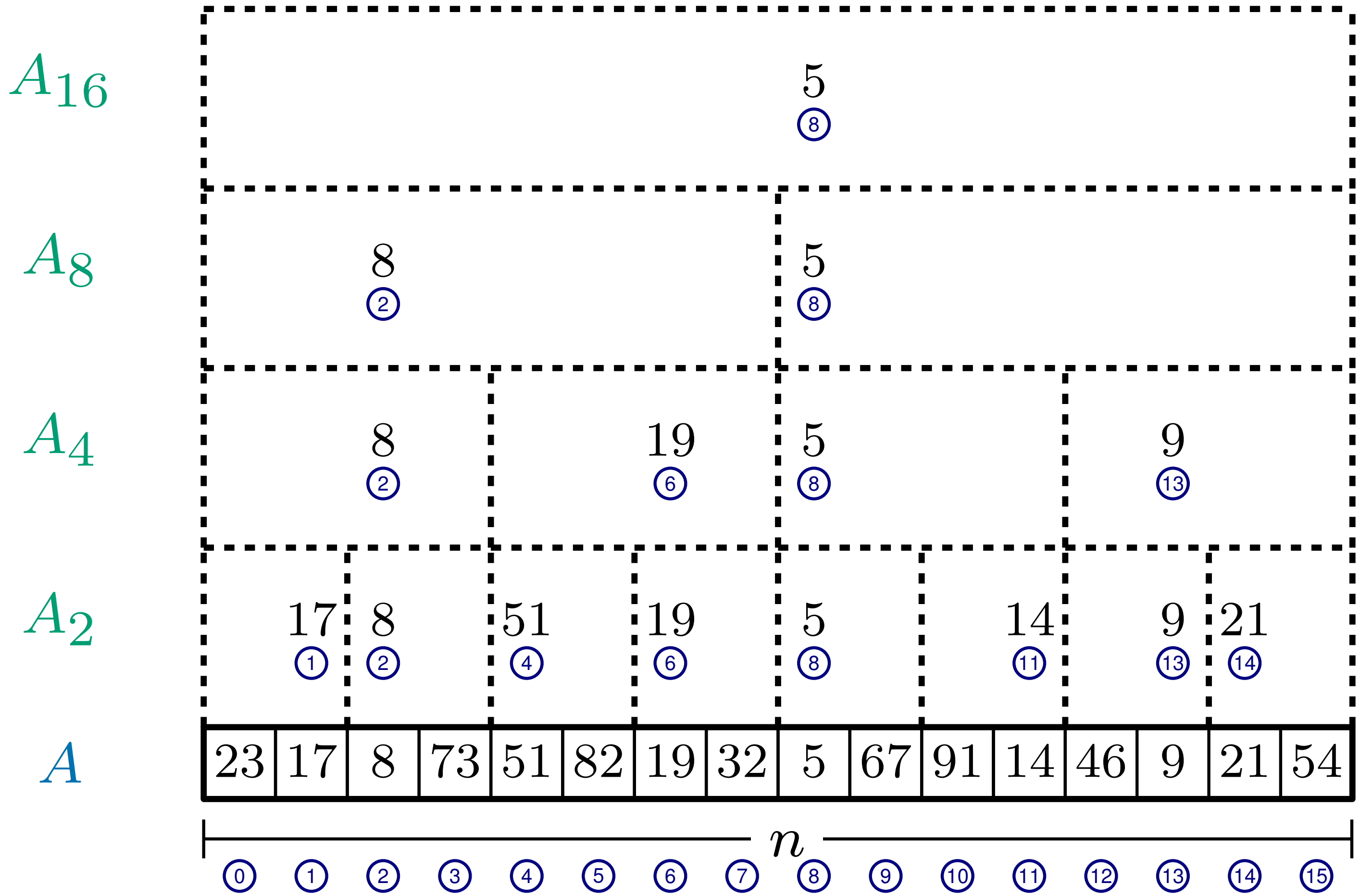

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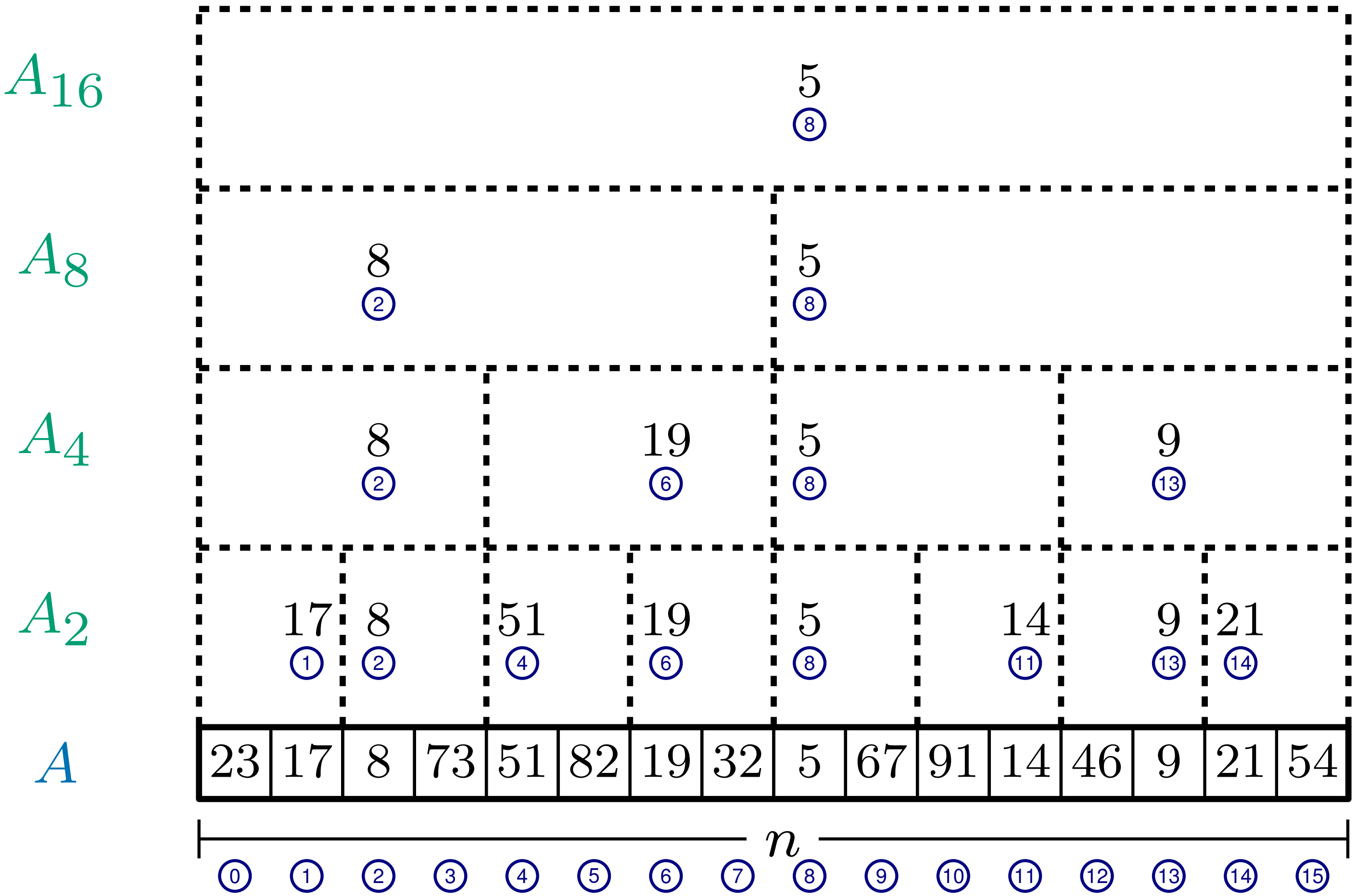

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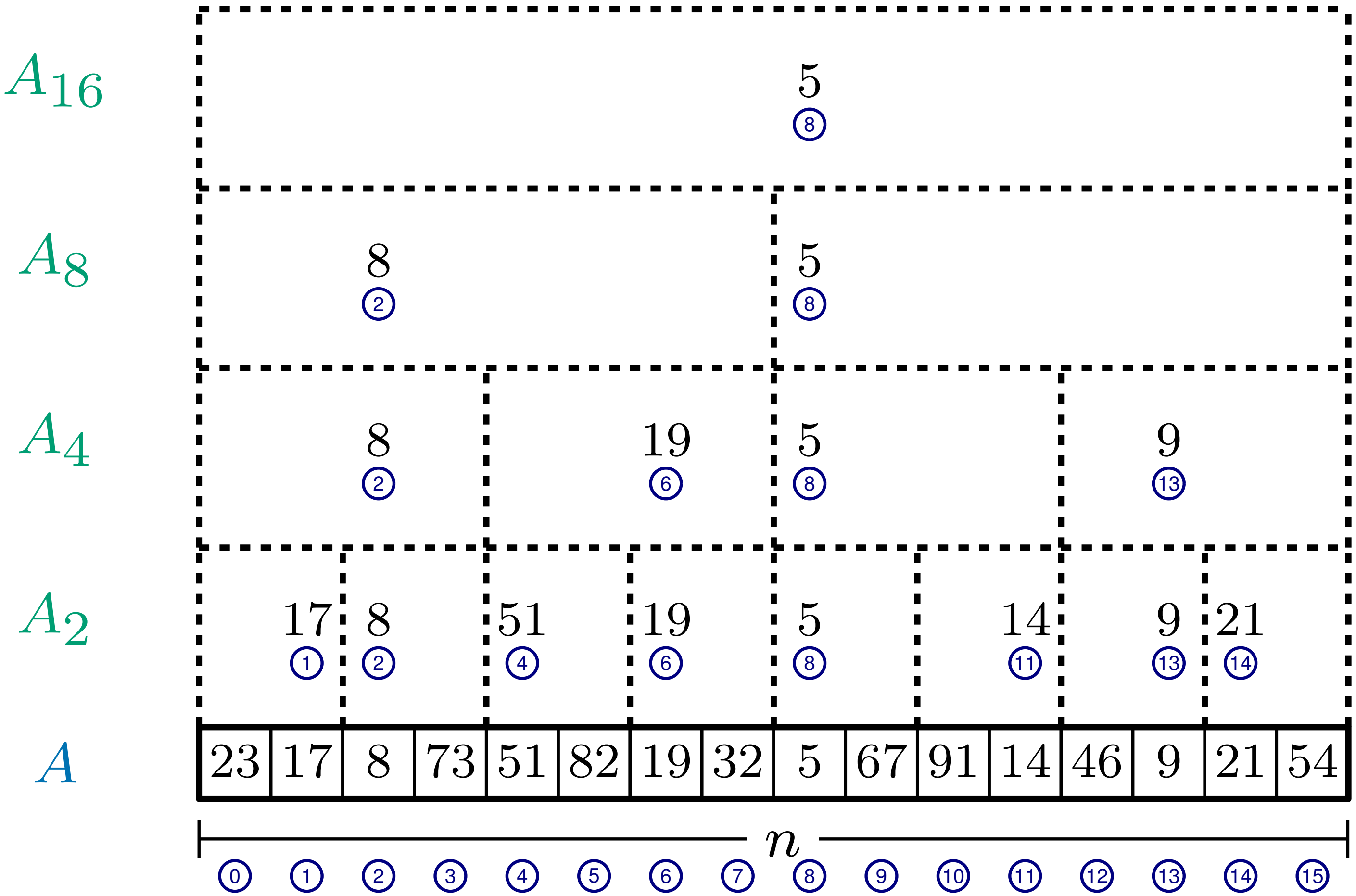

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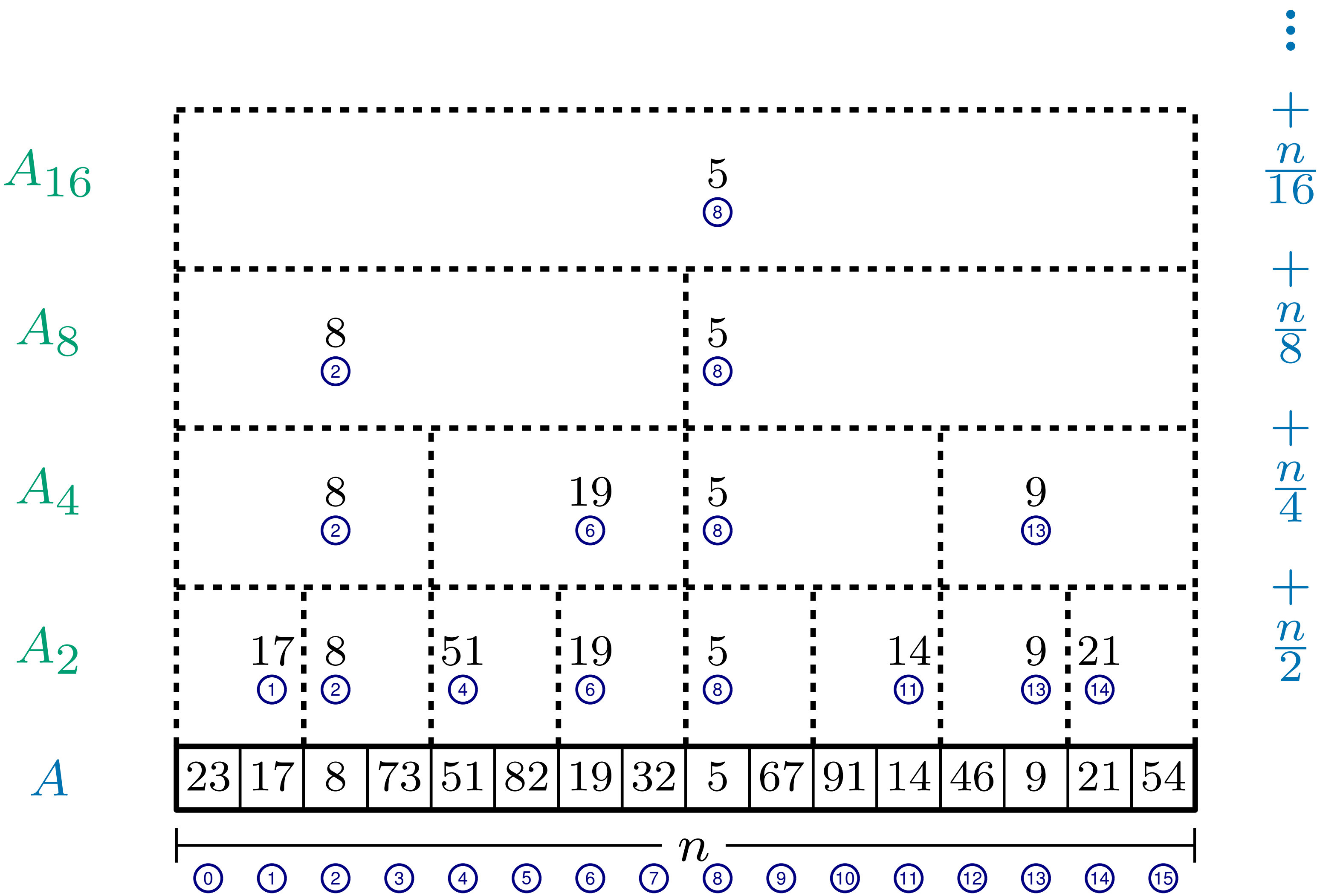

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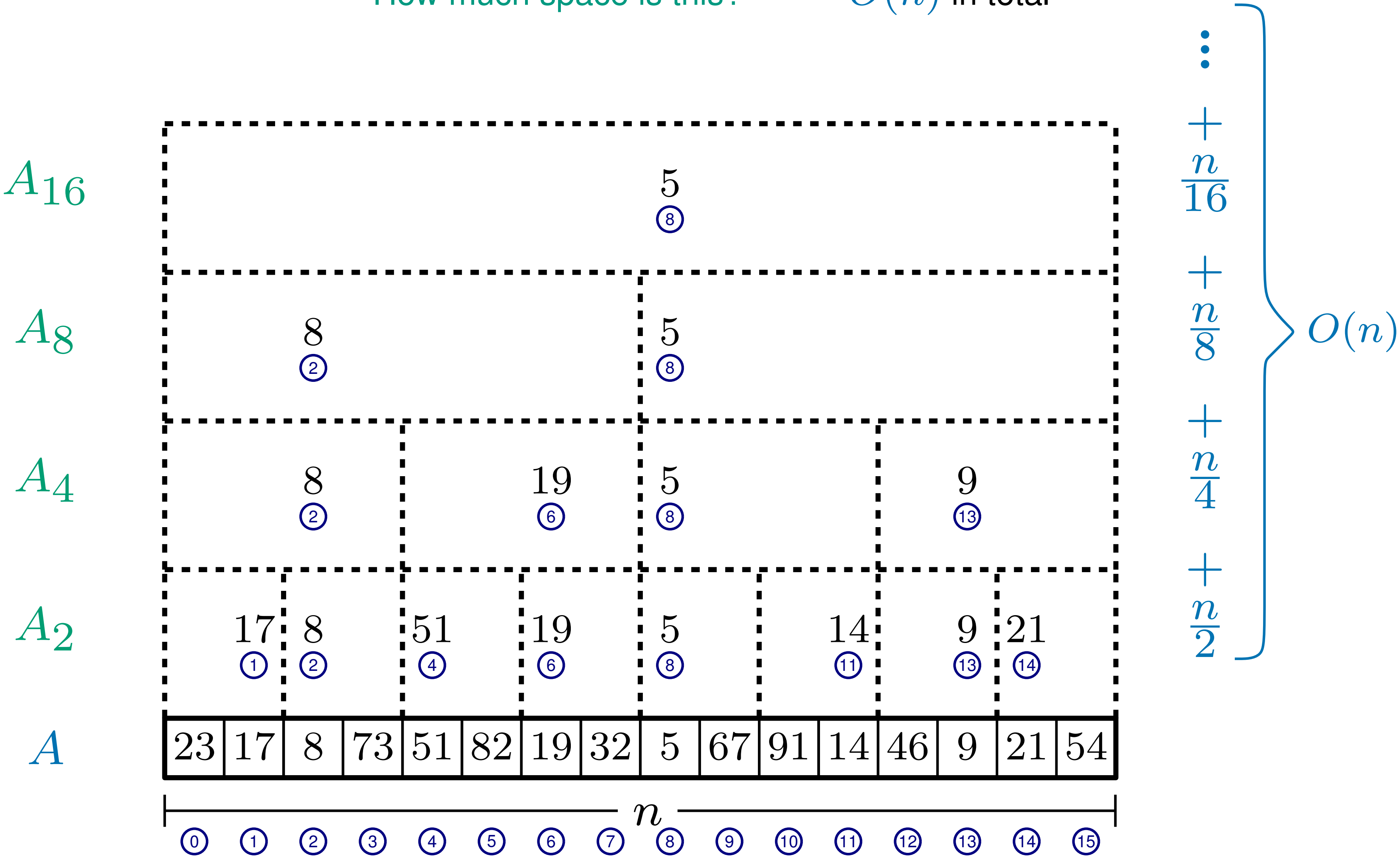

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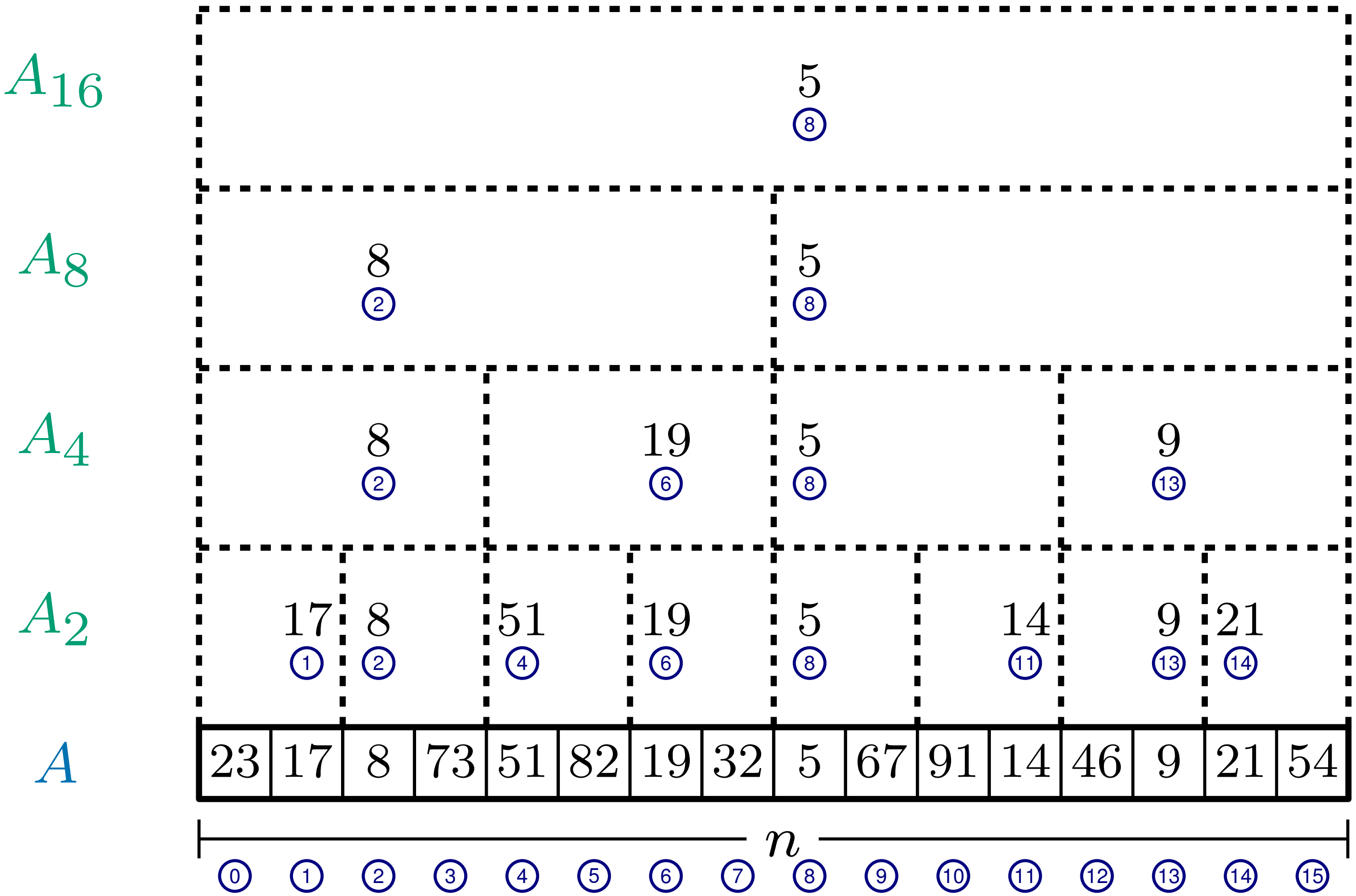

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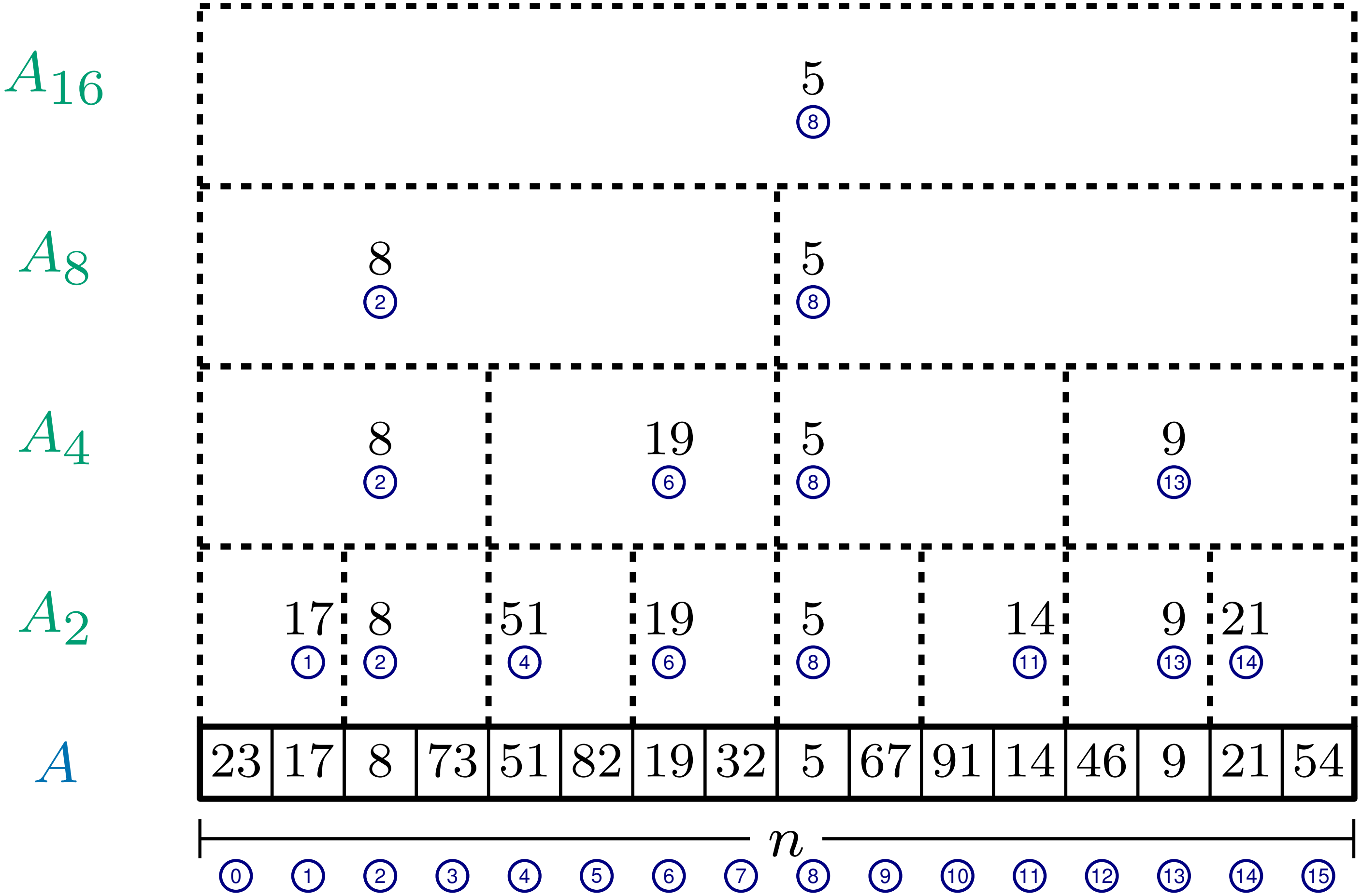

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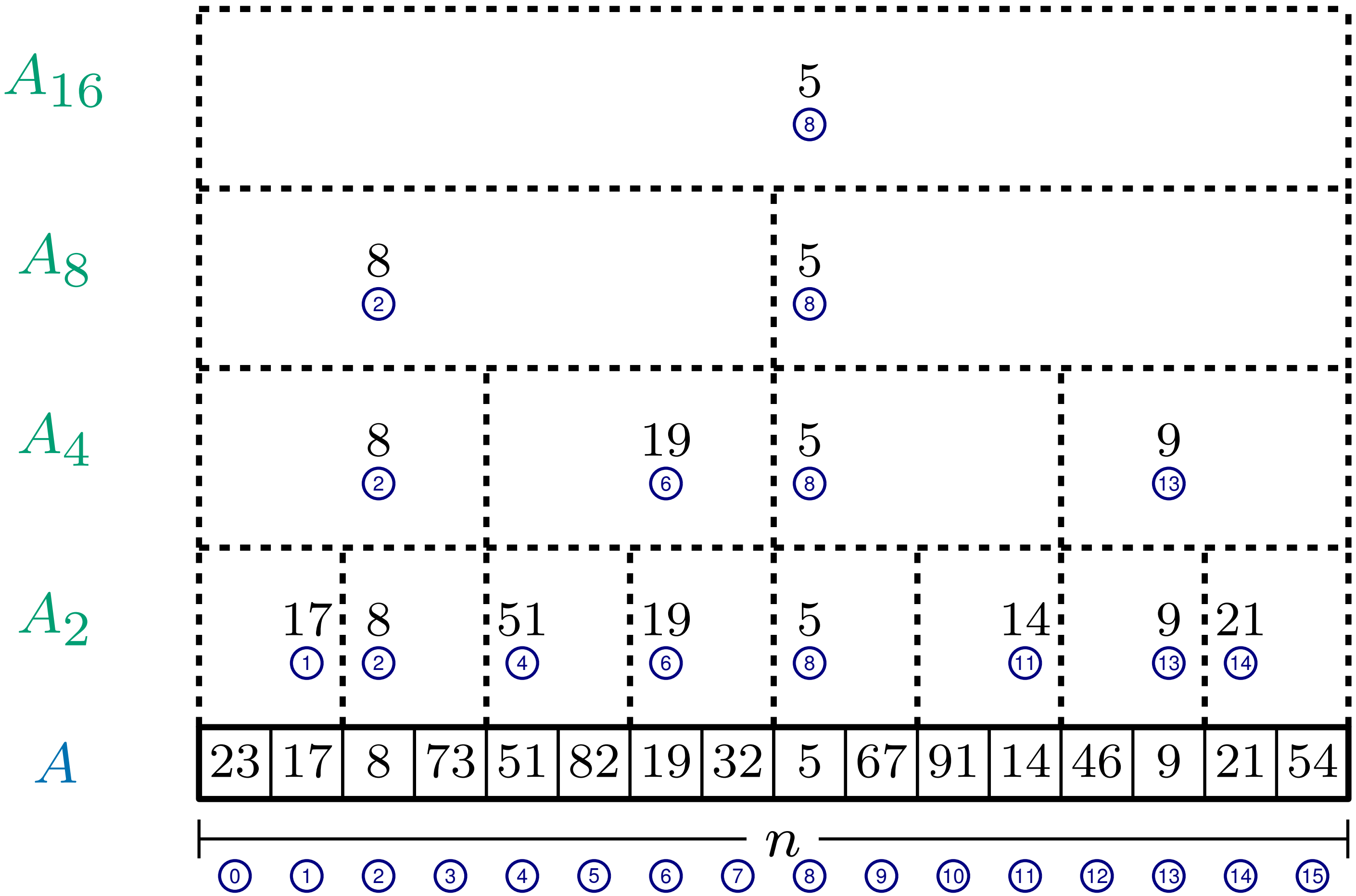

construct the A_k arrays bottom-up

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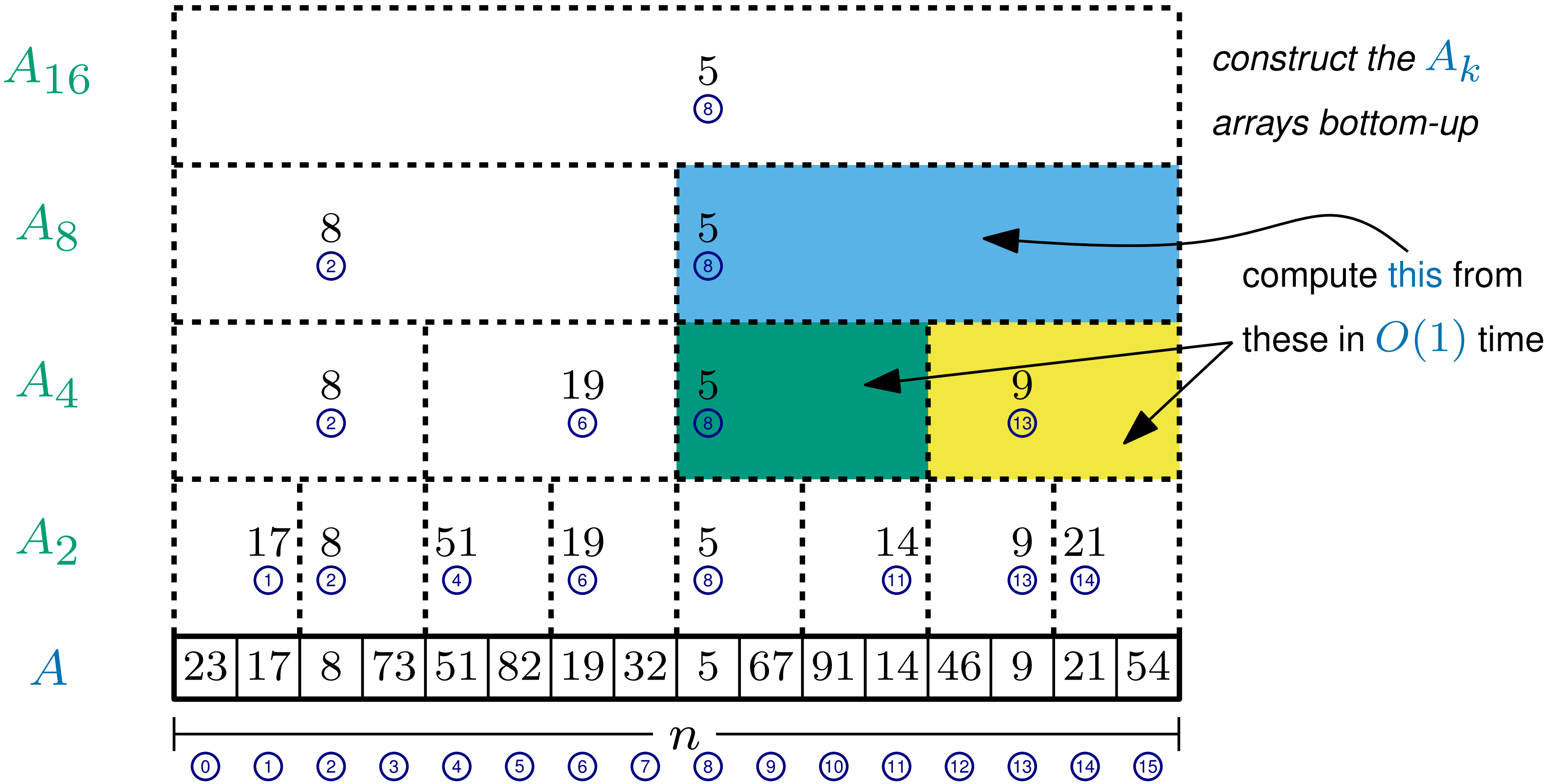

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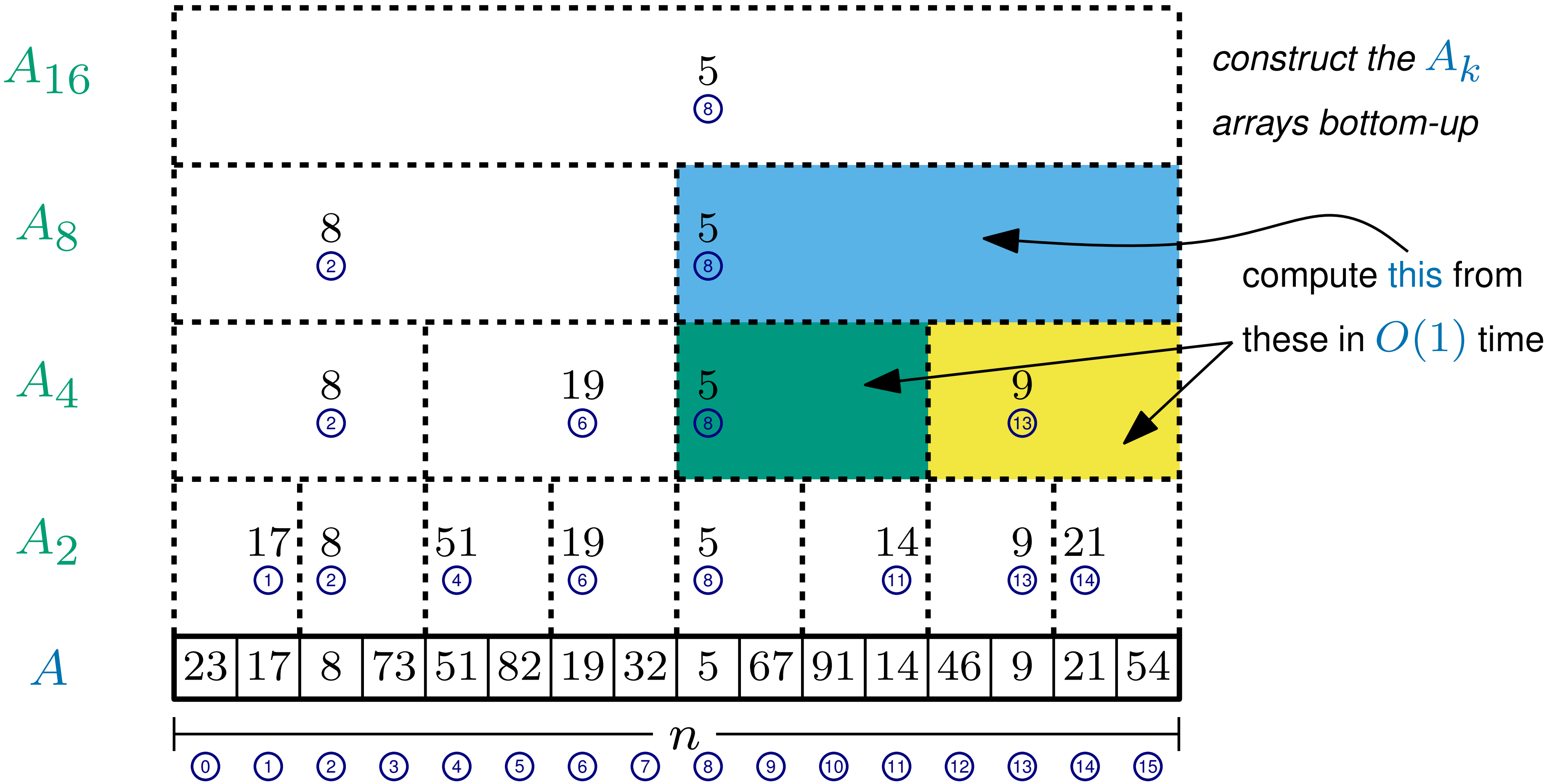

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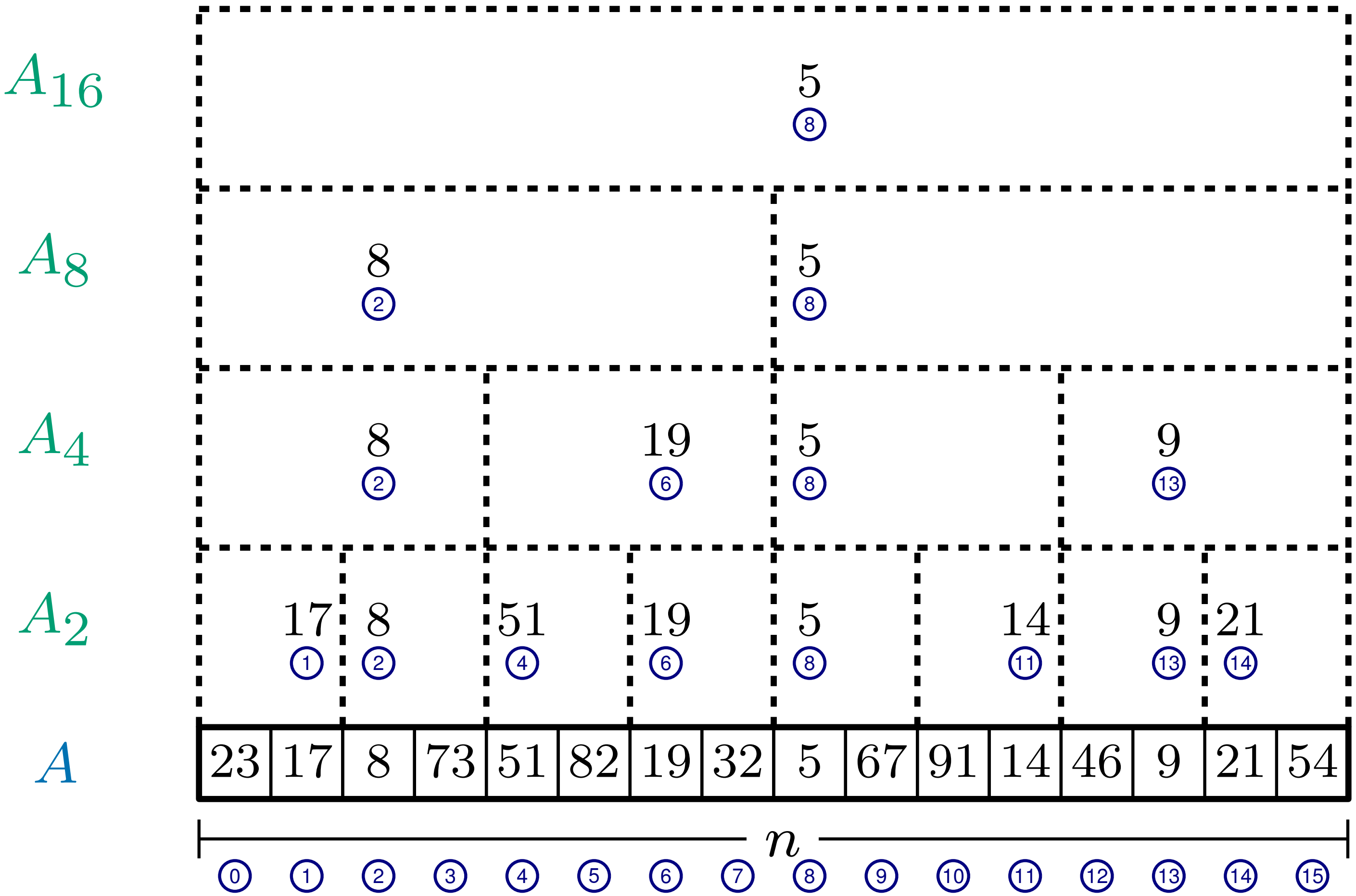

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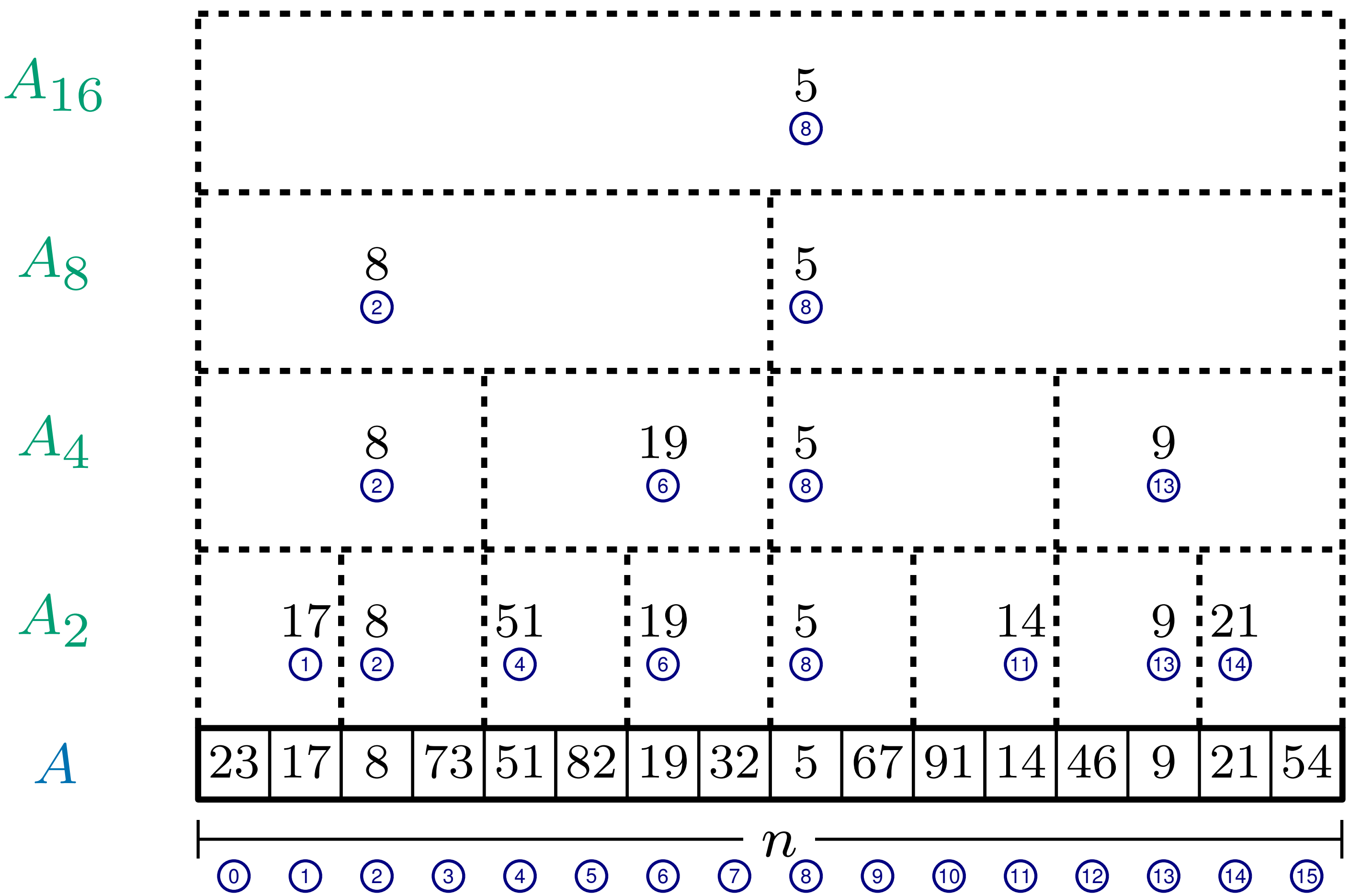

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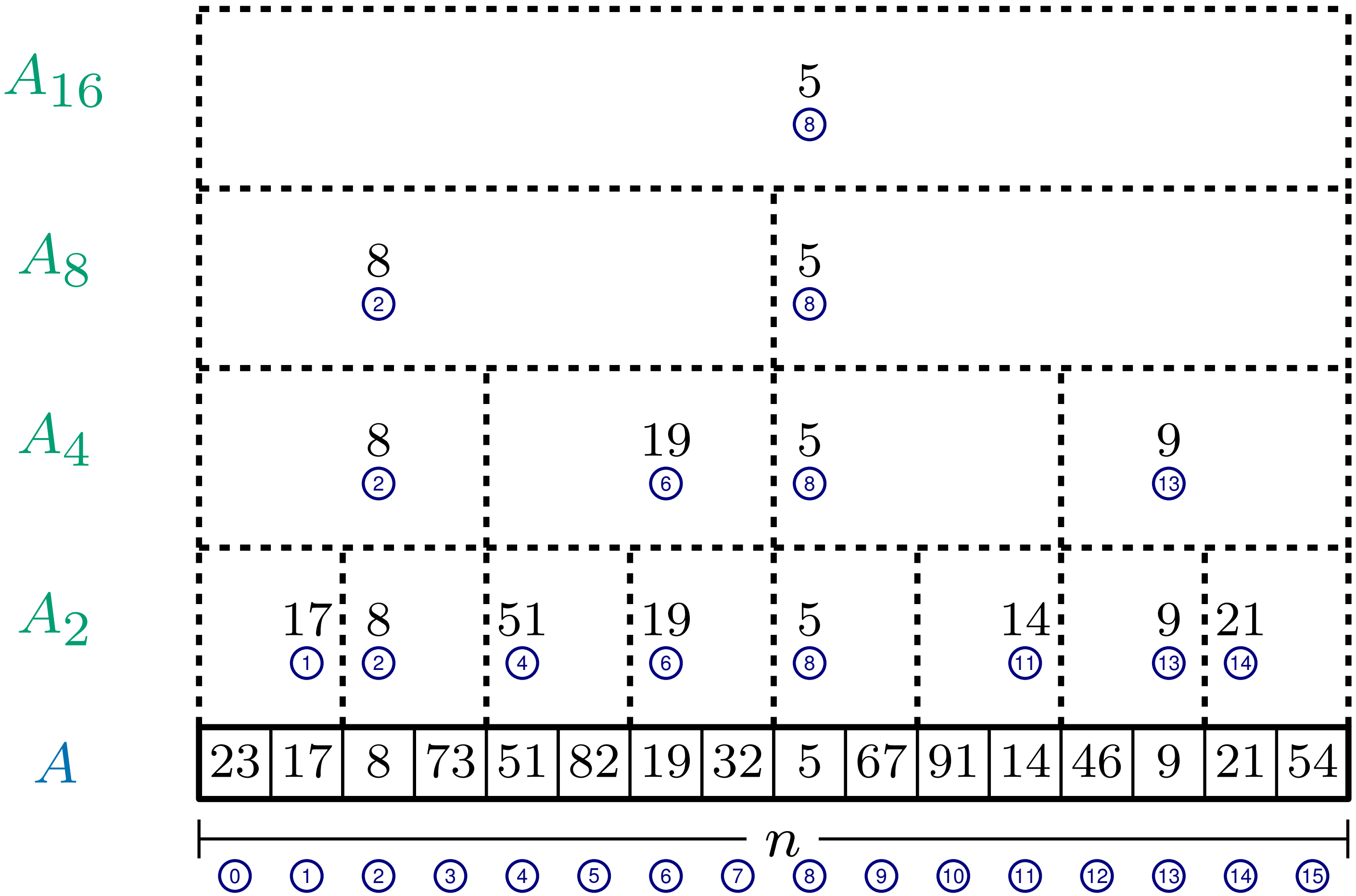

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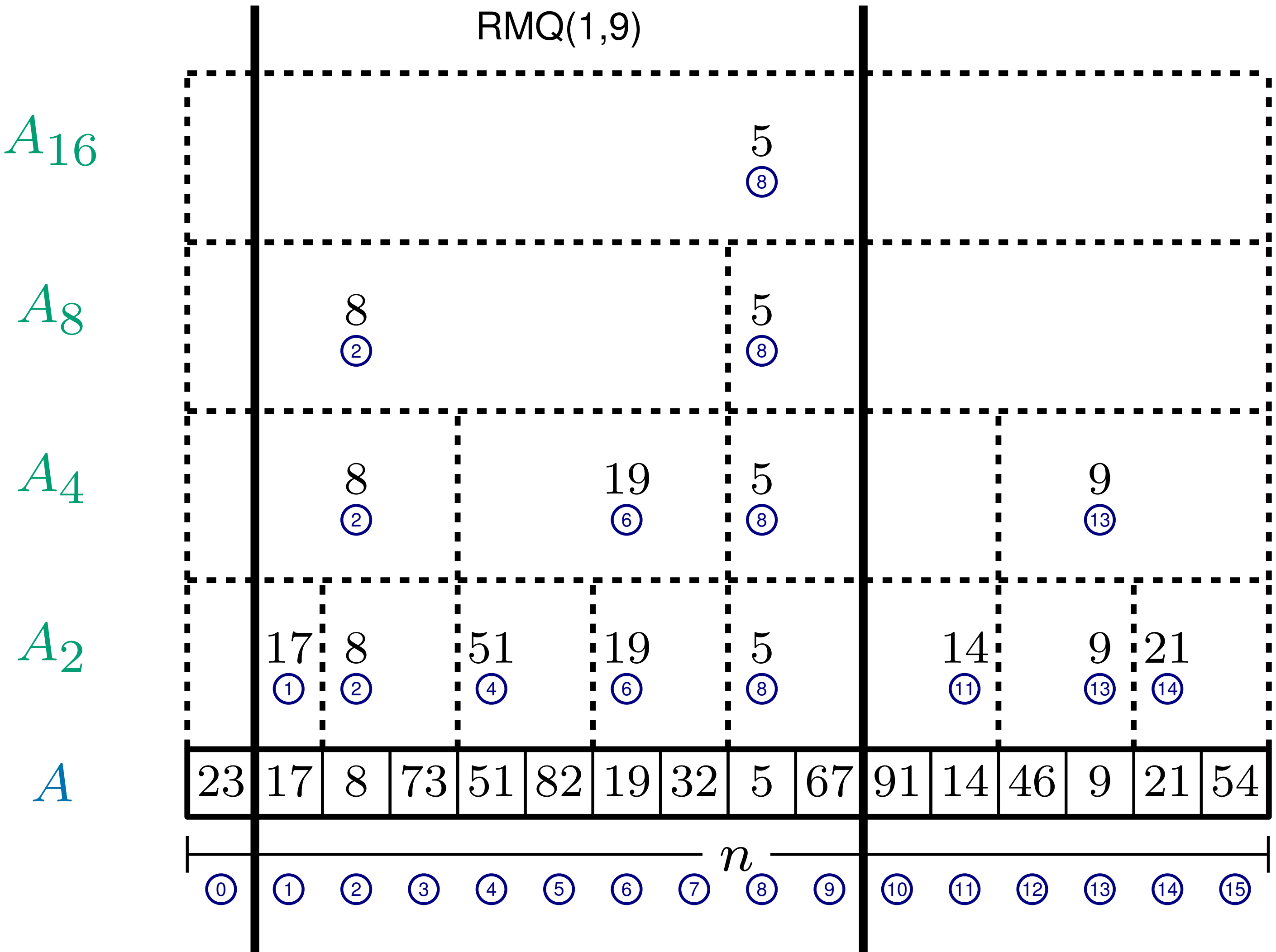

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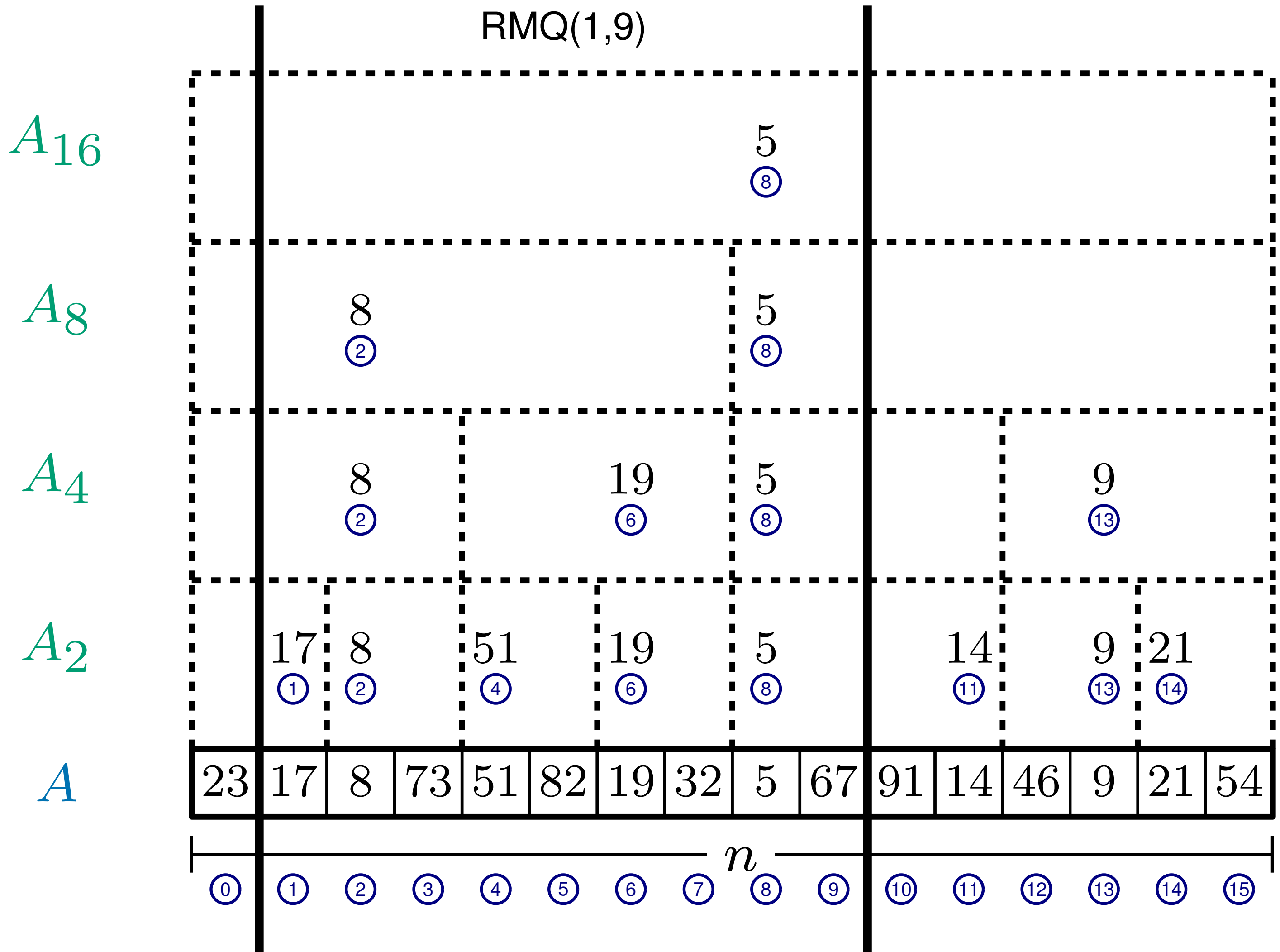

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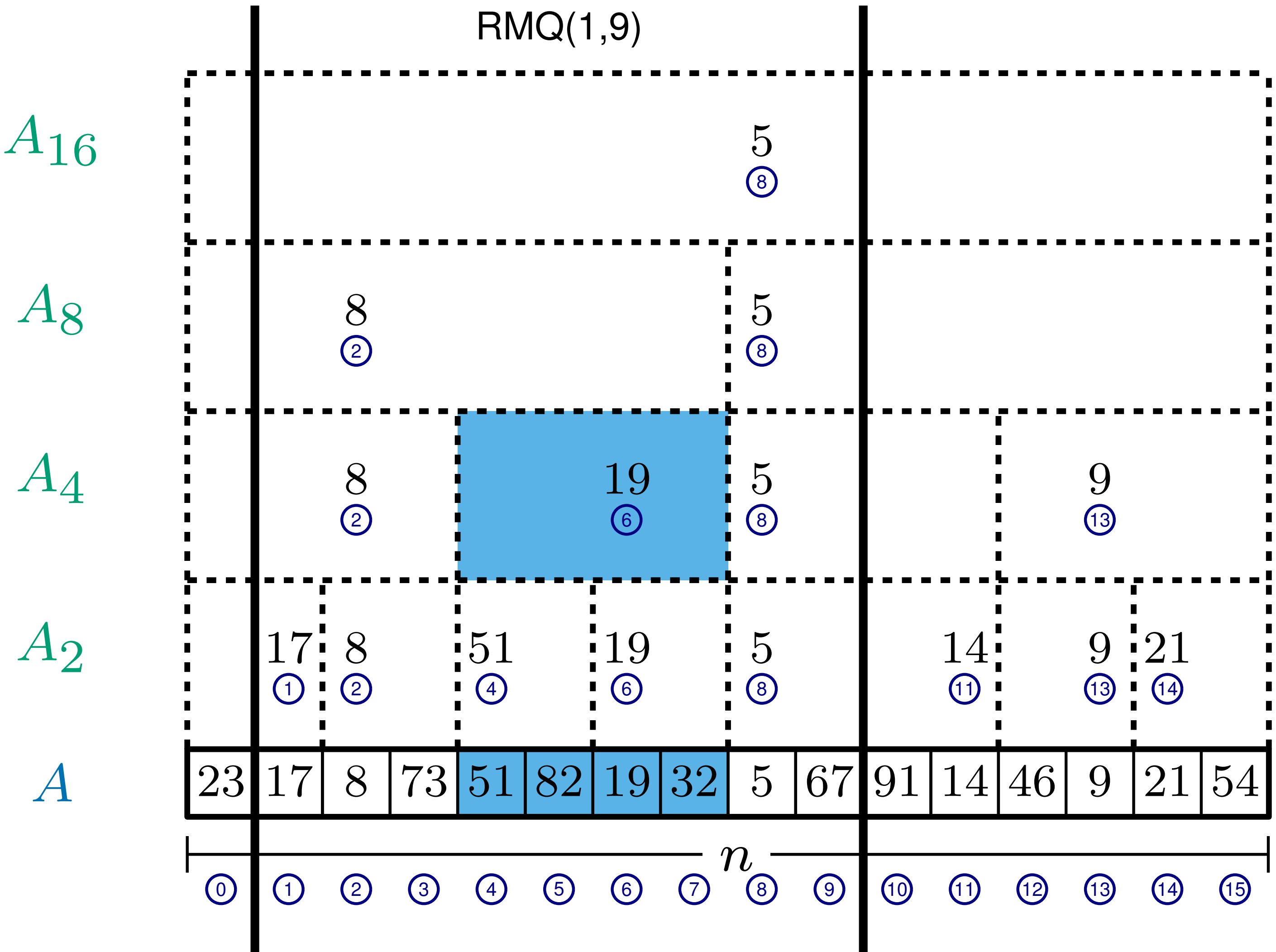

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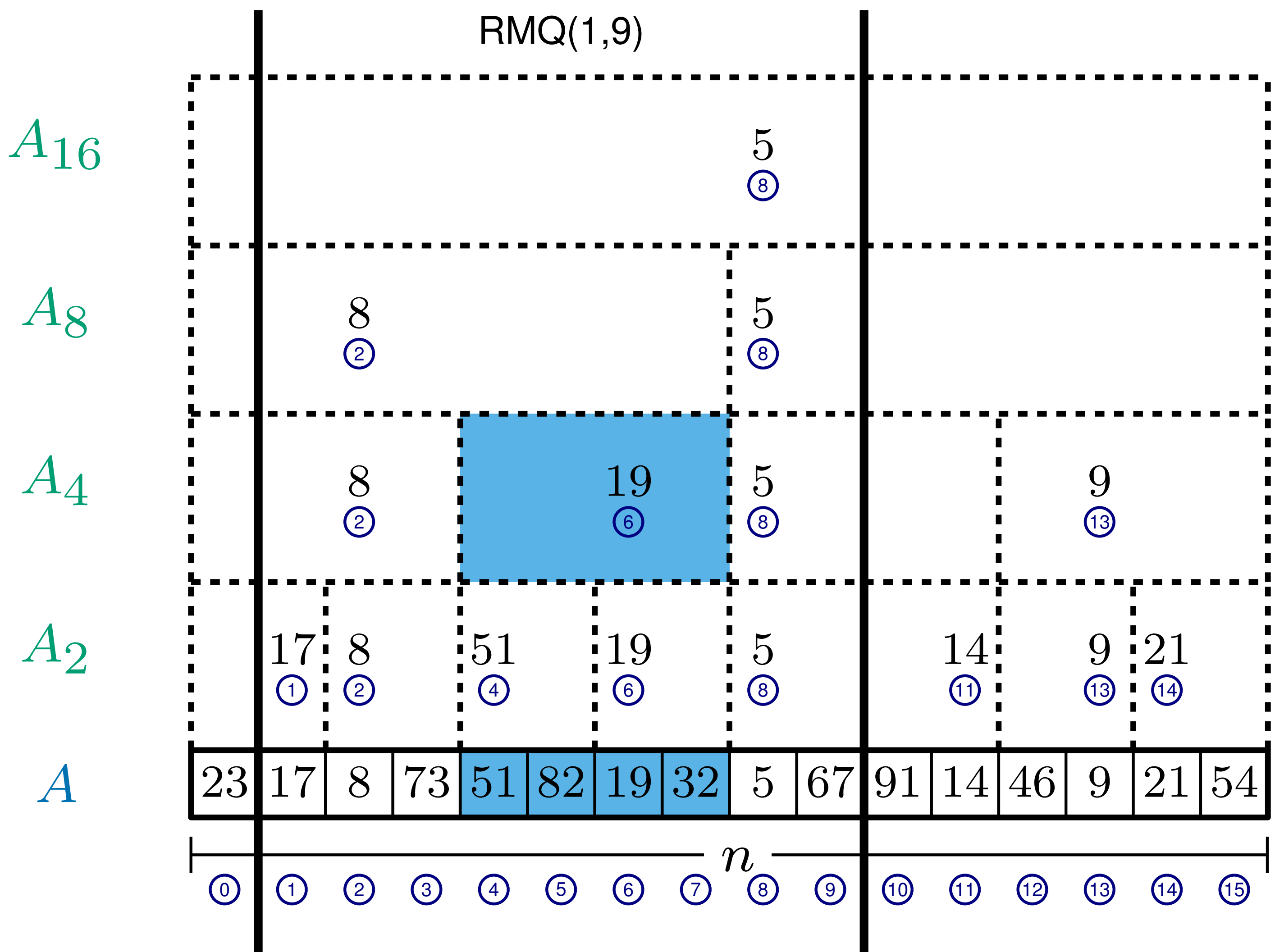

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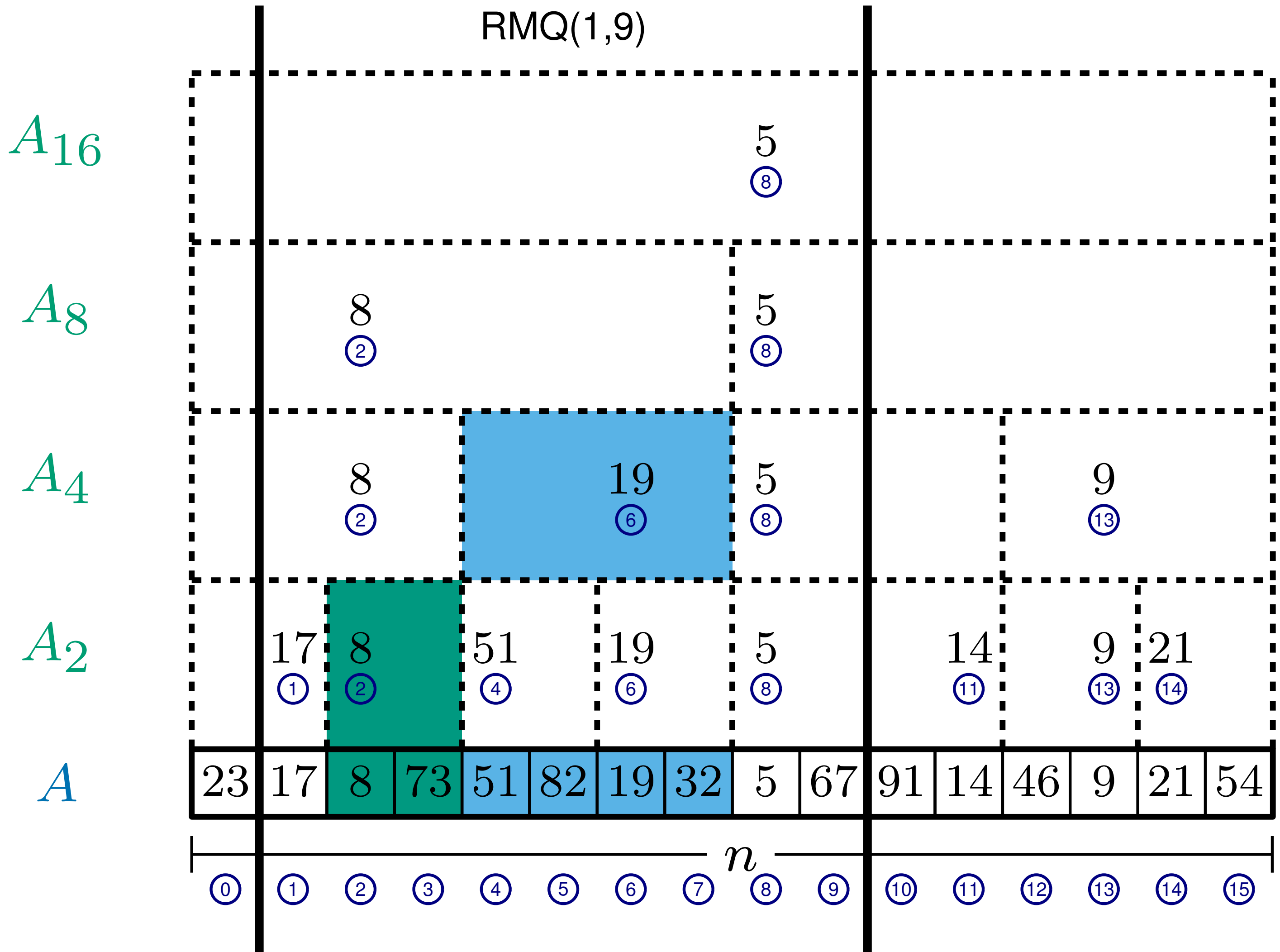

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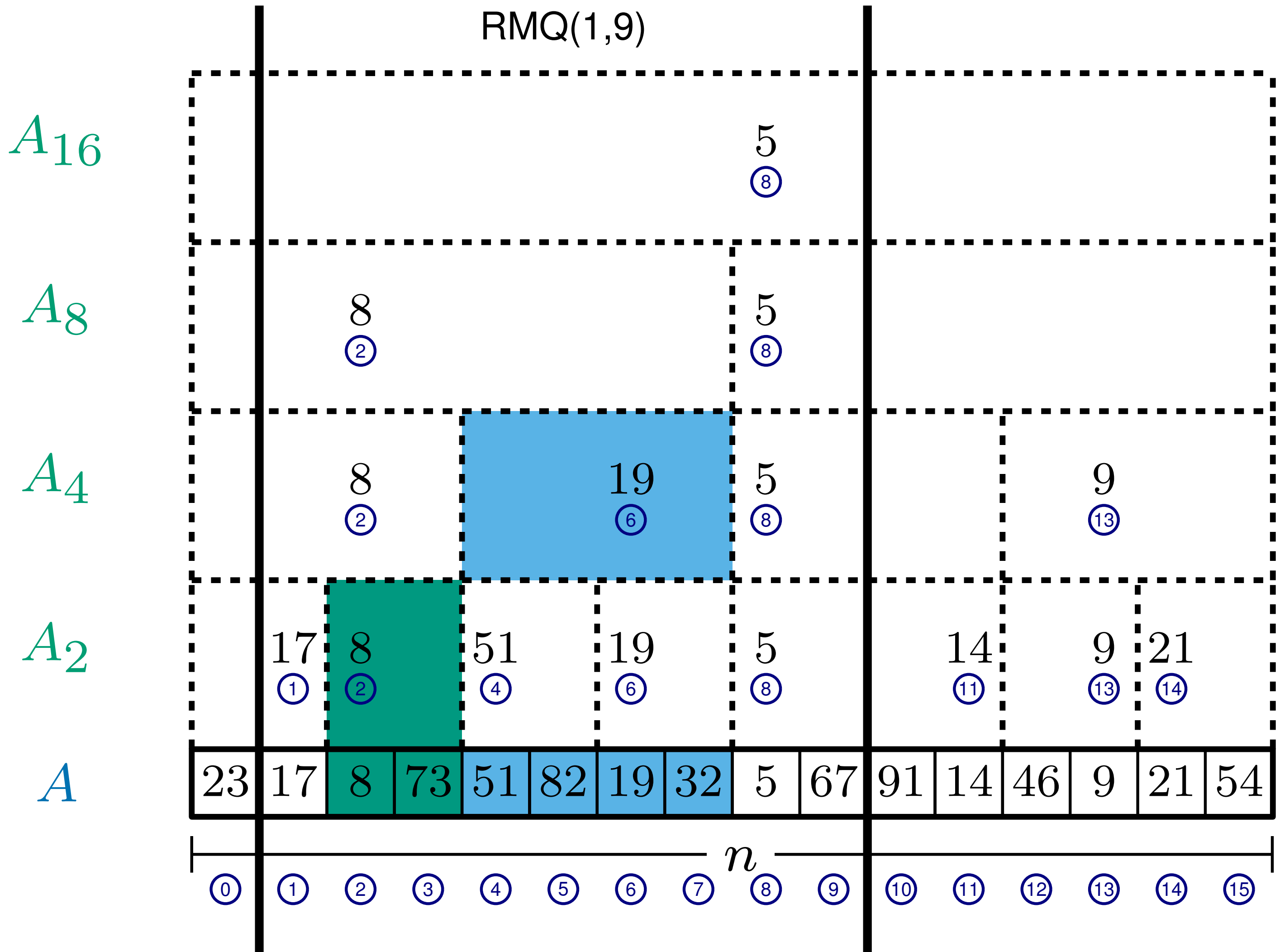
(break ties arbitrarily)



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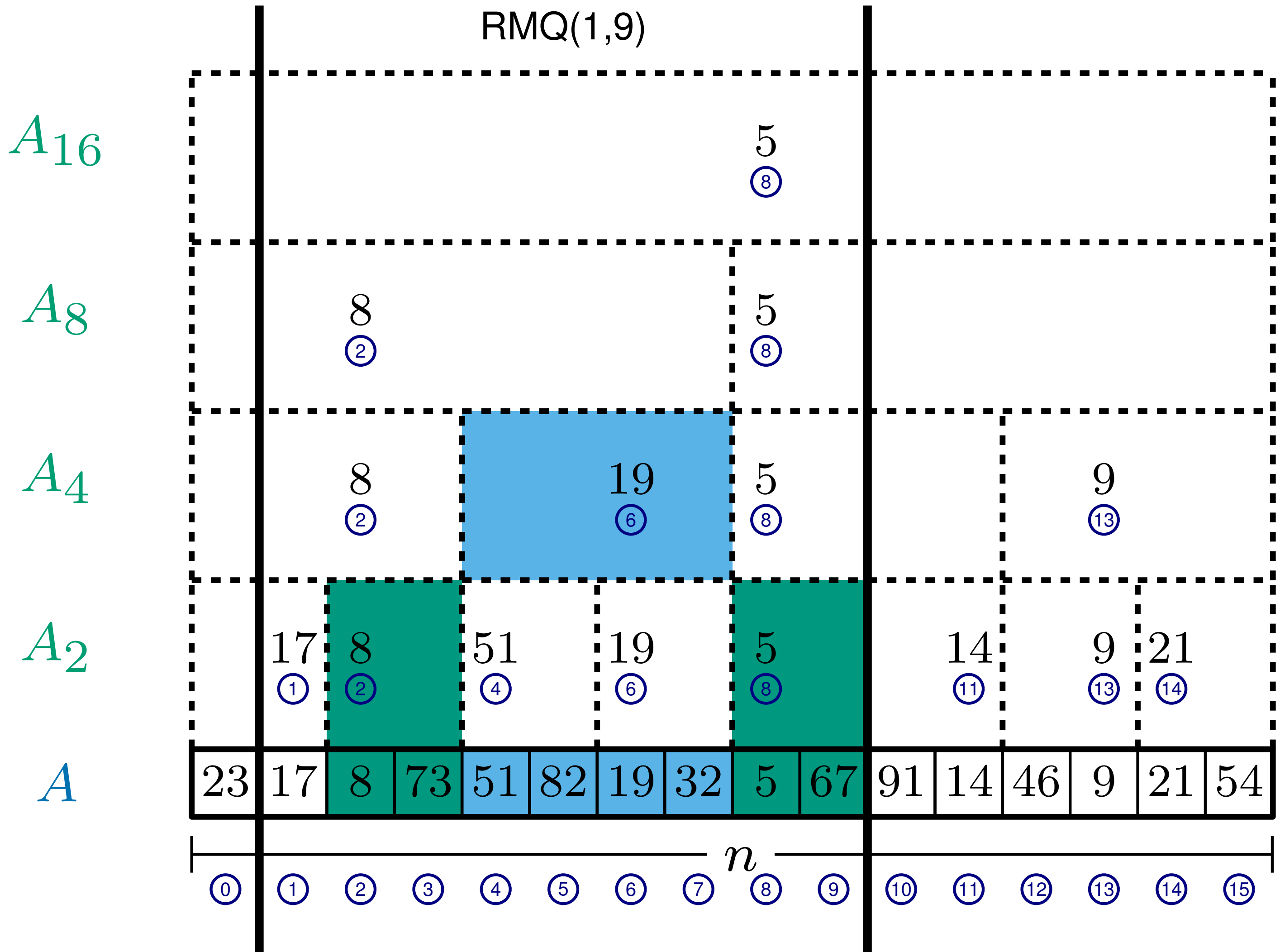

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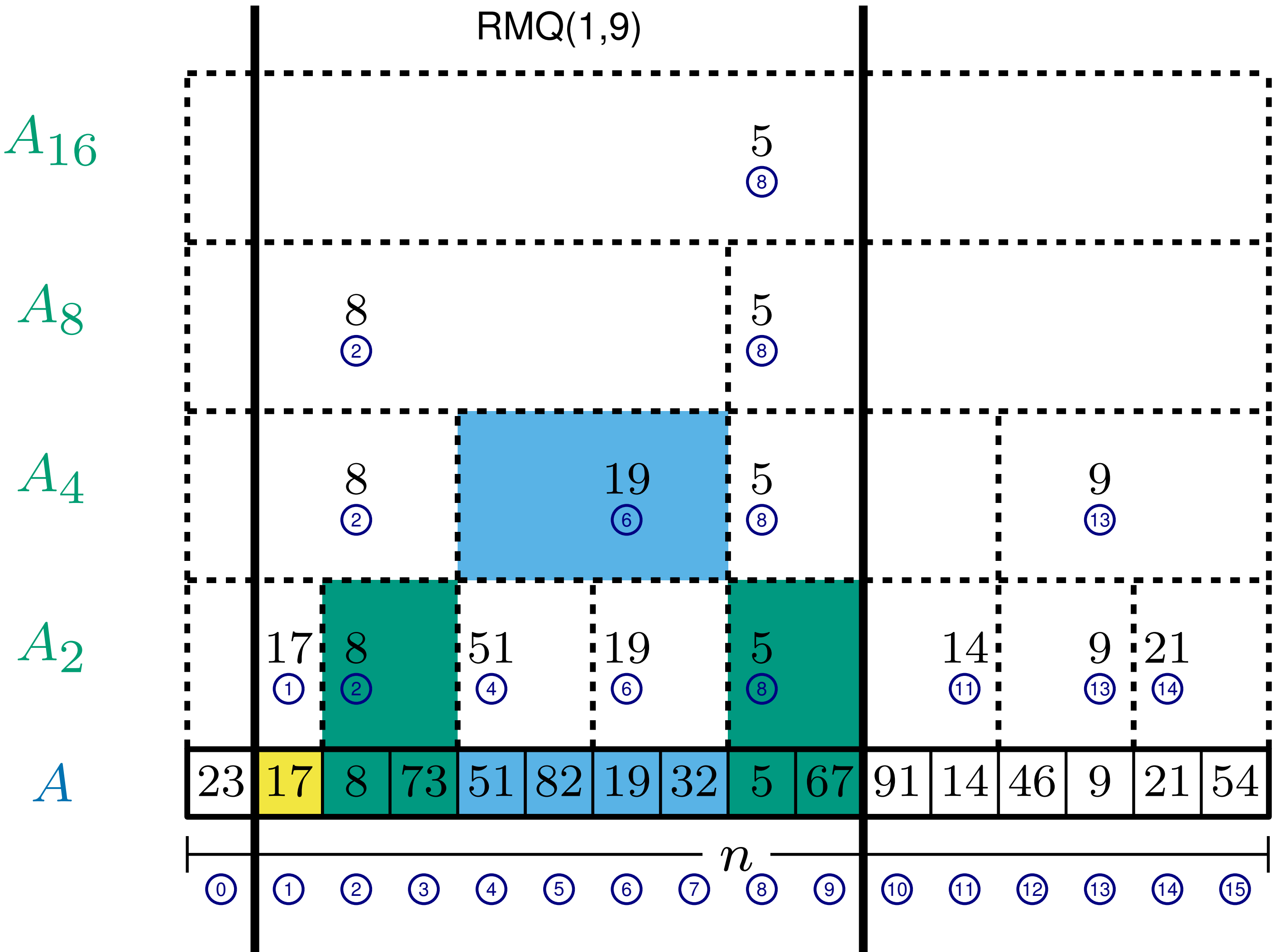

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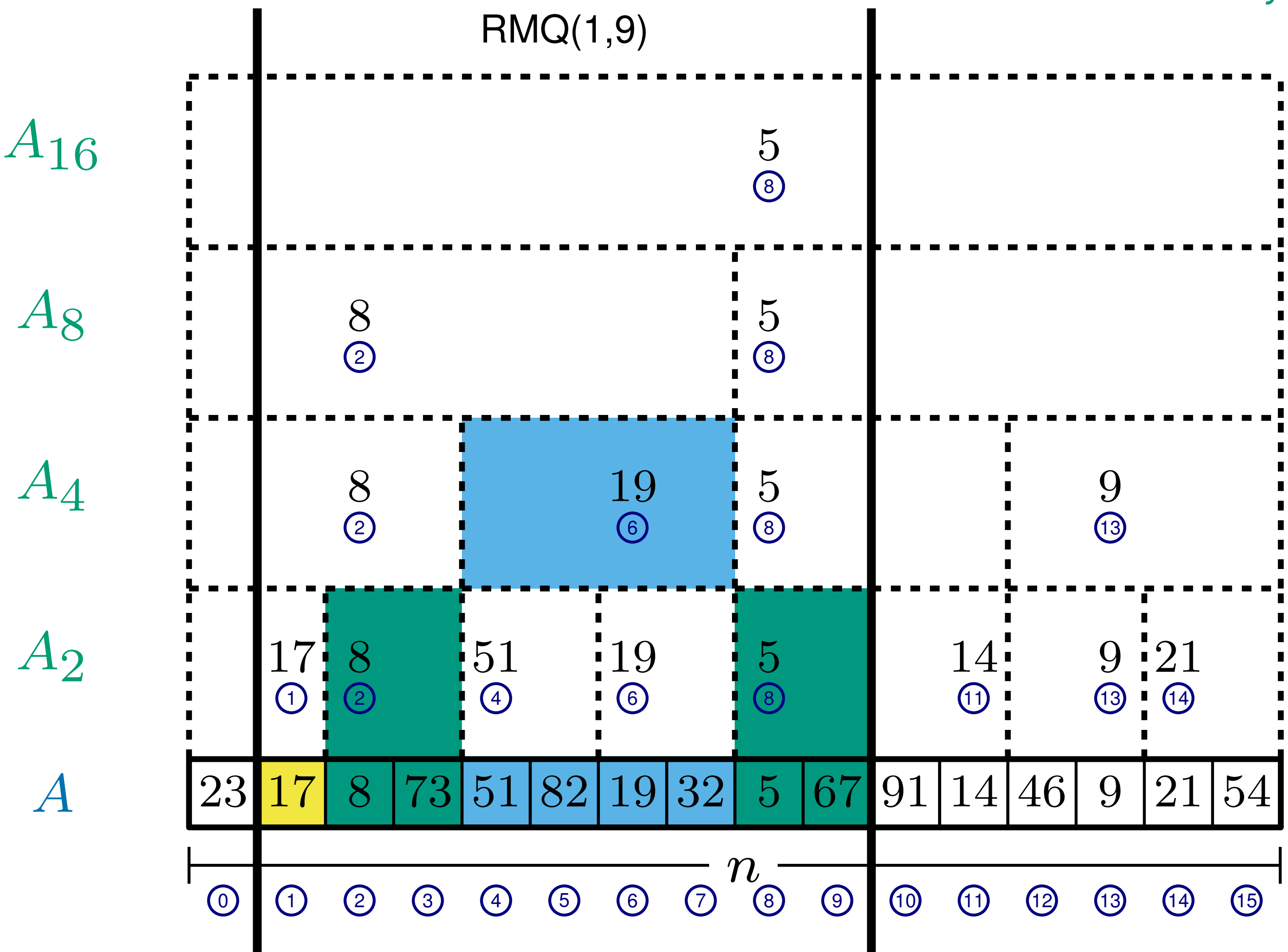
The minimum is the smallest in all these blocks

because they cover the query



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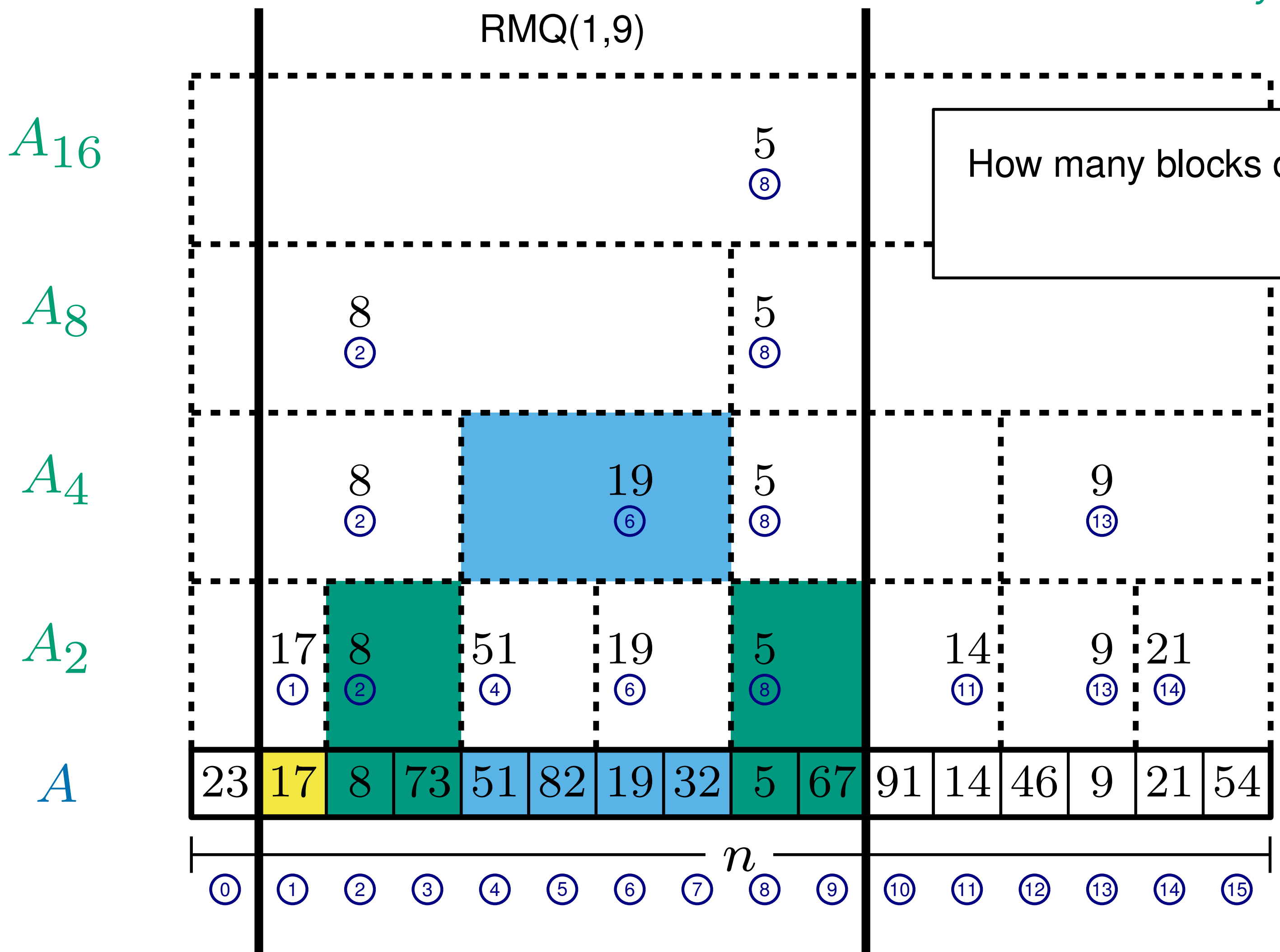

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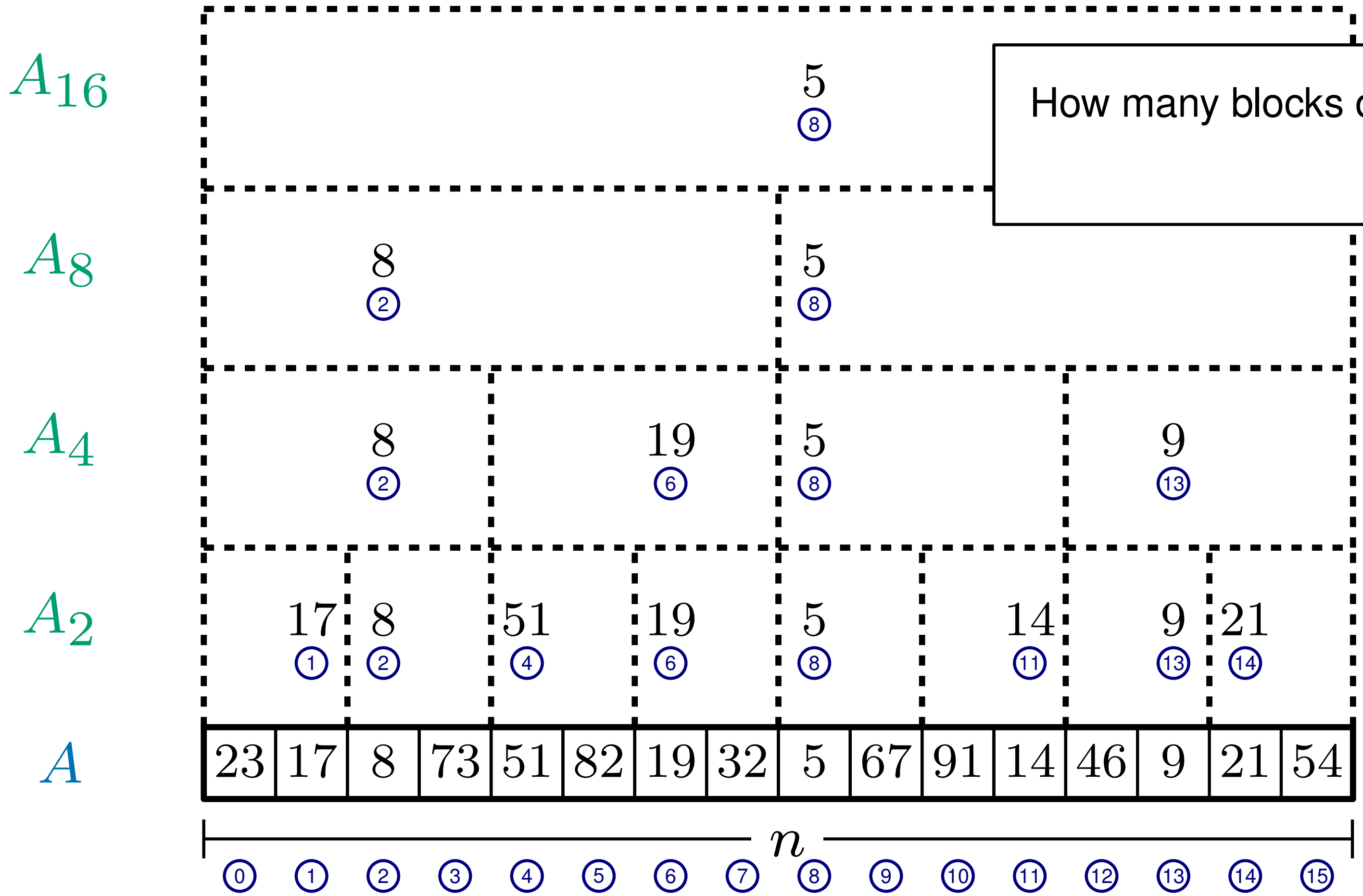

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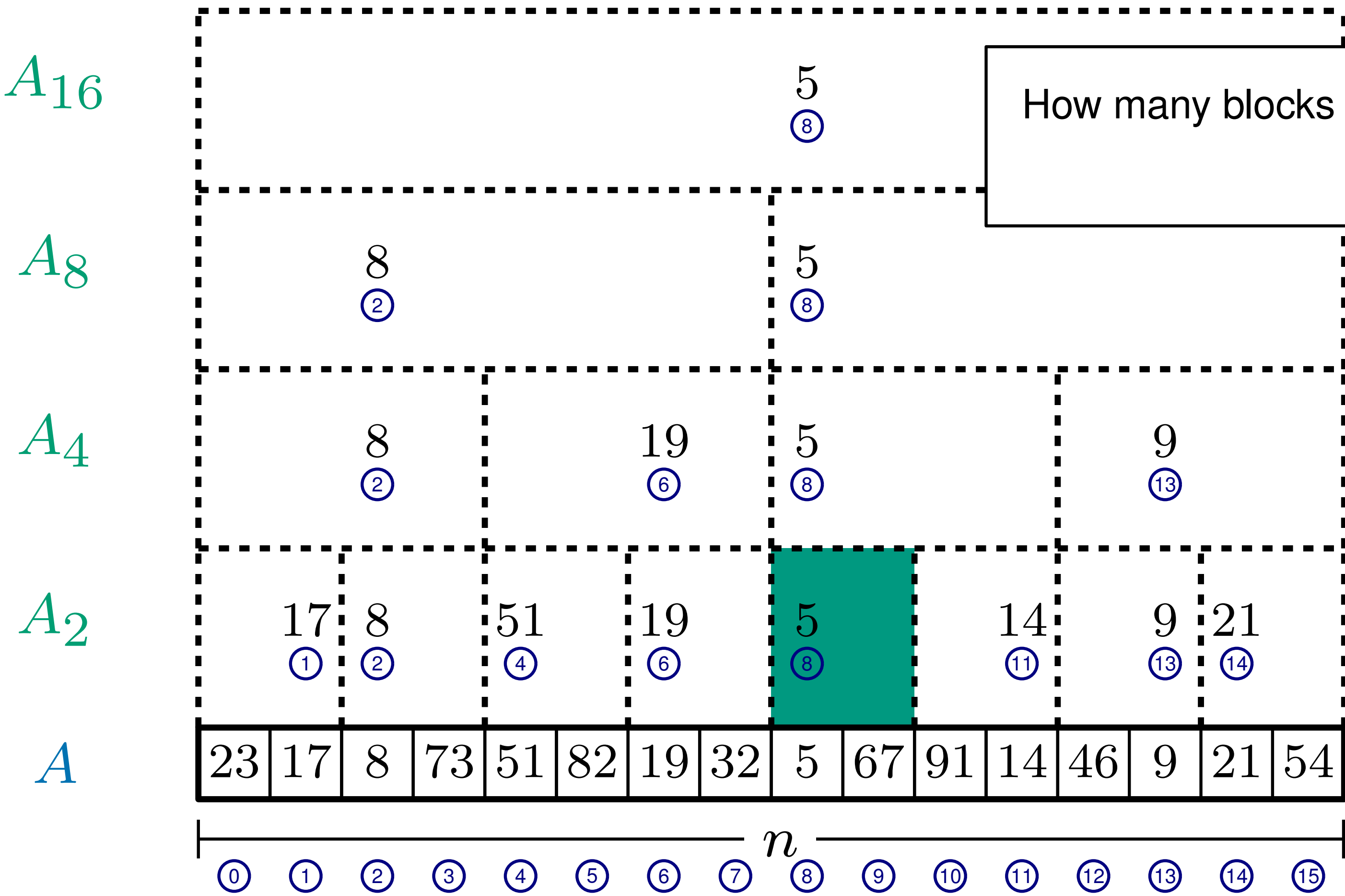

How many blocks do we pick?

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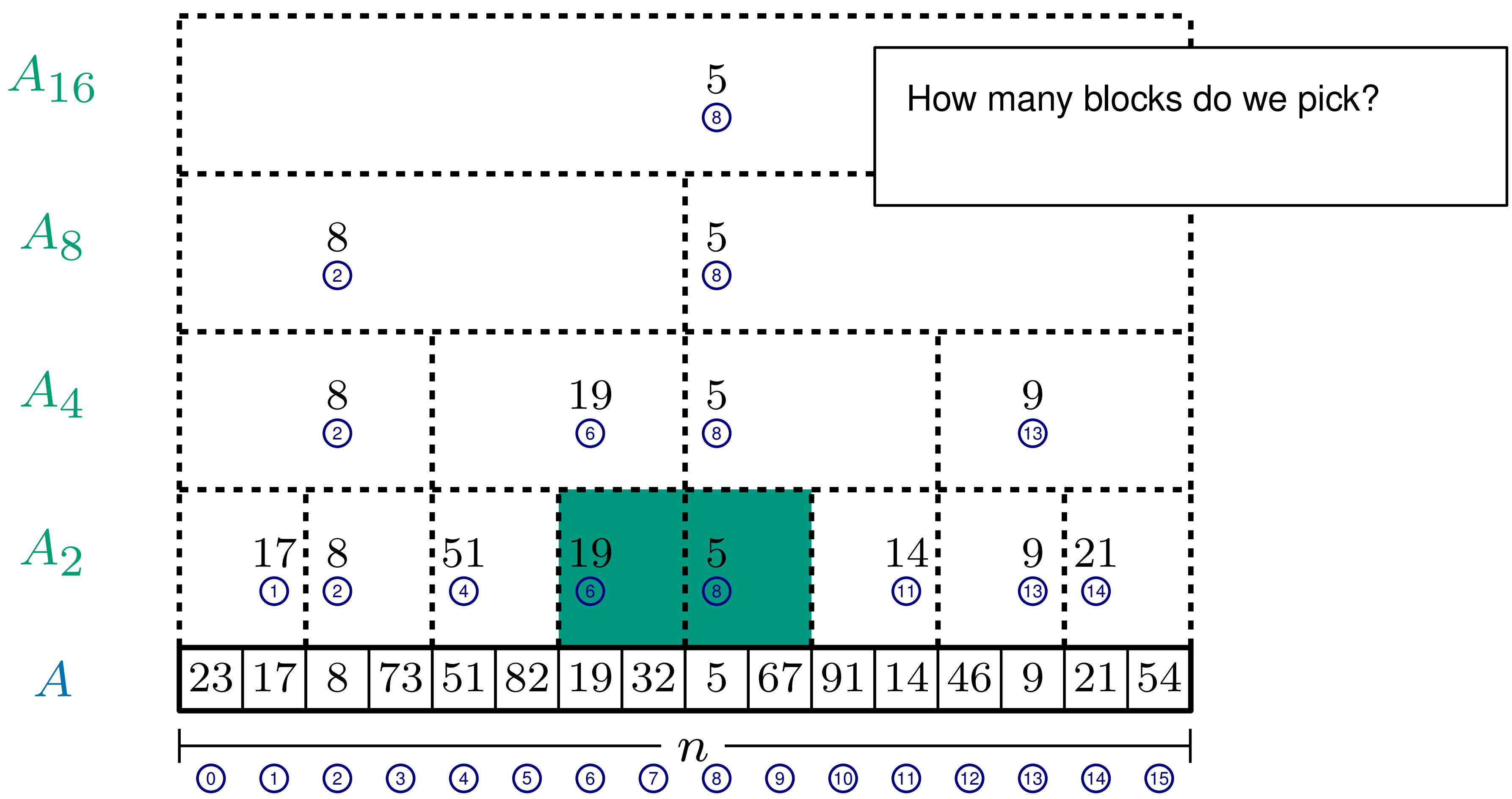

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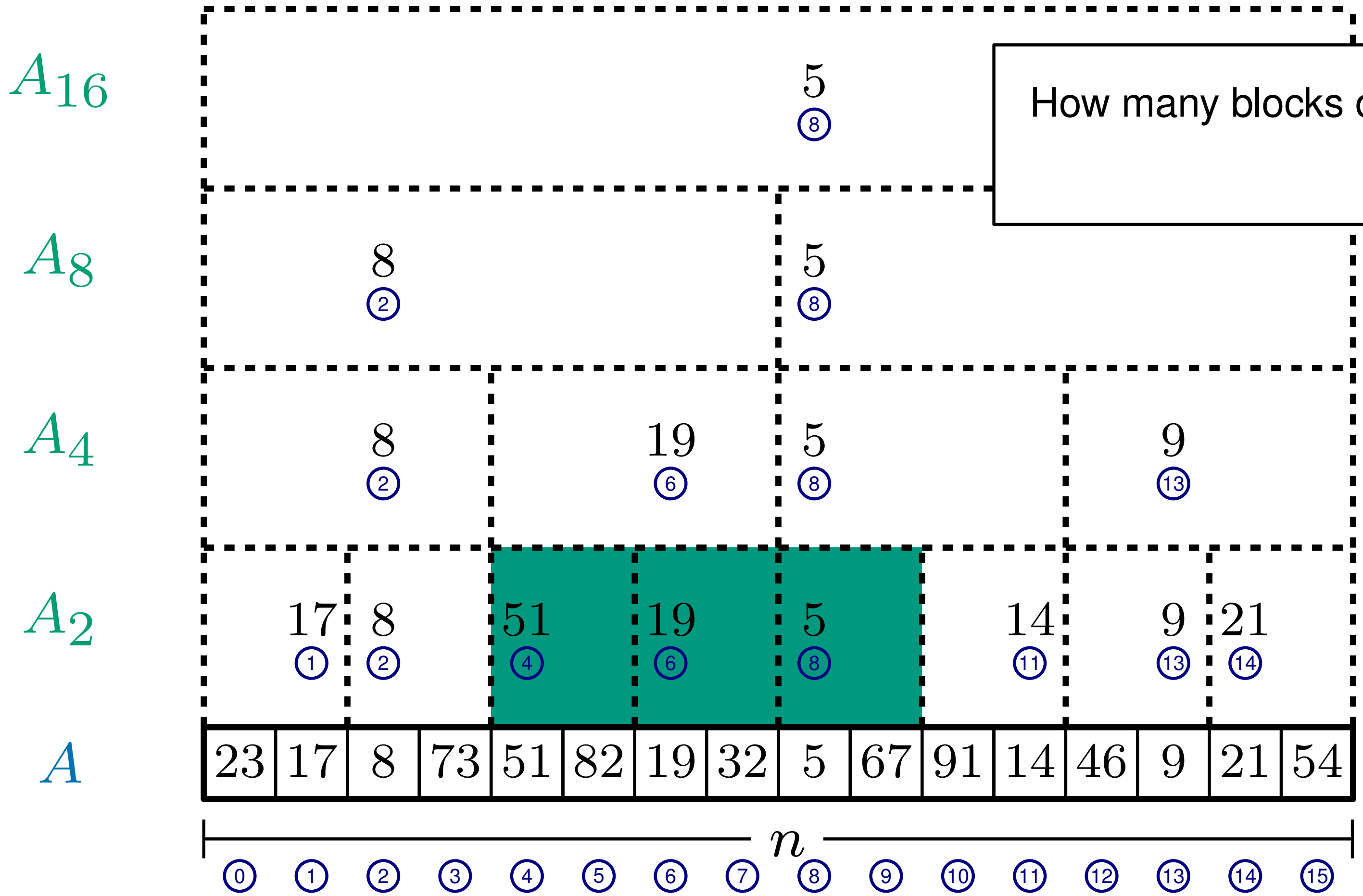

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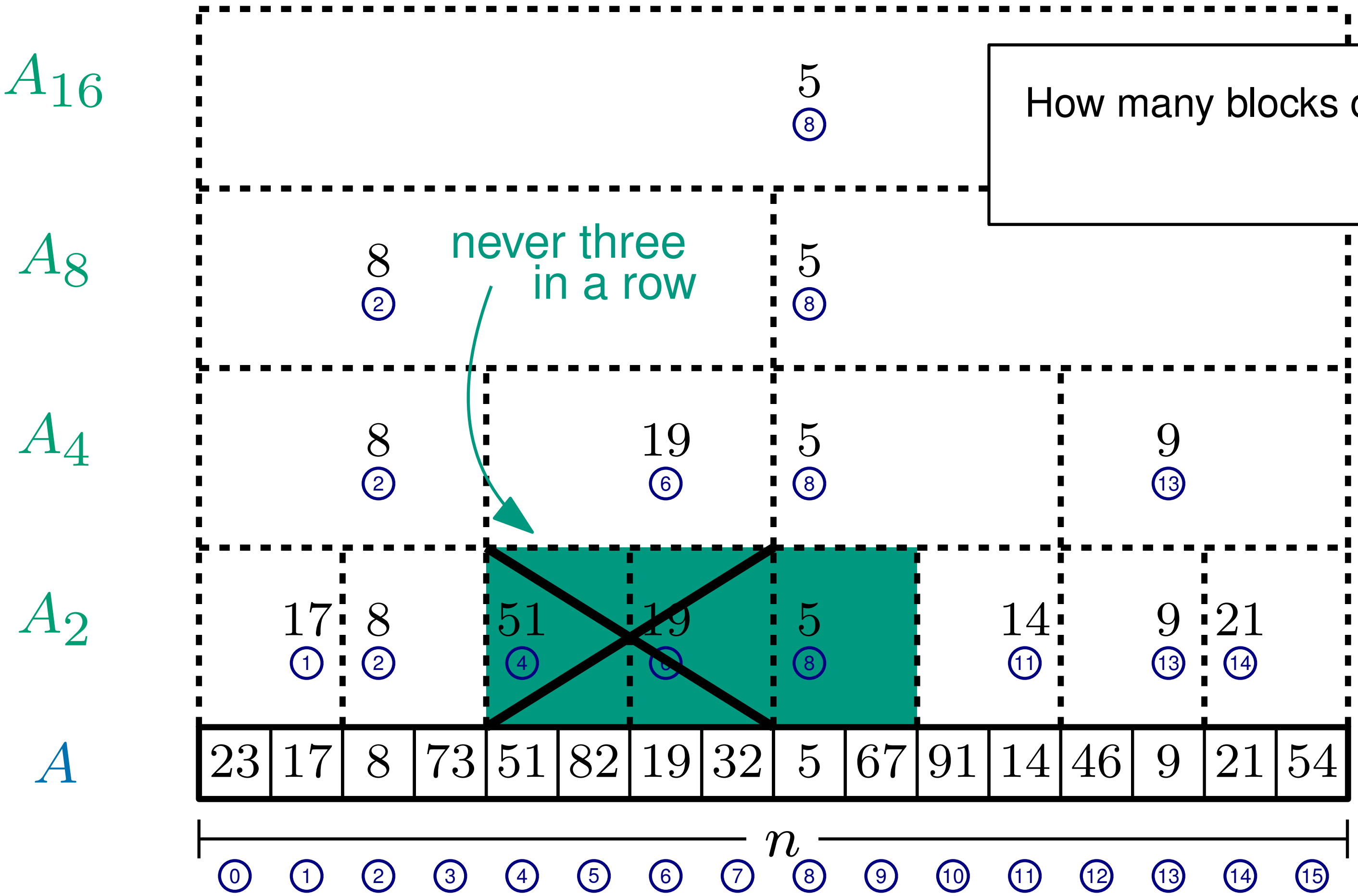

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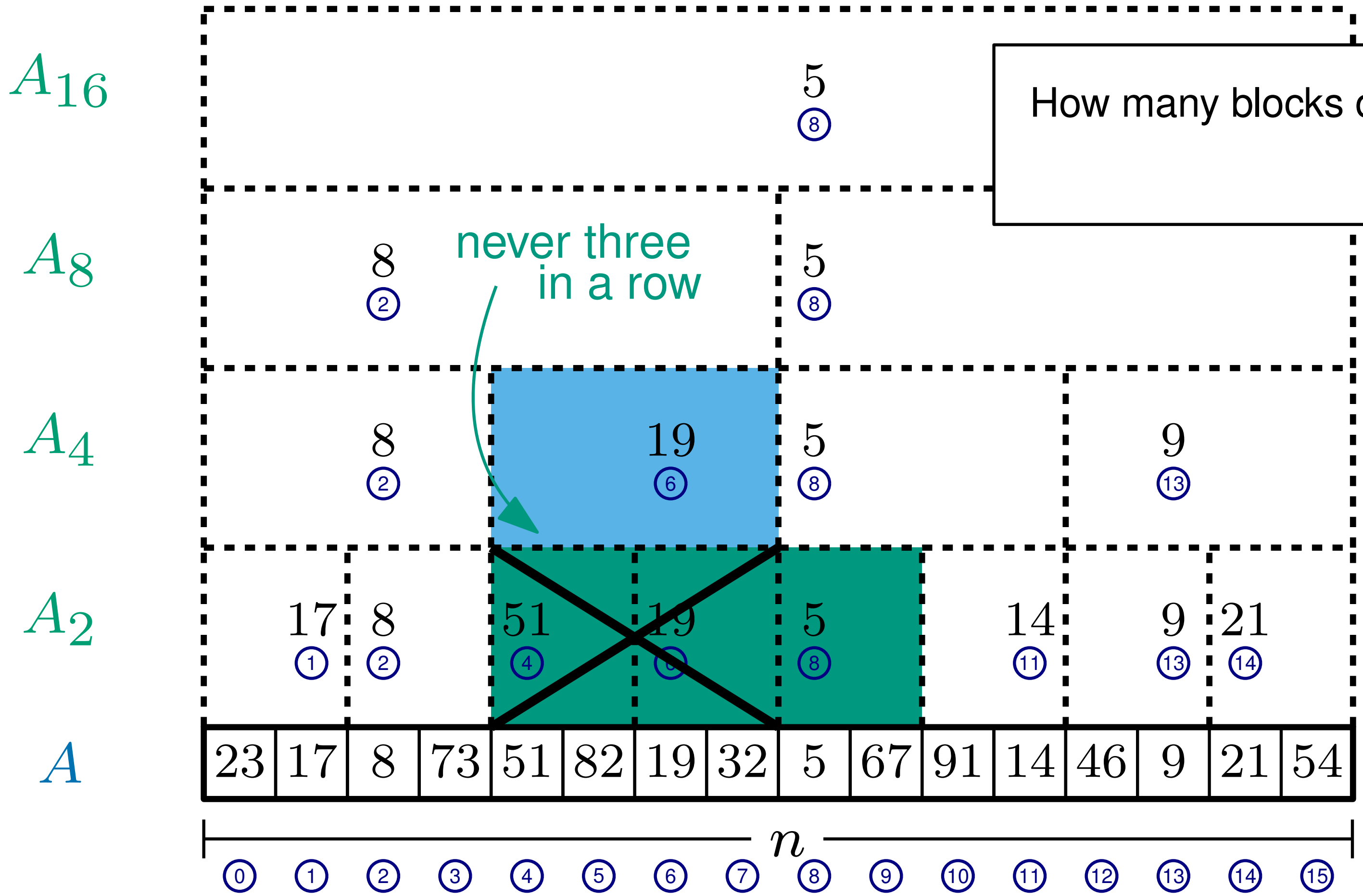

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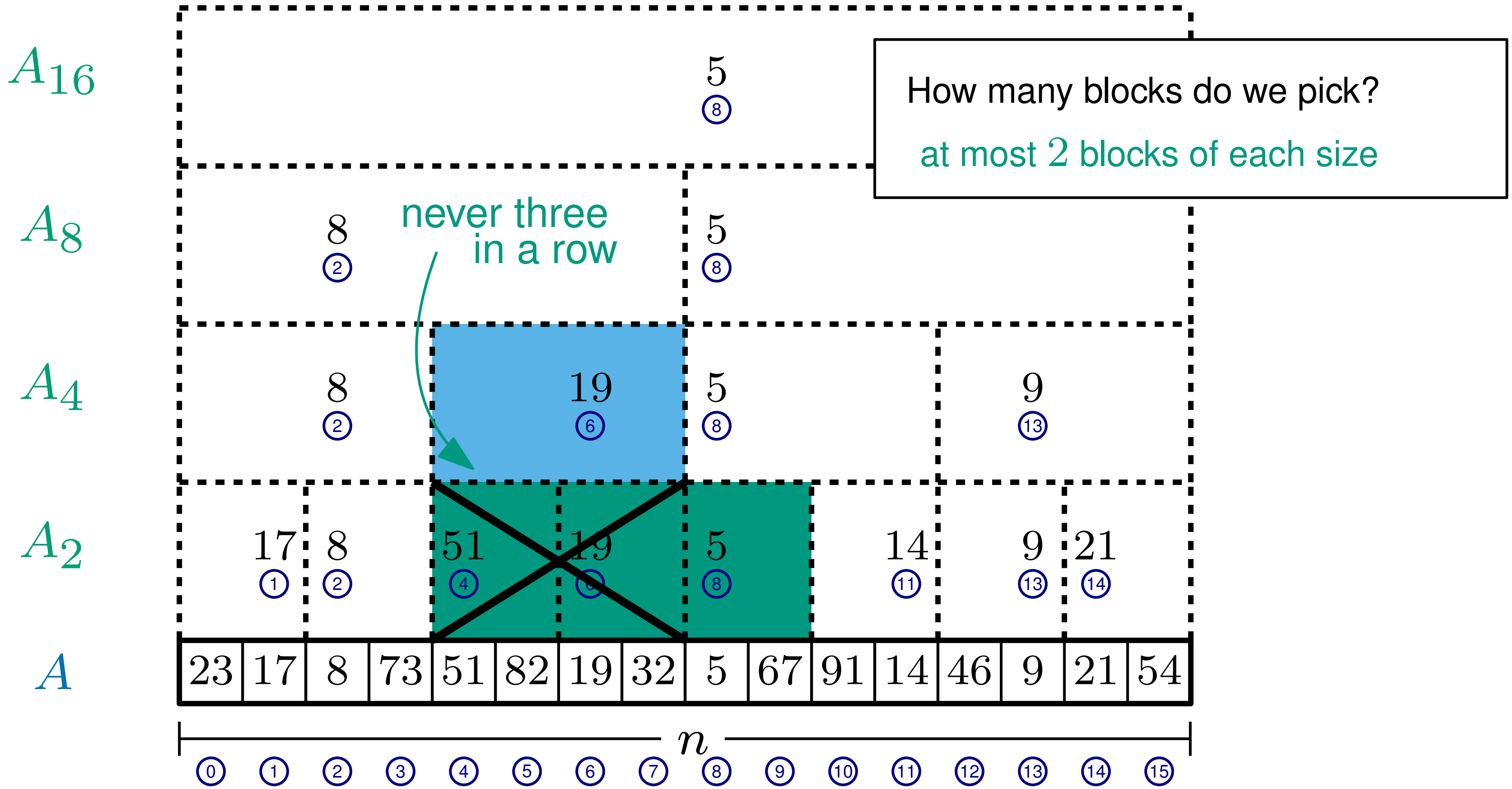

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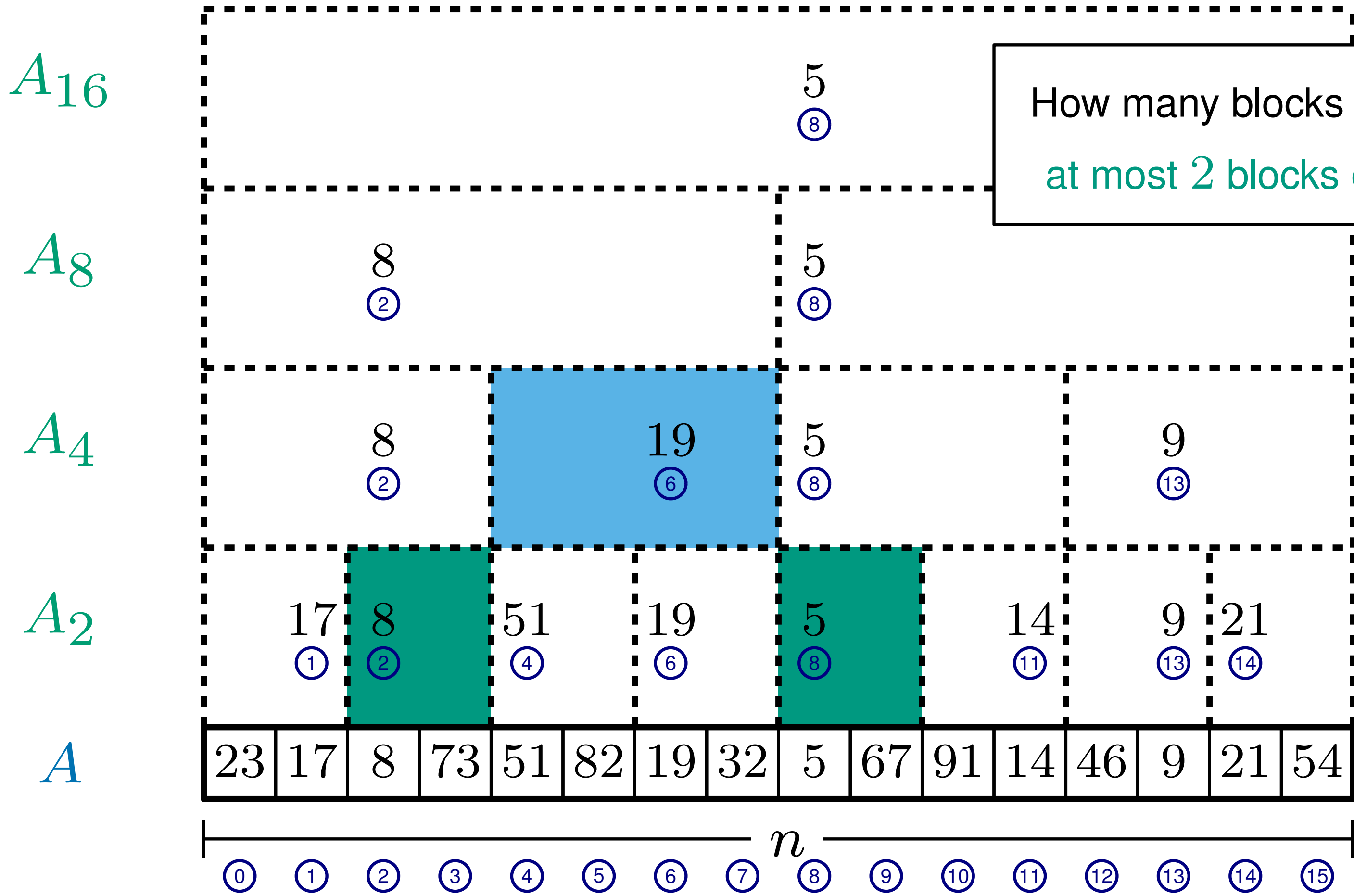

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at most 2 blocks of each size

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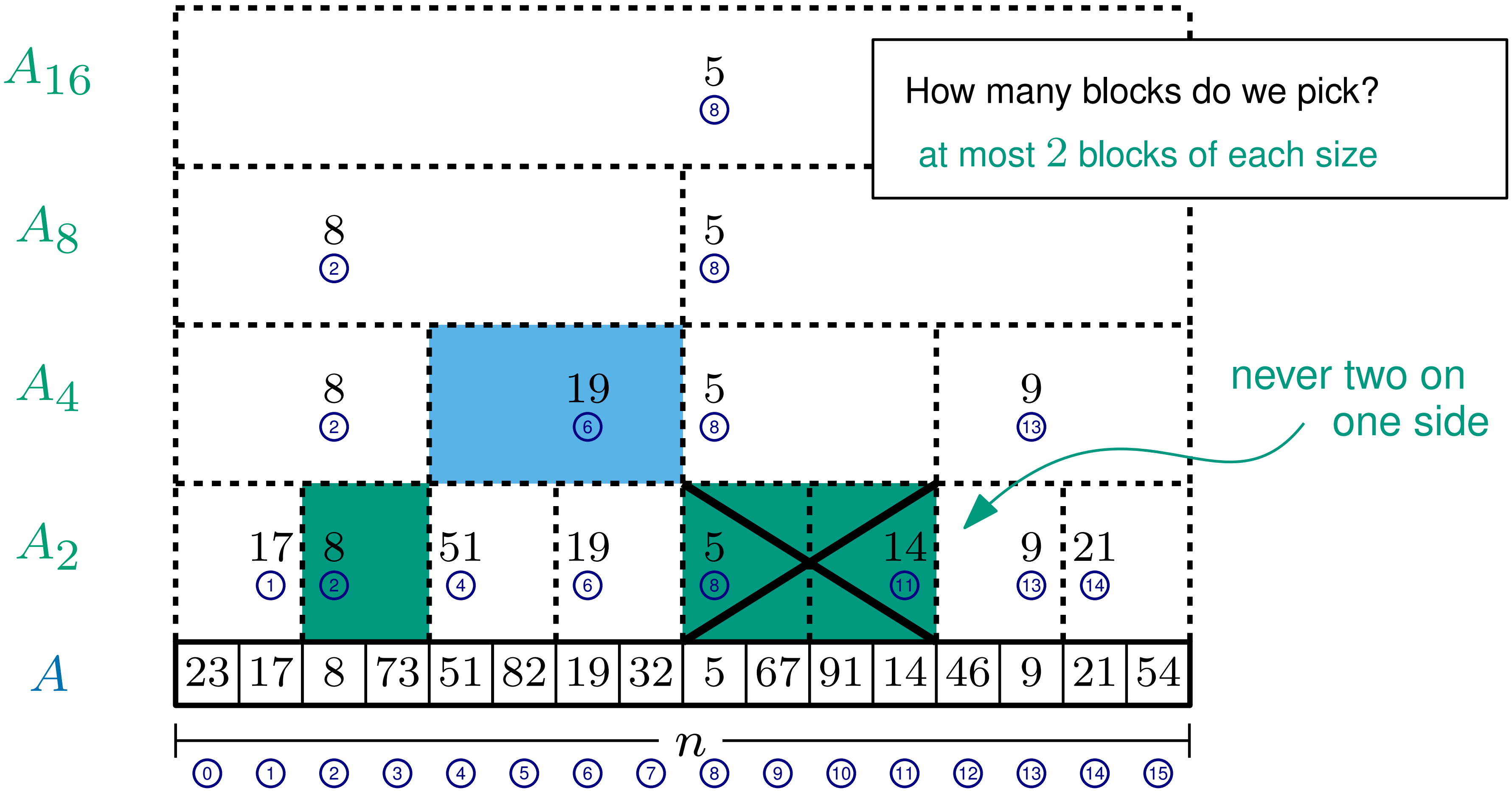

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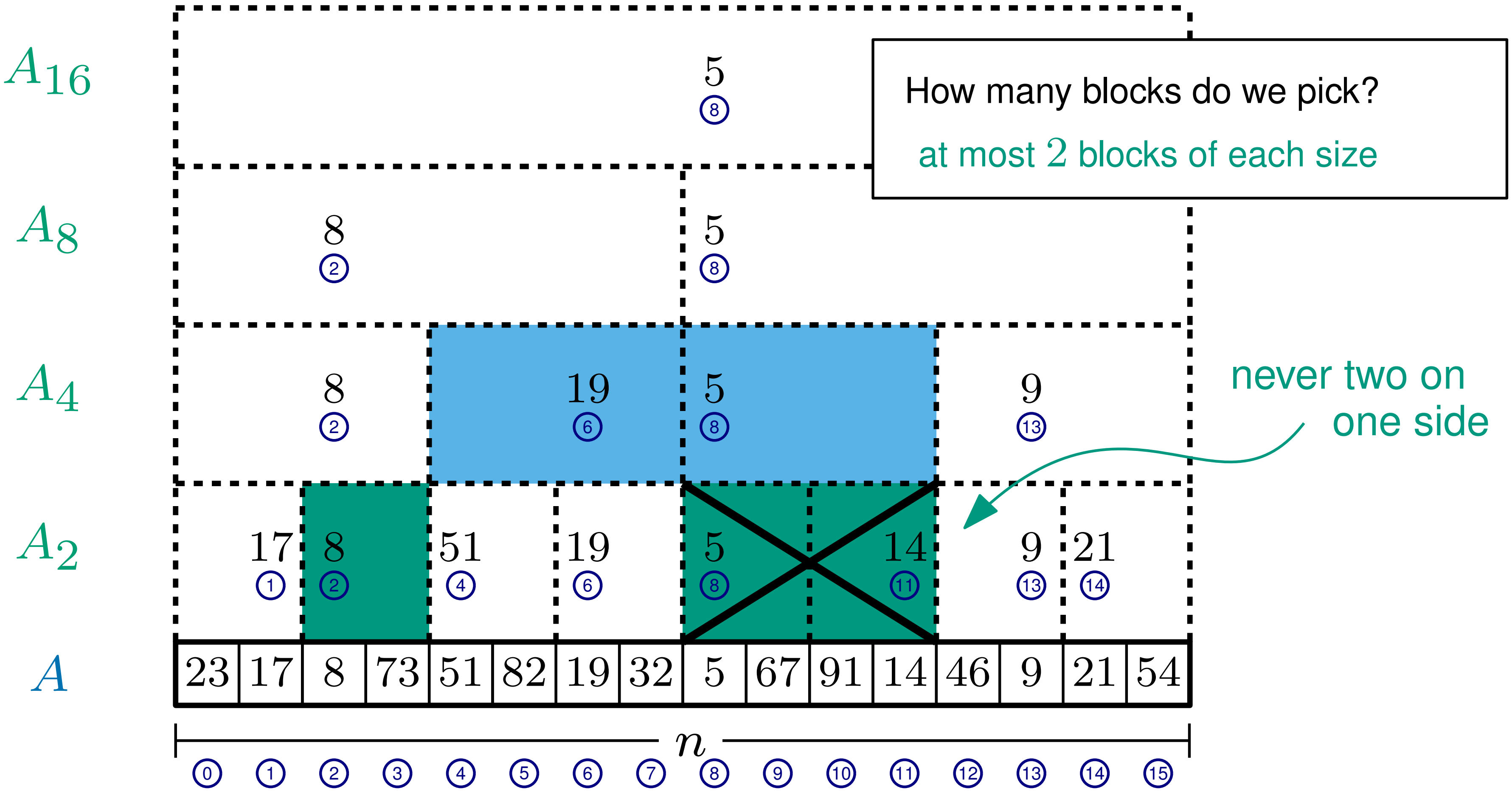

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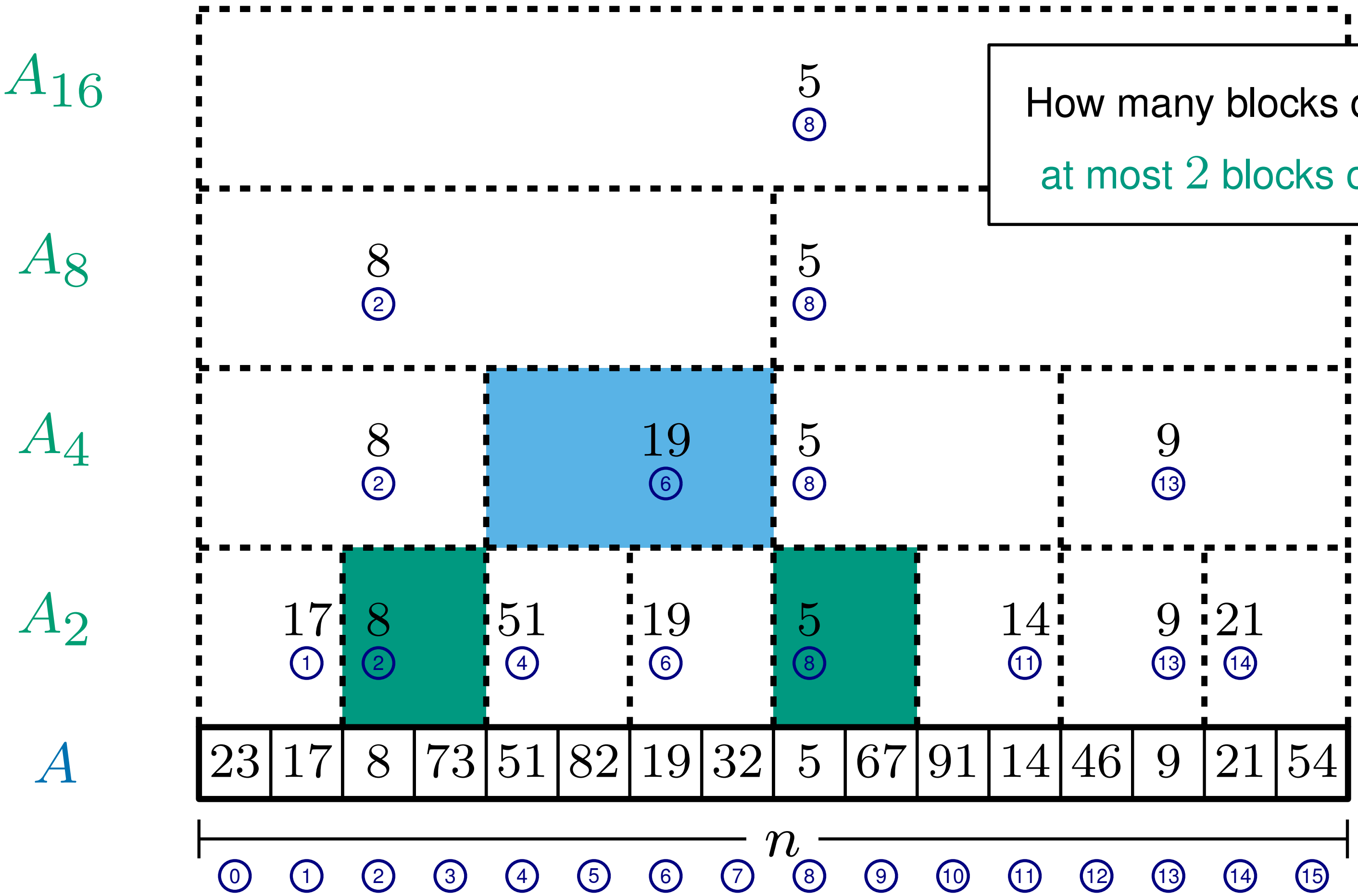

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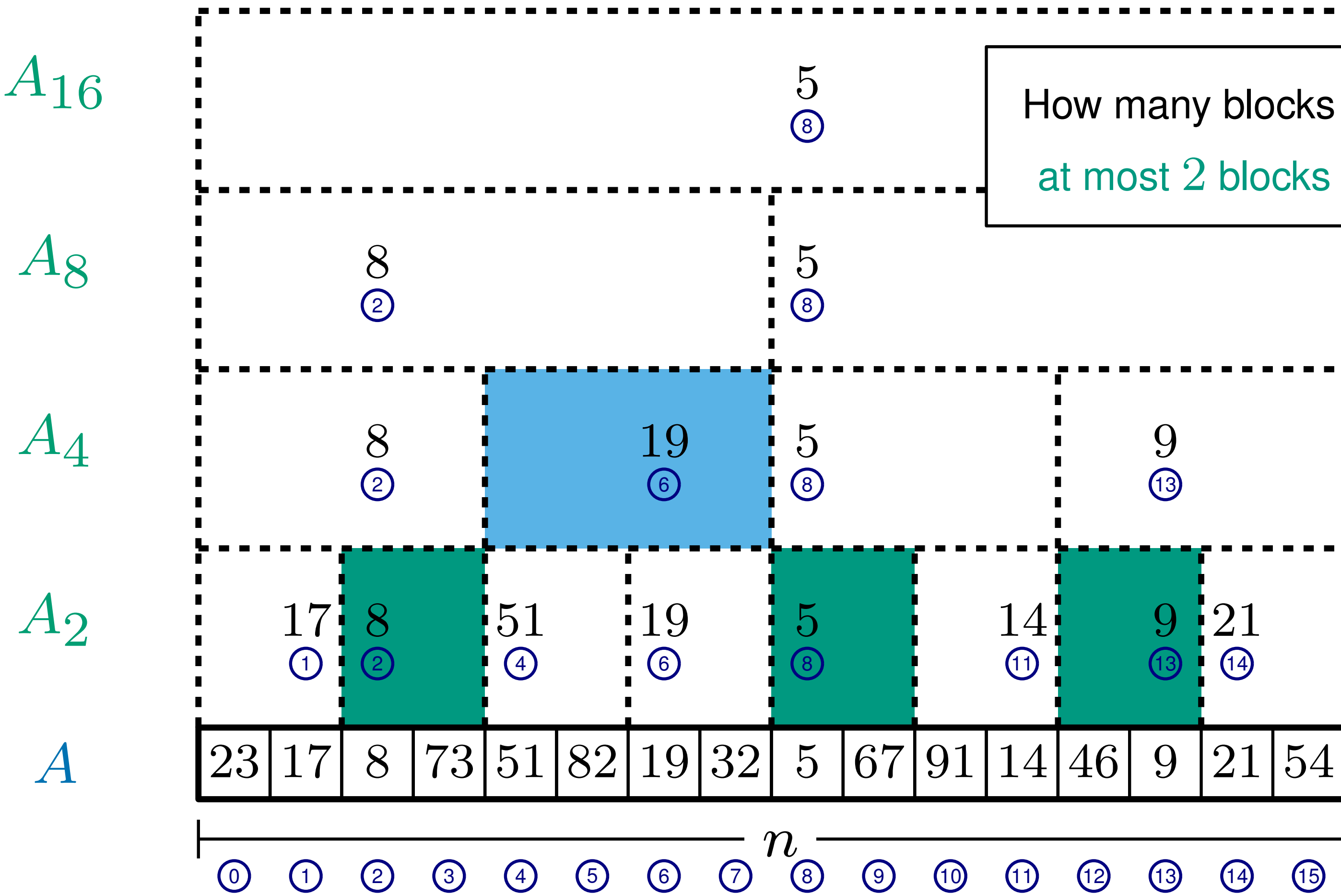

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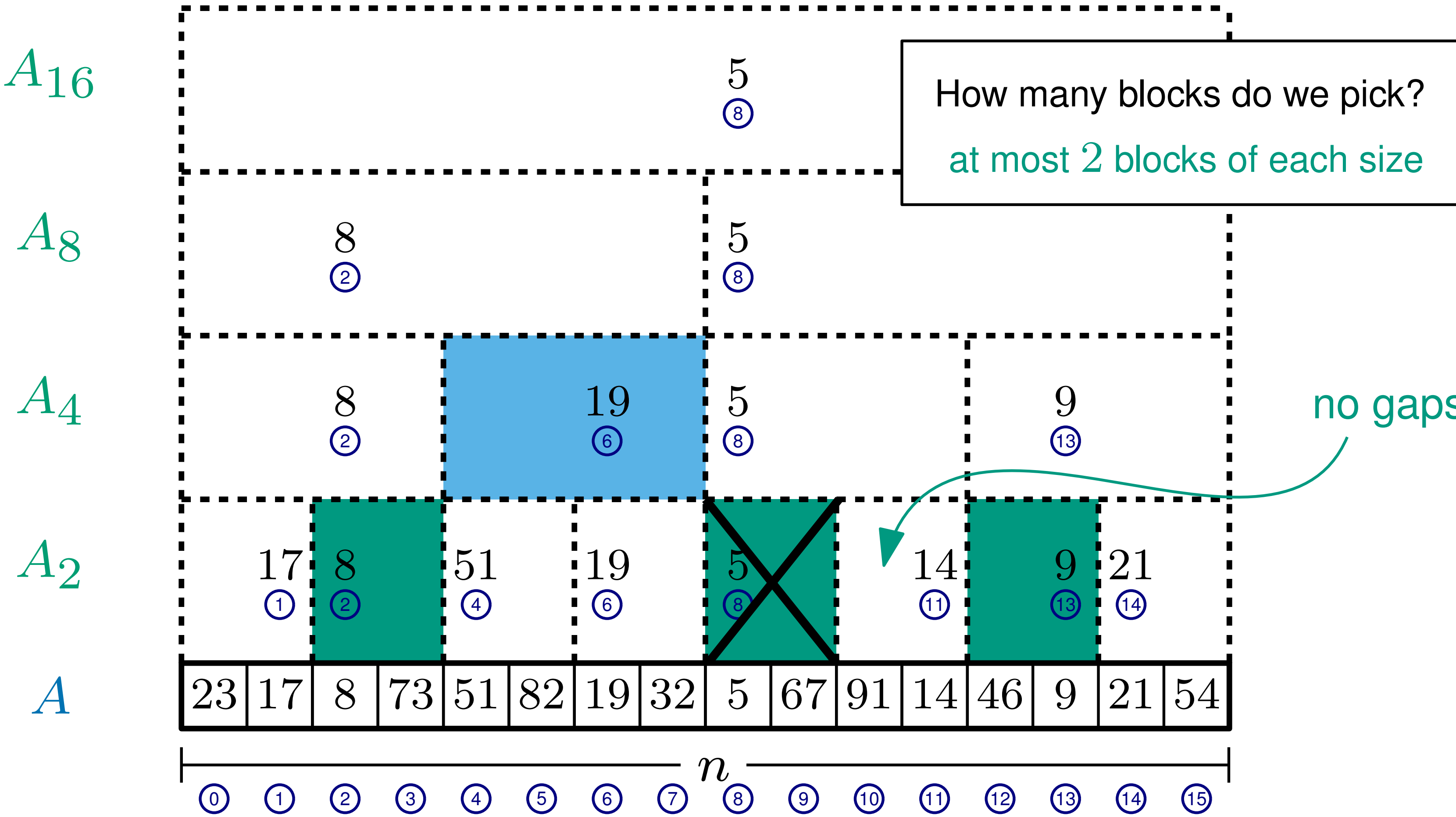

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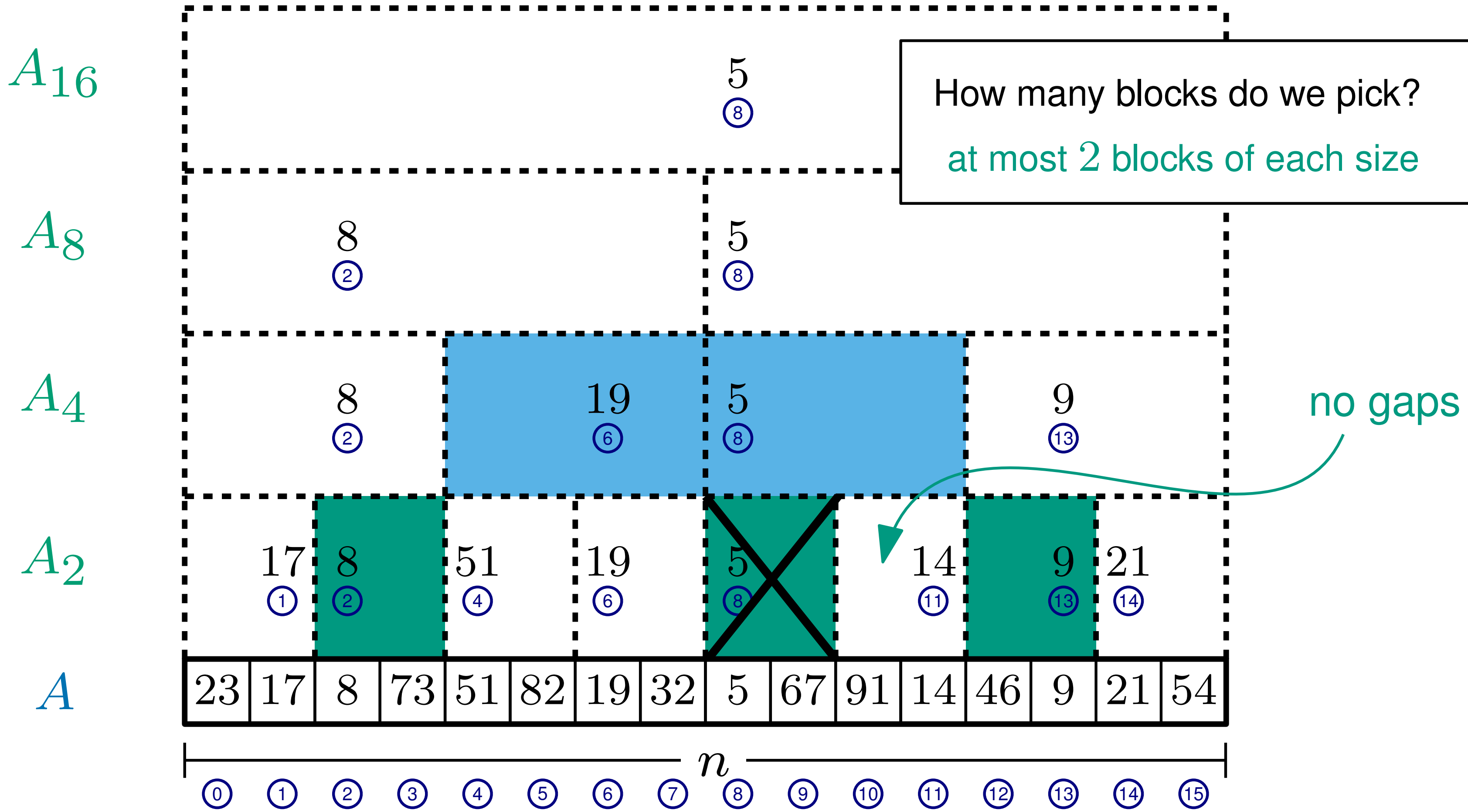

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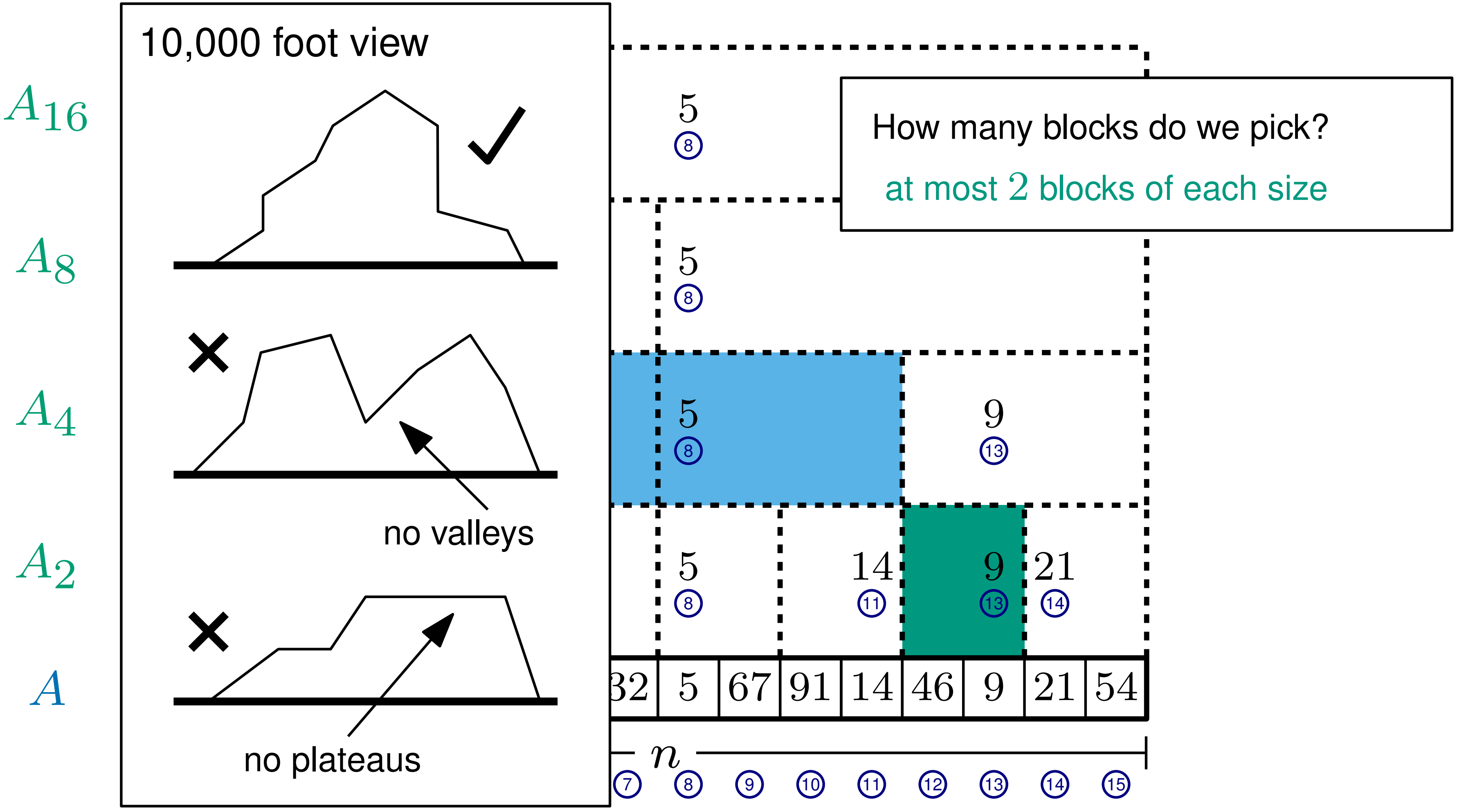

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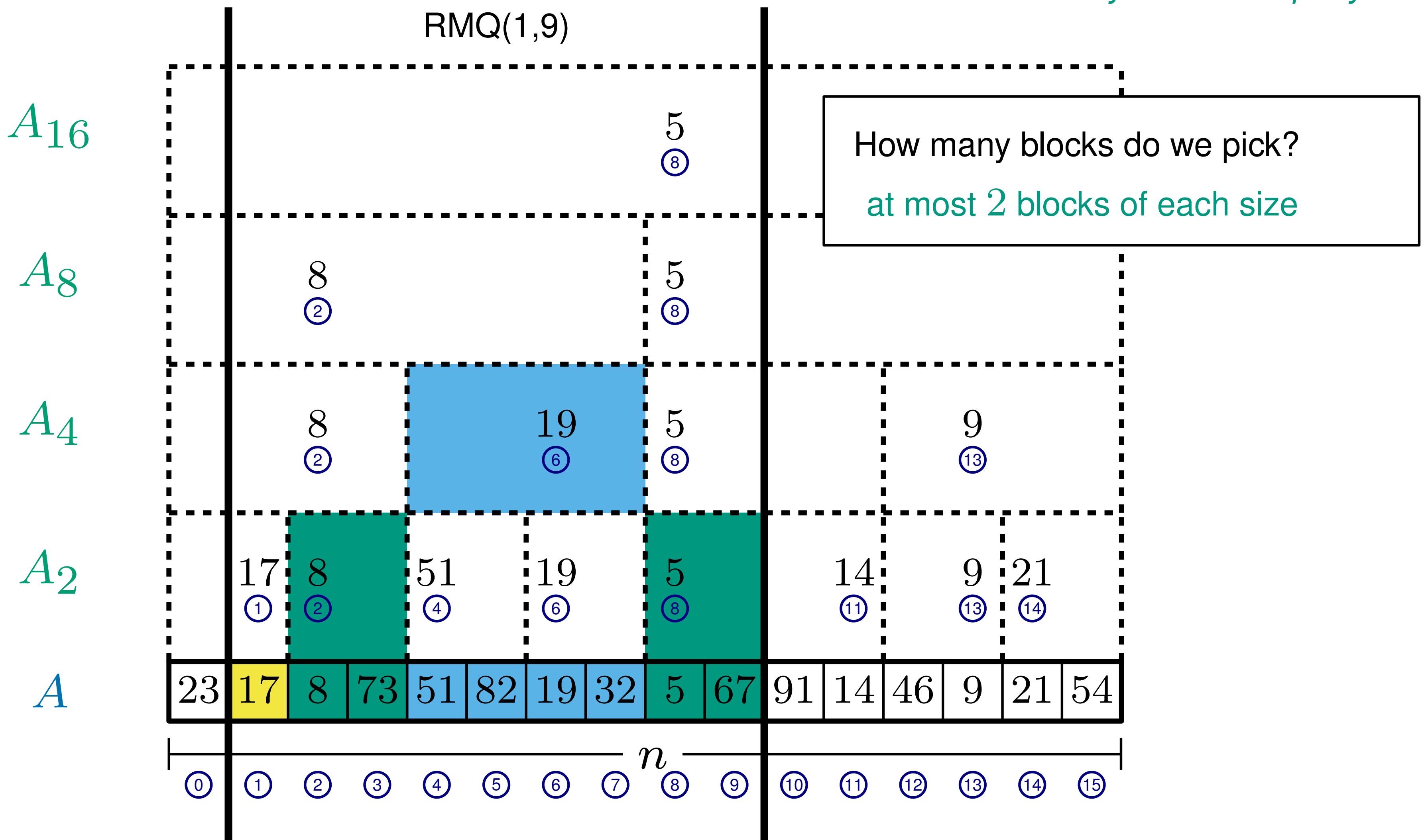

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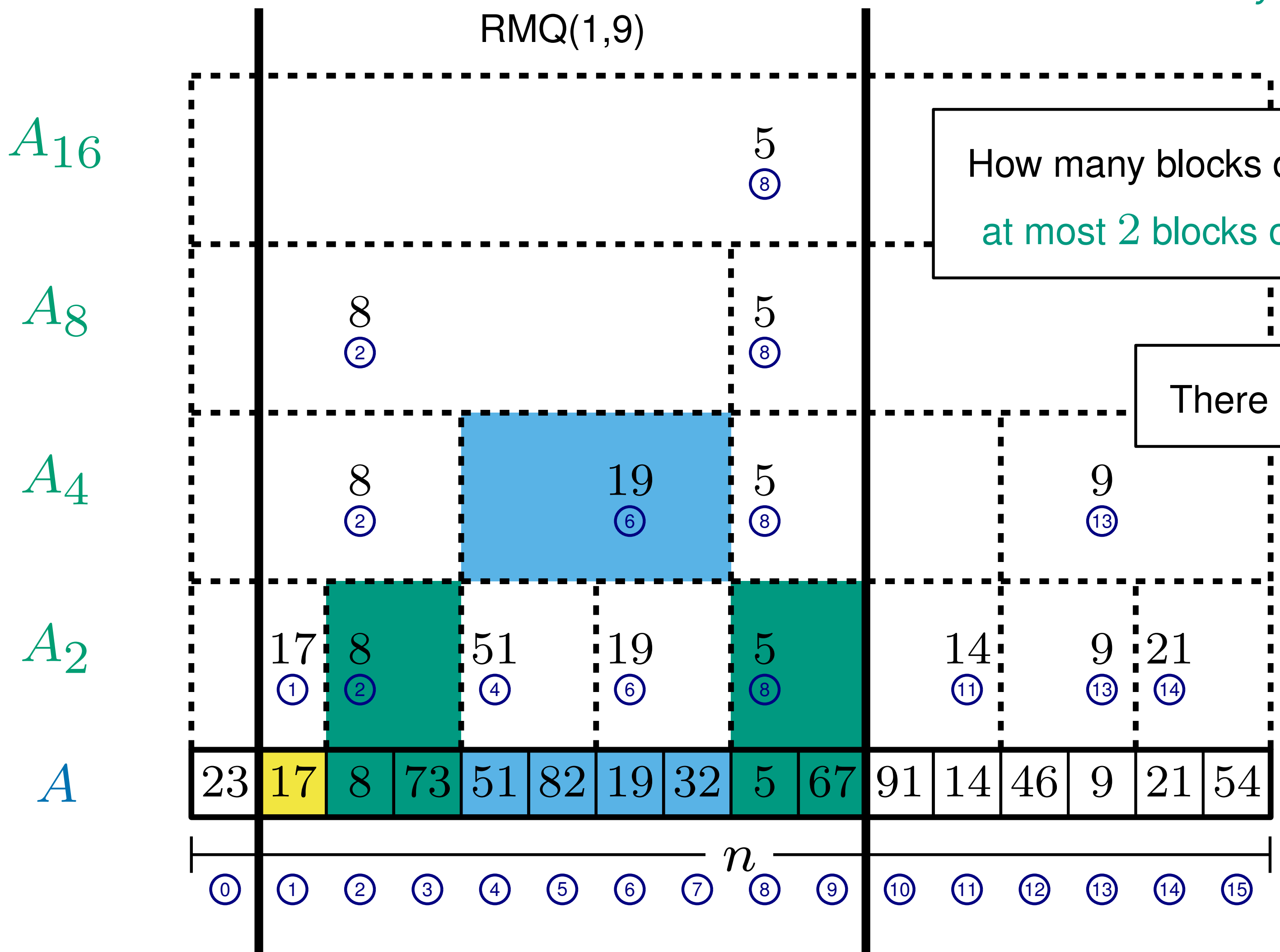

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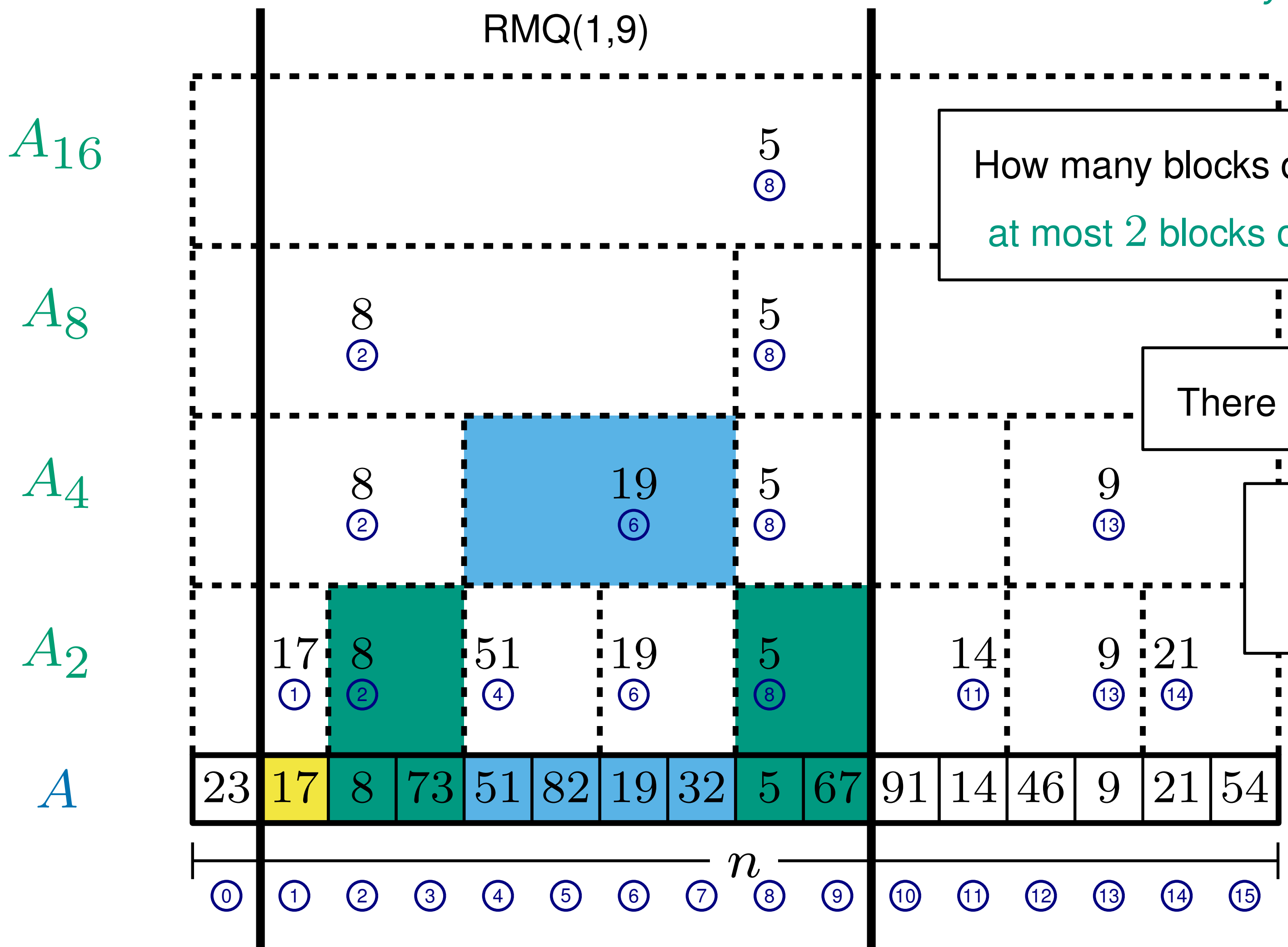

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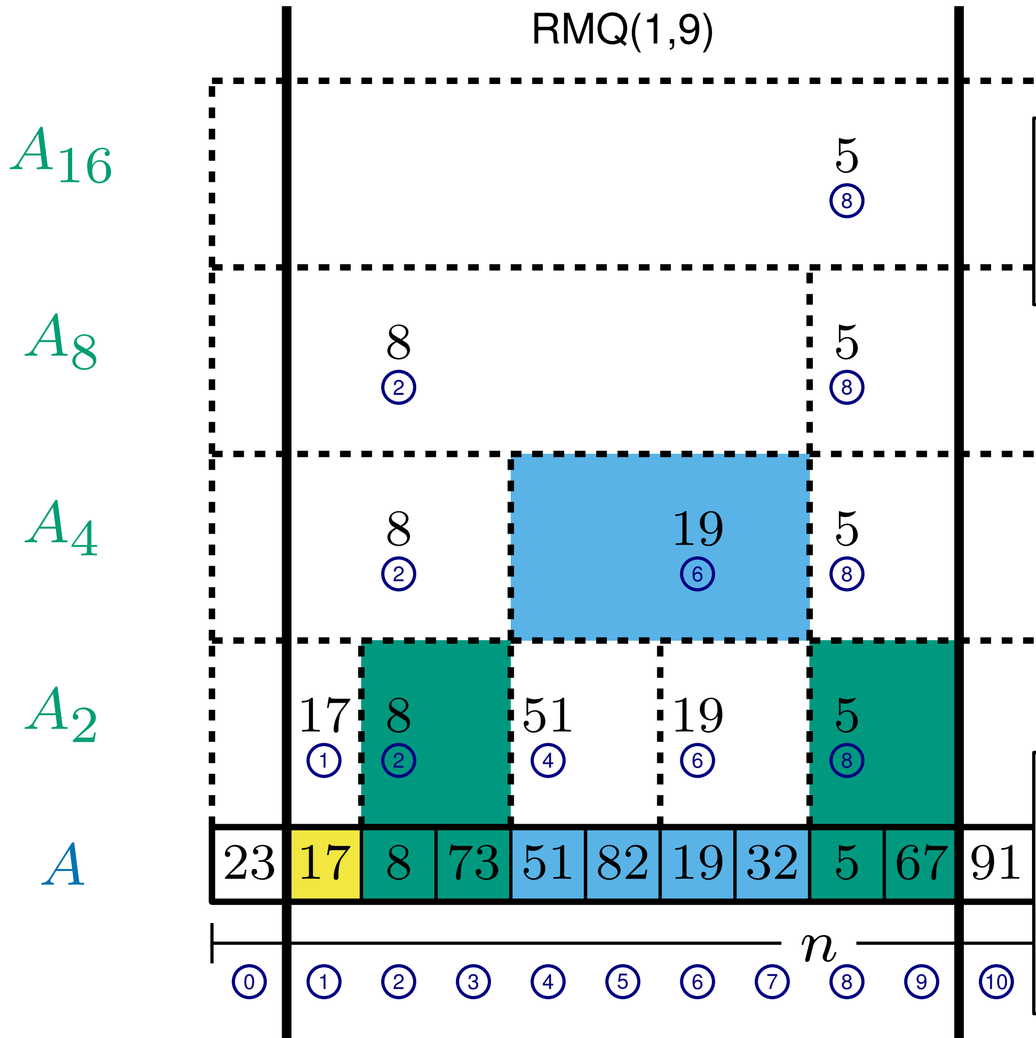

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There are $O(\log n)$ sizes

Picking the blocks from A_k takes $O(1)$ time

So we have . . . $O(n)$ space,

$O(n)$ prep time

$O(\log n)$ query time

More space, faster queries

Key Idea precompute the answers for every interval of length $2, 4, 8, 16 \ldots$

A

The array R_2 stores $\text{RMQ}(i, i+1)$ for all i

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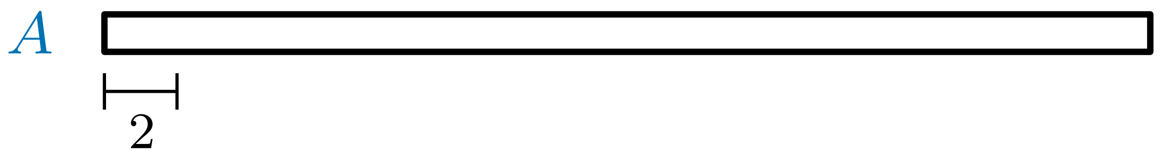

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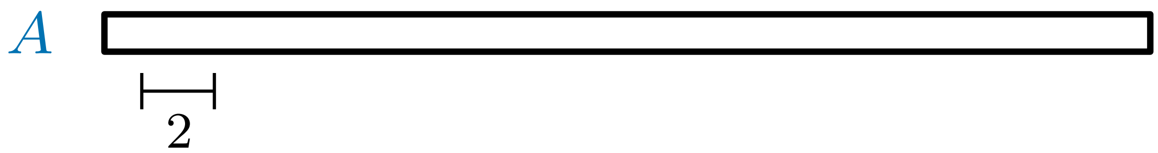

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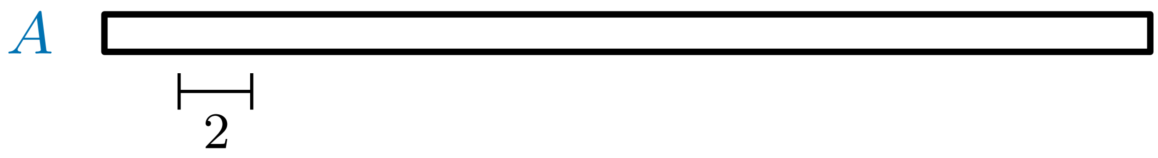

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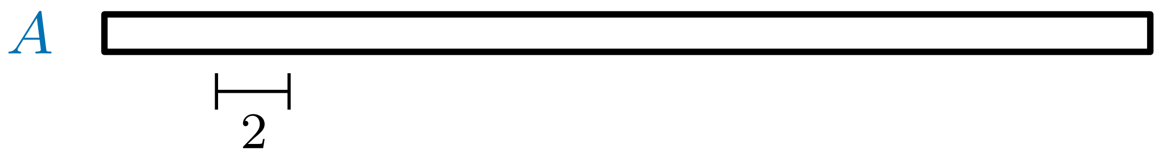

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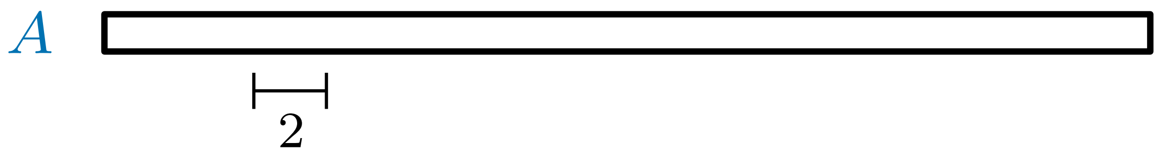

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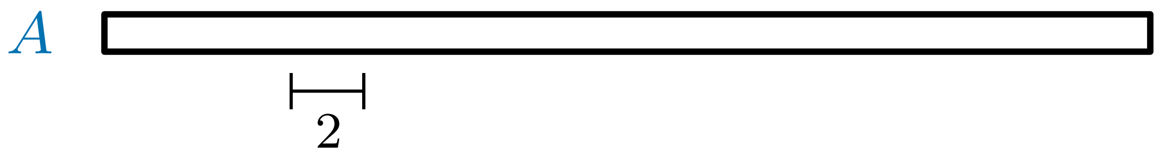

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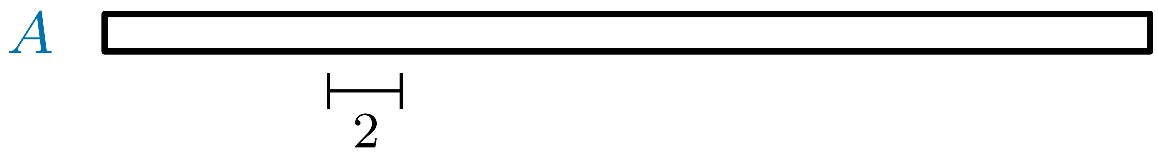

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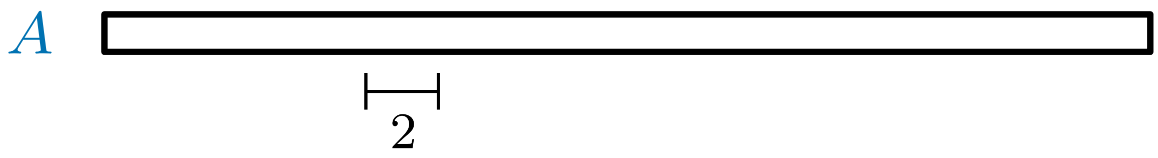

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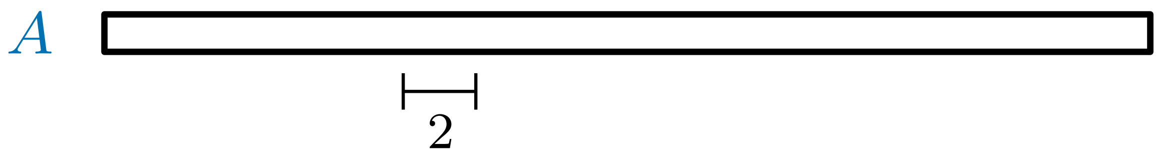

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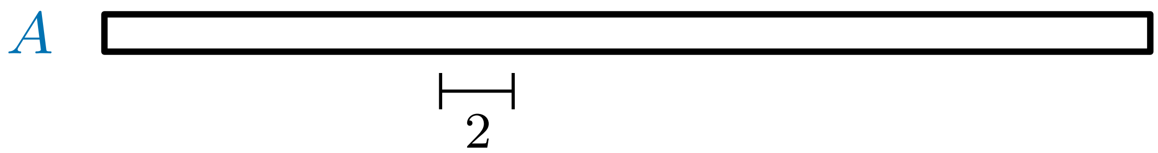

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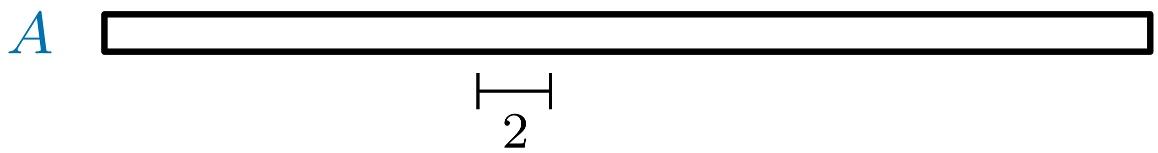

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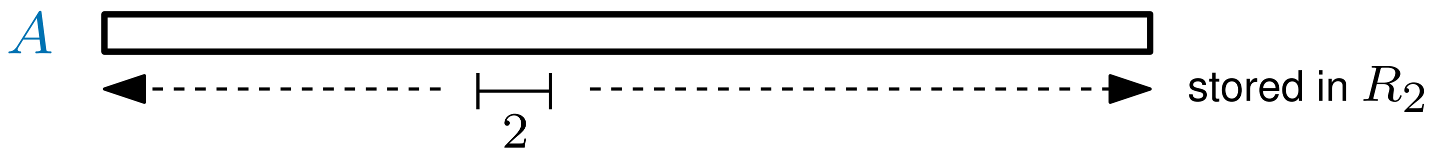

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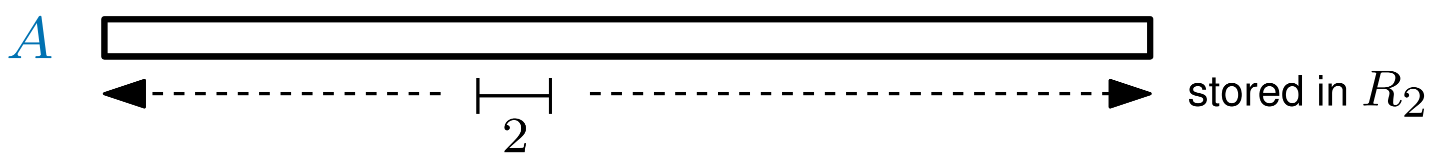

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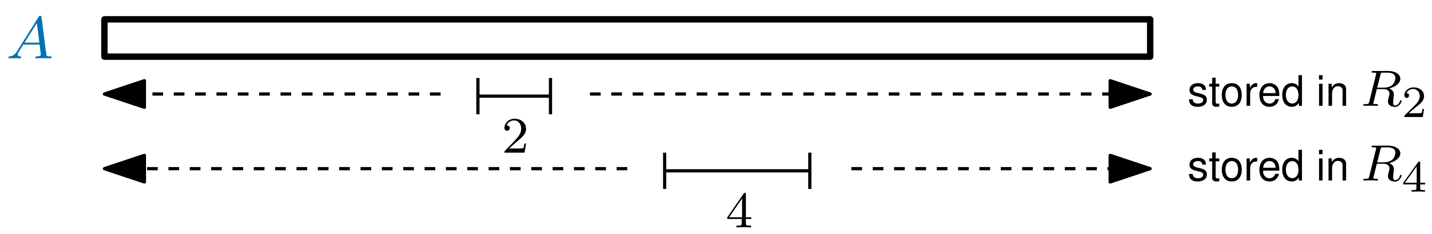

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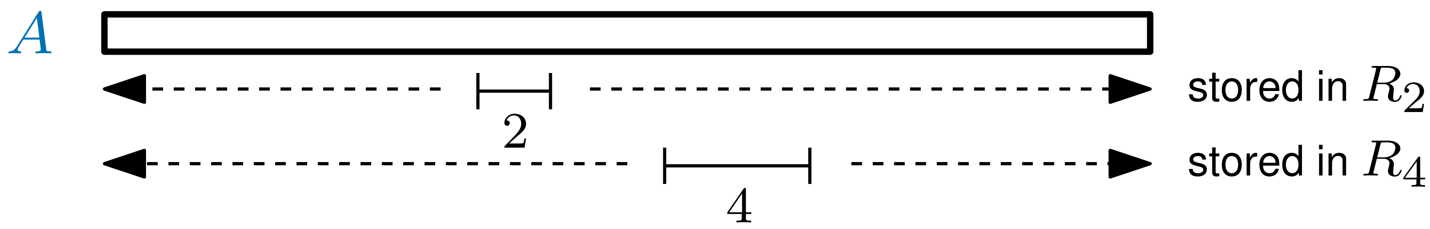

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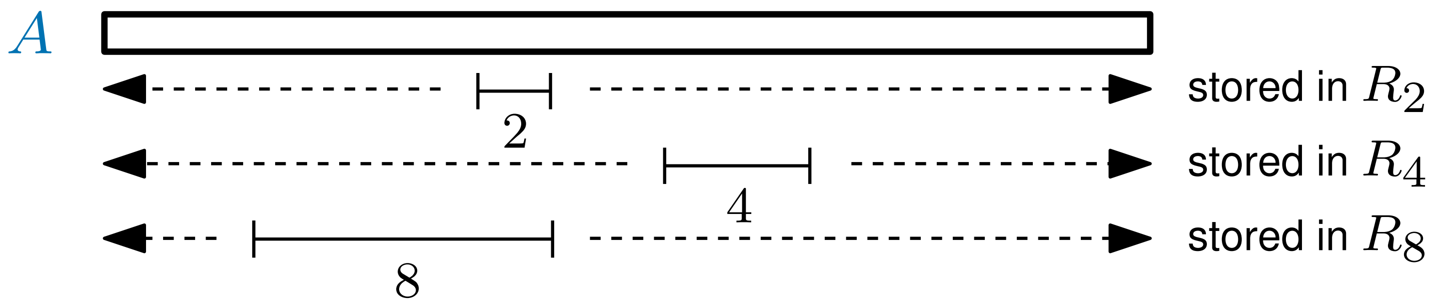

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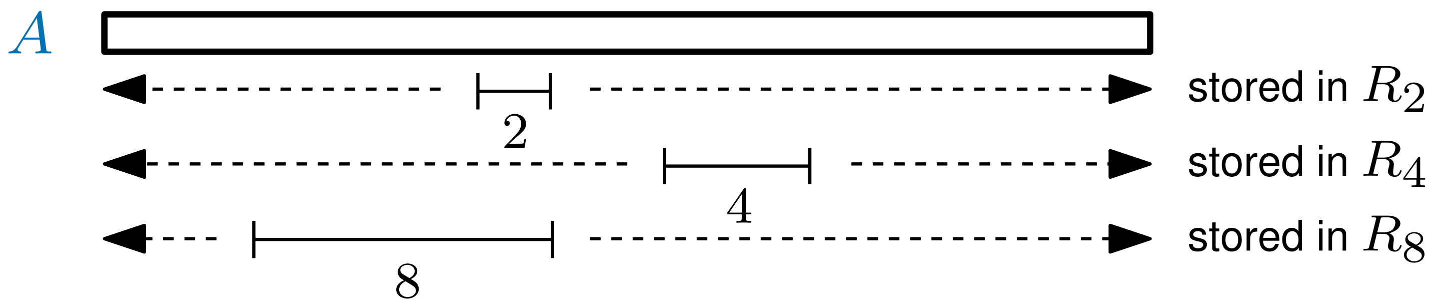

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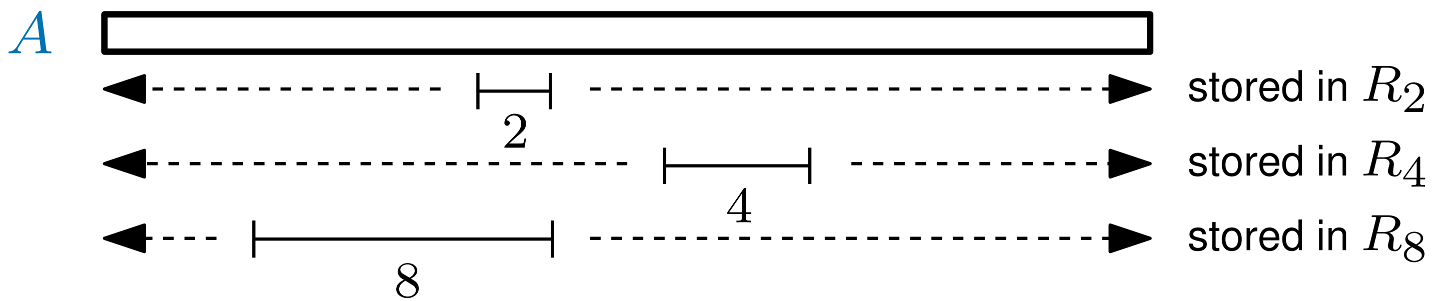

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We build R_k for $k = 2, 4, 8, 16 \ldots \leqslant n$

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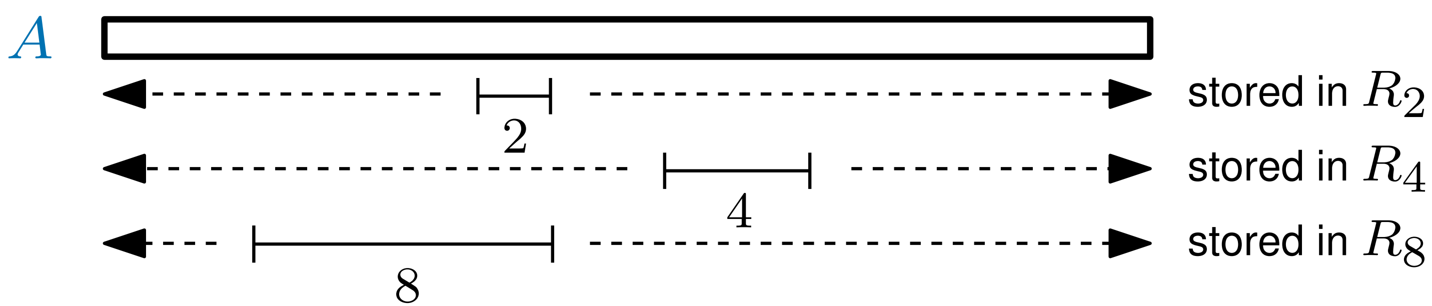

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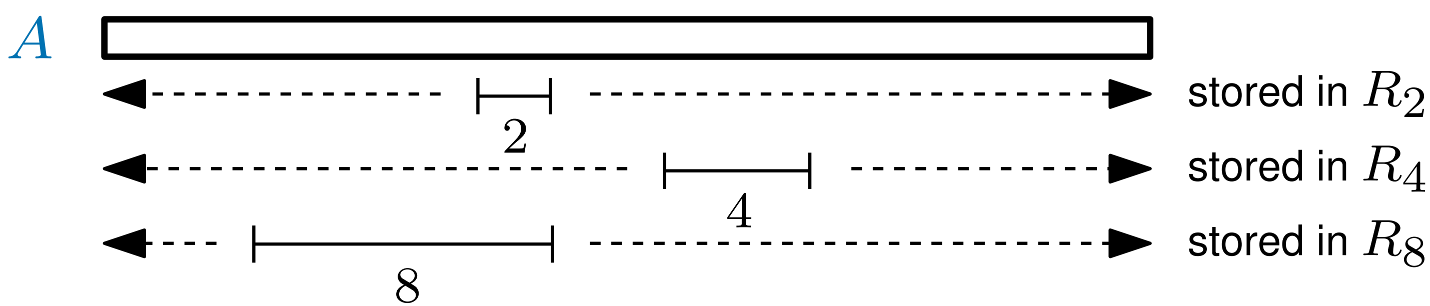

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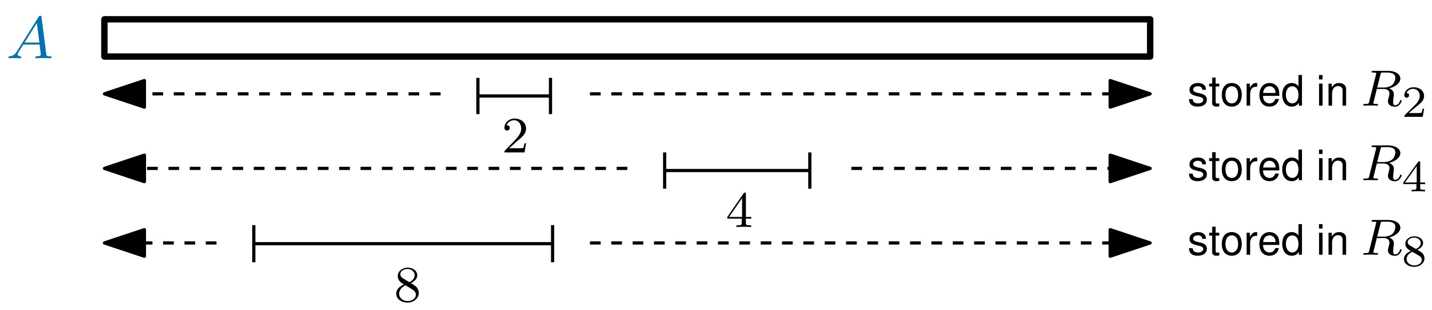

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We build R_2 from A in $O(n)$ time

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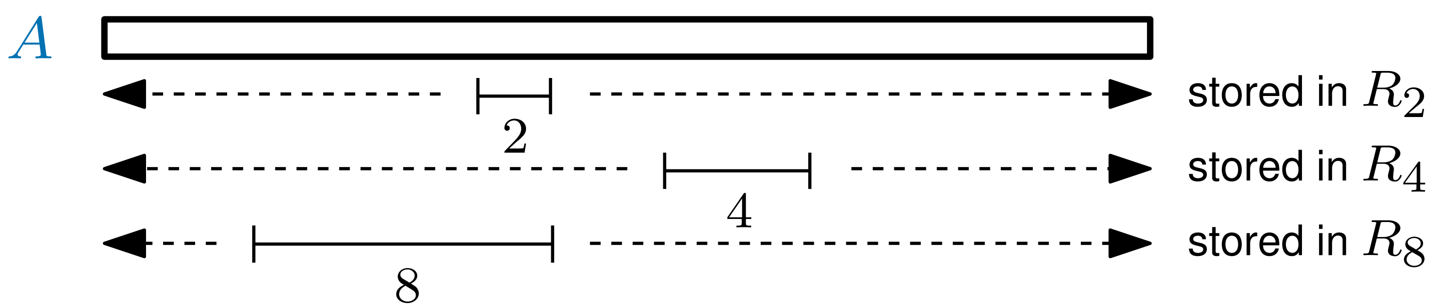

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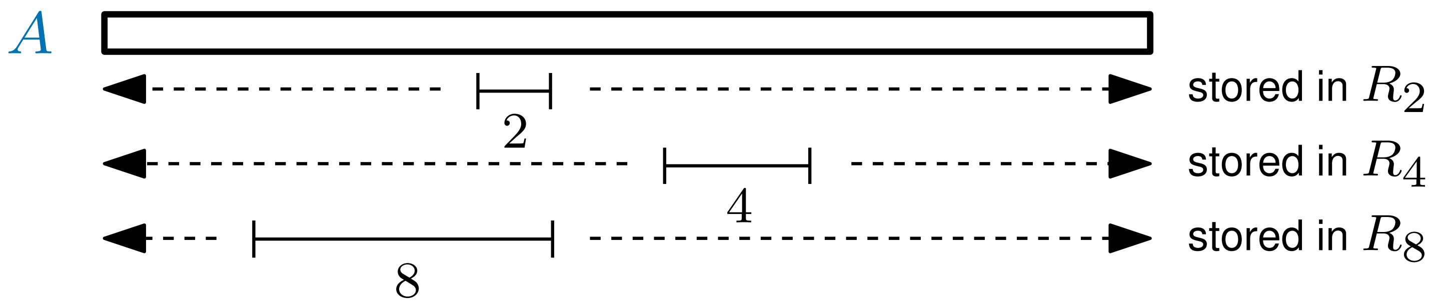

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how?

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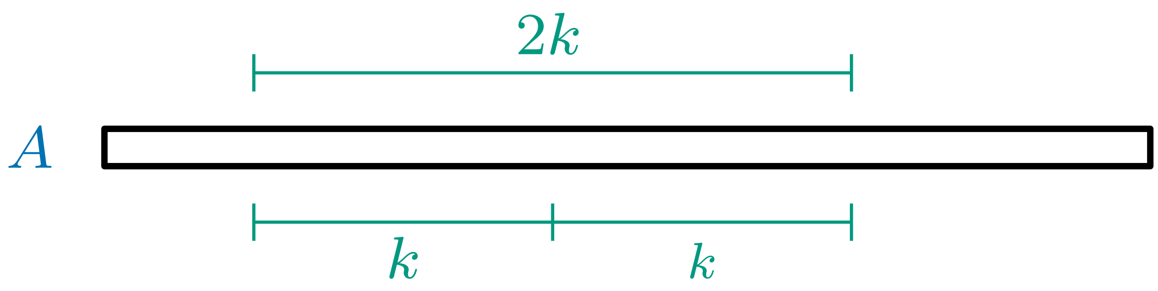
Key Idea precompute the answers for every interval of length $2, 4, 8, 16\ldots$



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so $O(n \log n)$ total space

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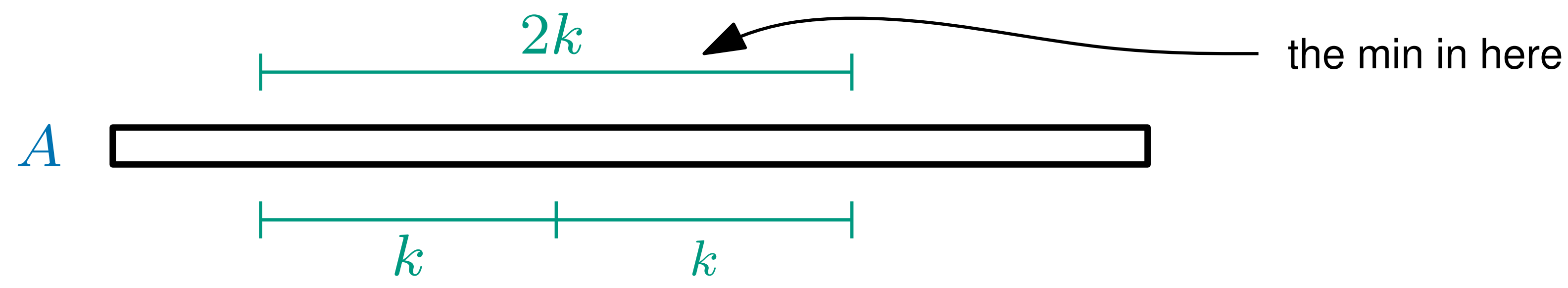

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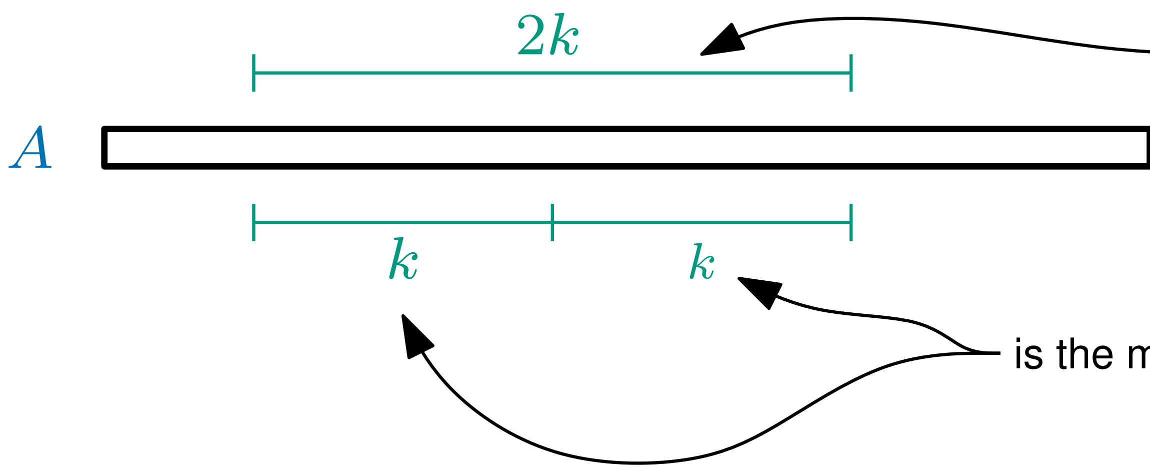

the min in here

is the min of these two mins
(which we have in R_k)

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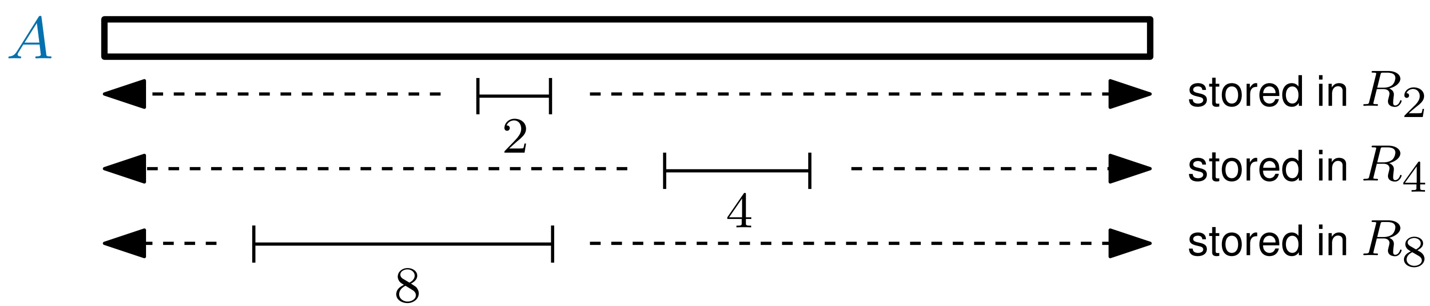

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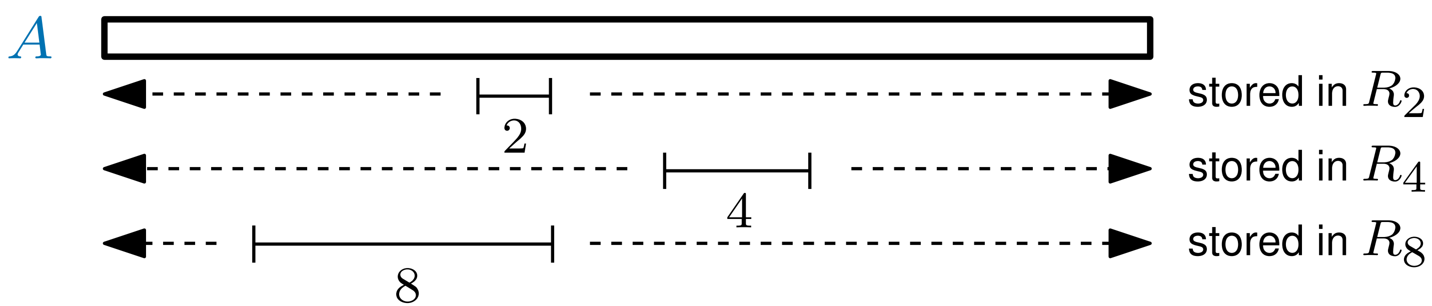

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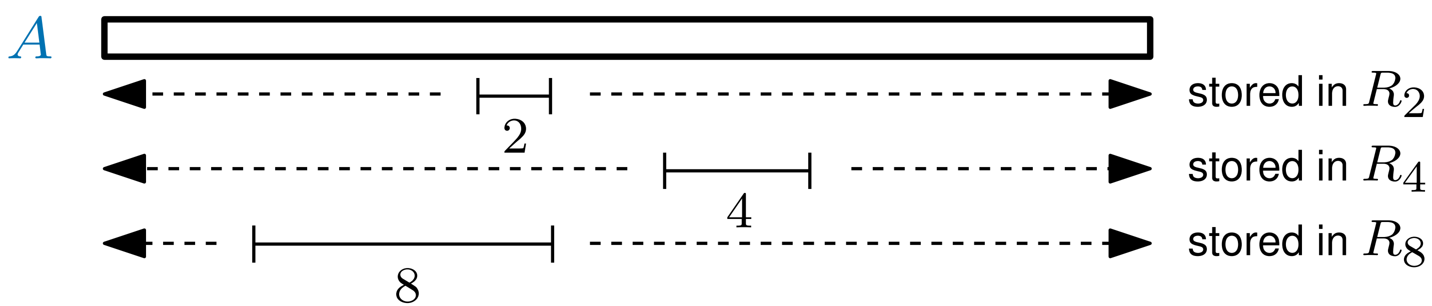

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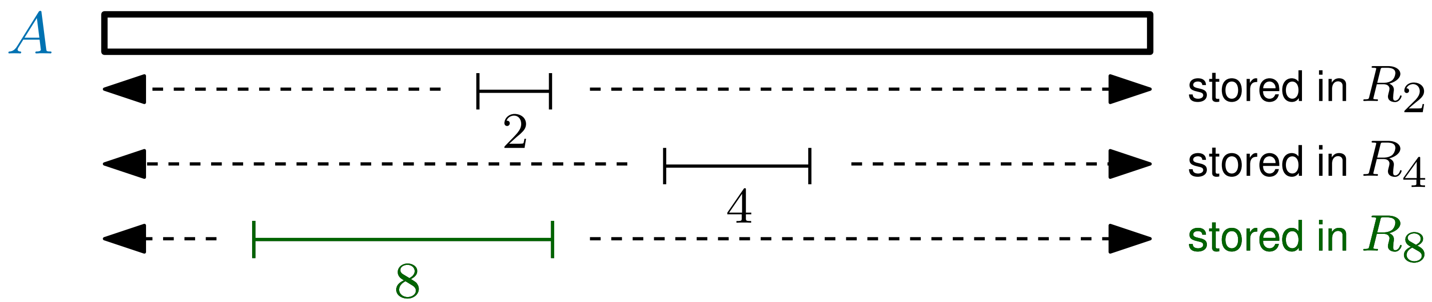

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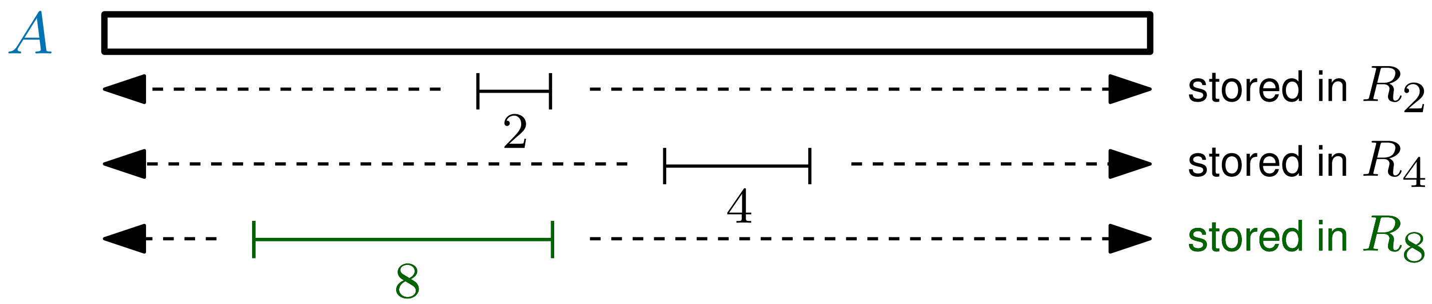

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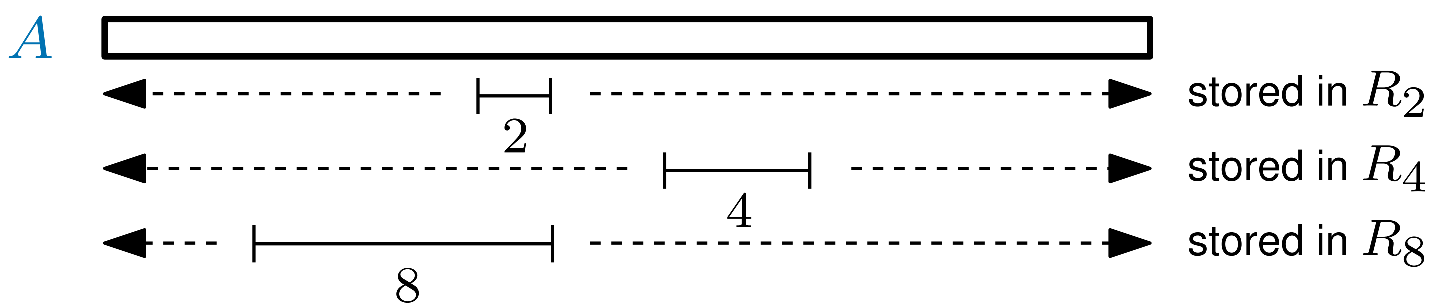

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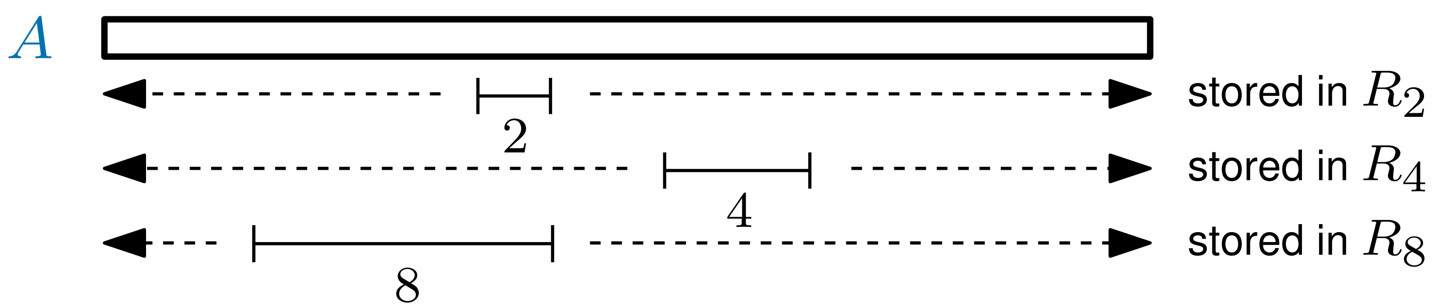

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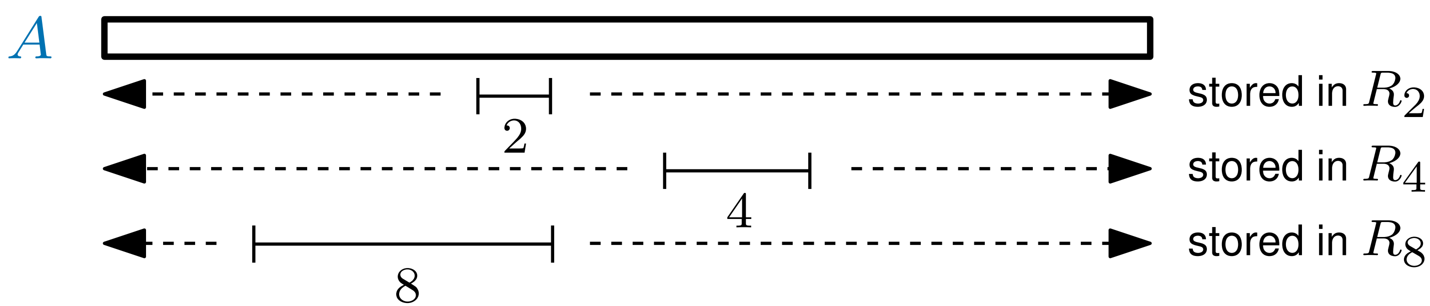

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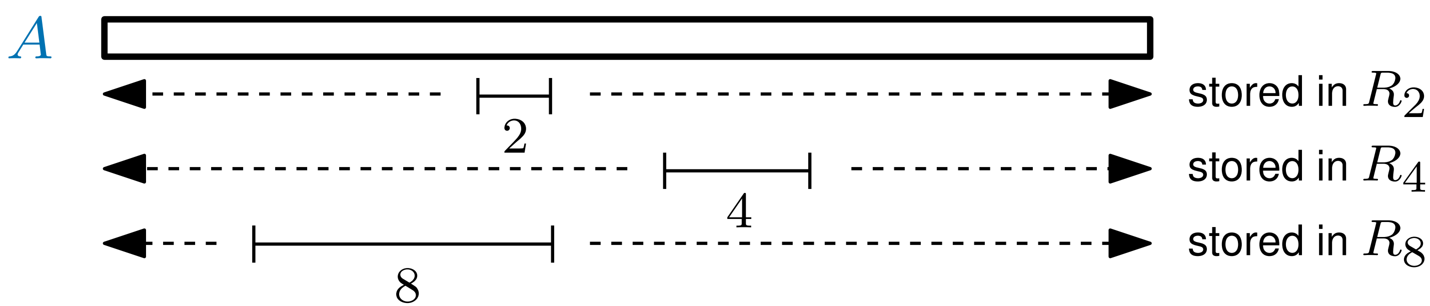

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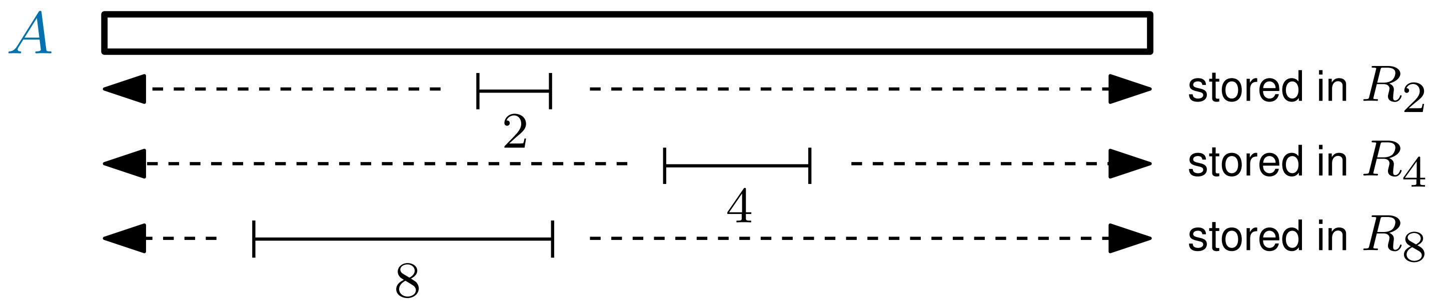

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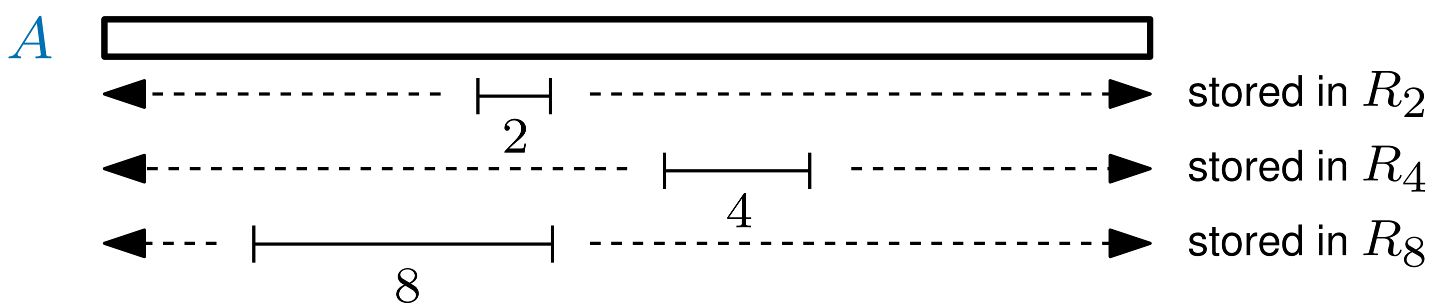

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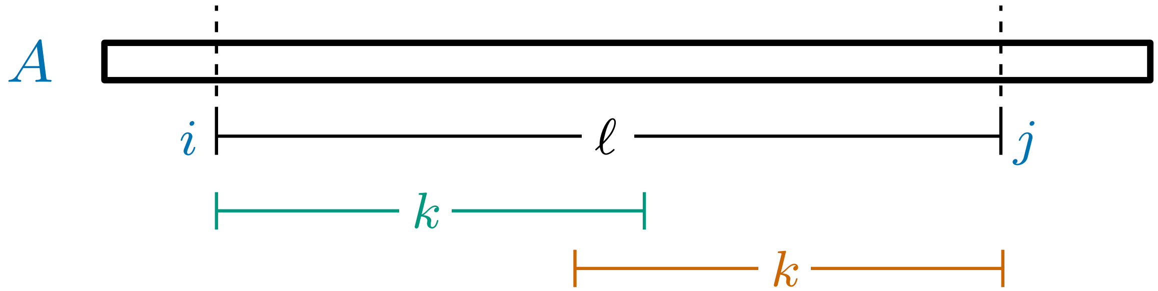

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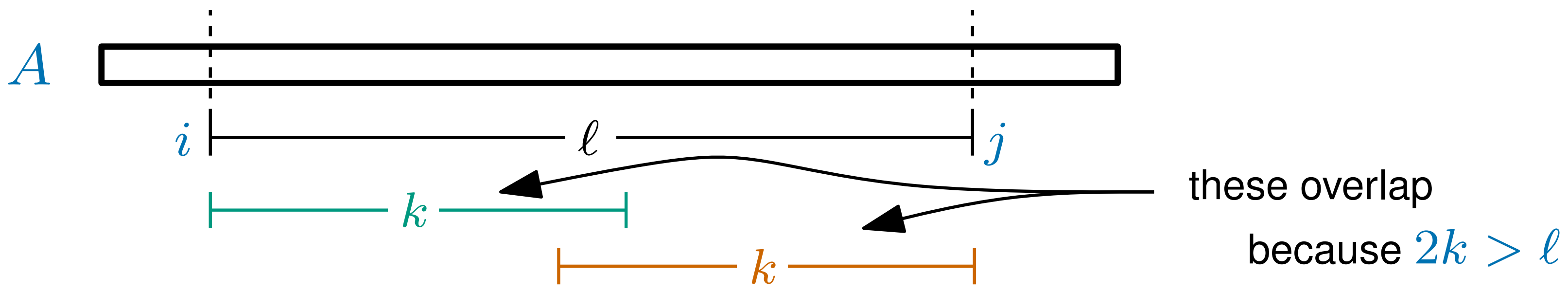

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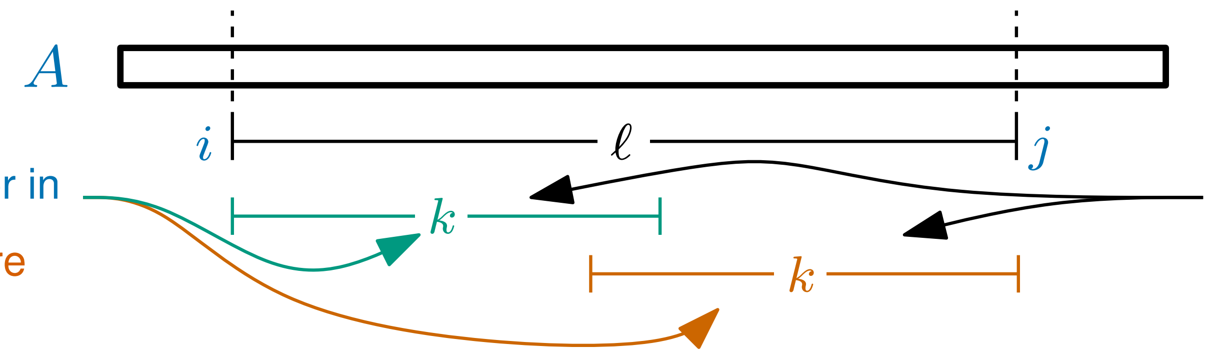

the min is either in here or in here

these overlap because $2k > \ell$

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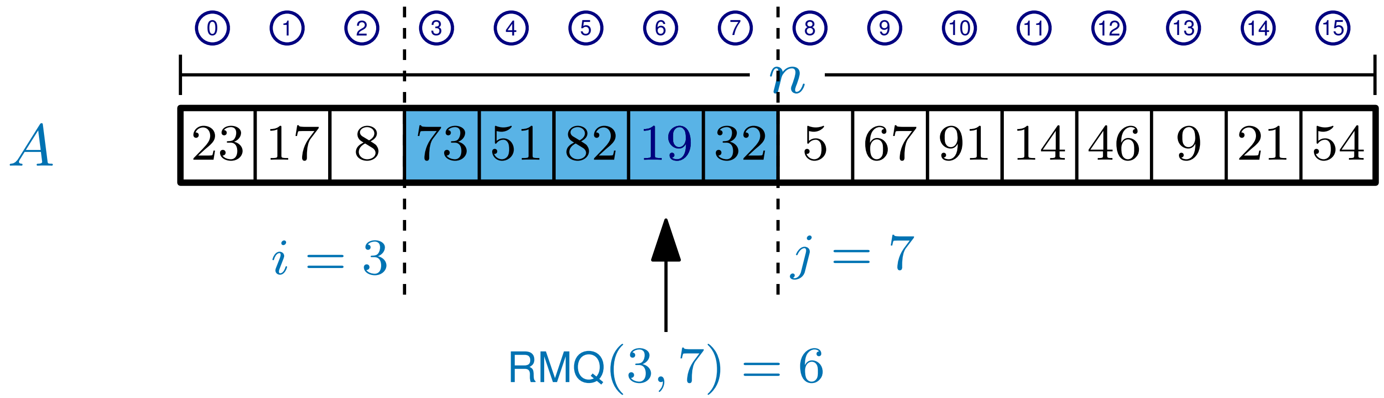
Range minimum query (intermediate) summary

Preprocess an integer array A (length n) to answer range minimum queries...



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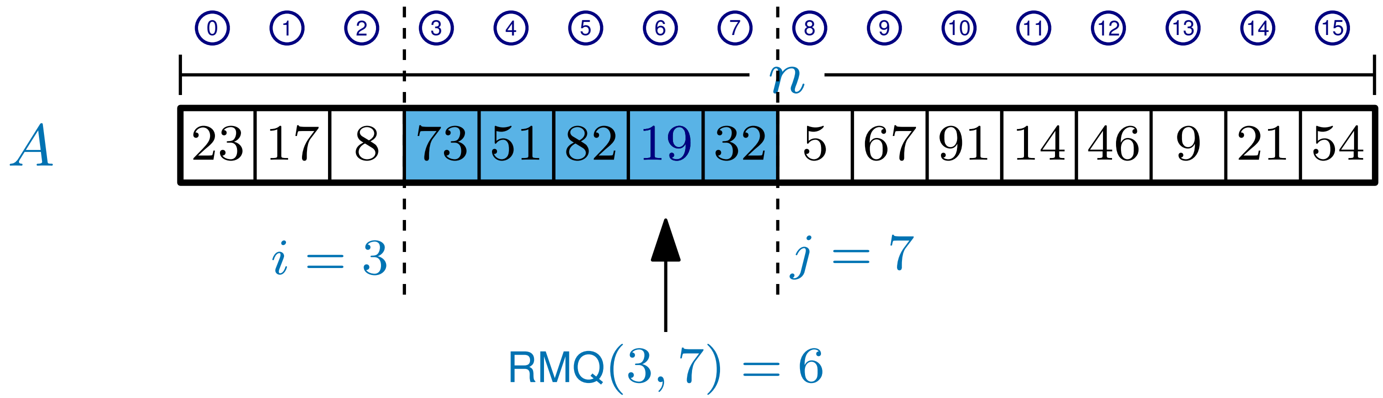

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$O(n)$ space

$O(n)$ prep time

$O(\log n)$ query time

Solution 2

$O(n \log n)$ space

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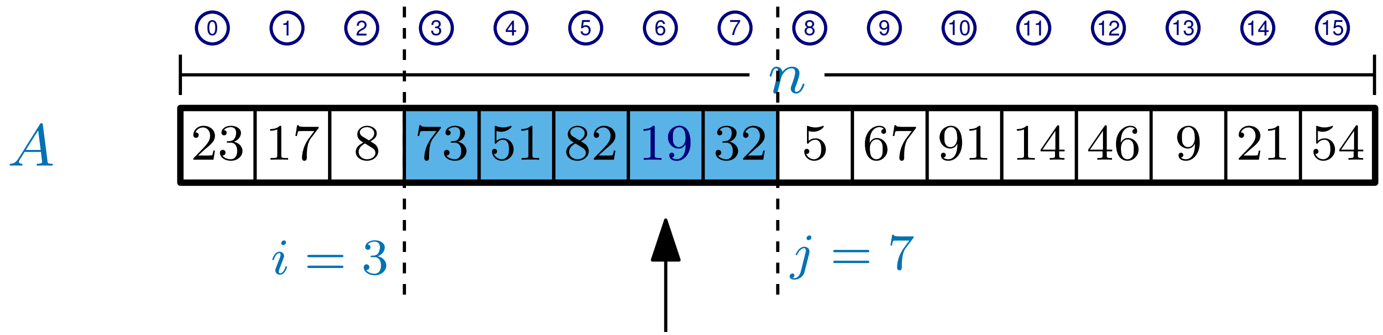

RMQ$(3, 7) = 6$

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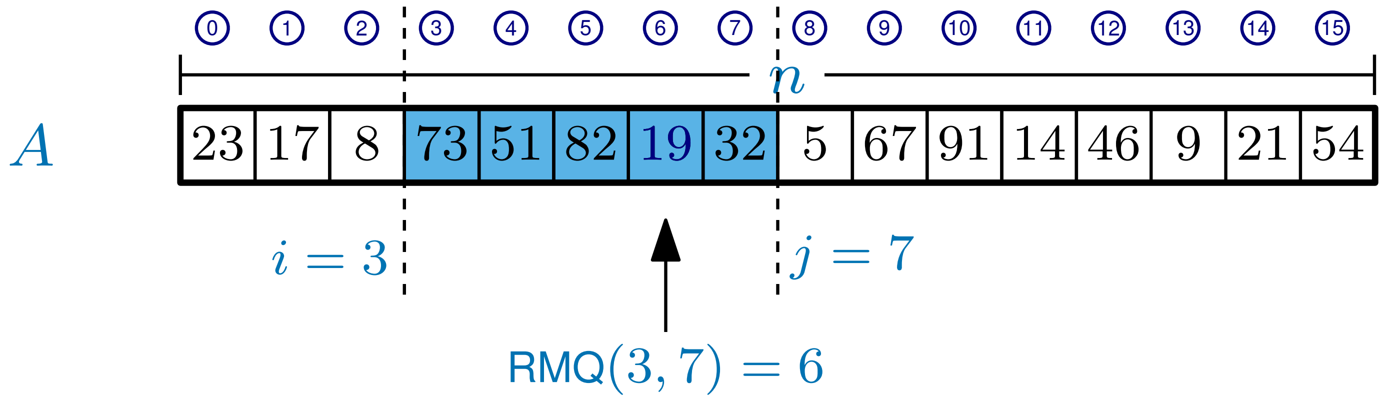

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Can we do better?

(yes)

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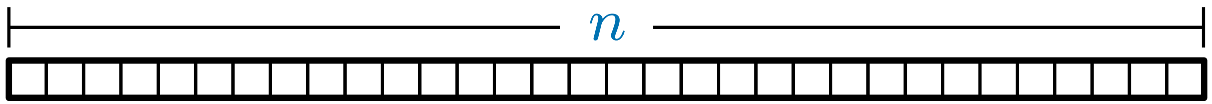
Low-resolution RMQ

Key Idea replace A with a smaller, *'low resolution'* array H

A



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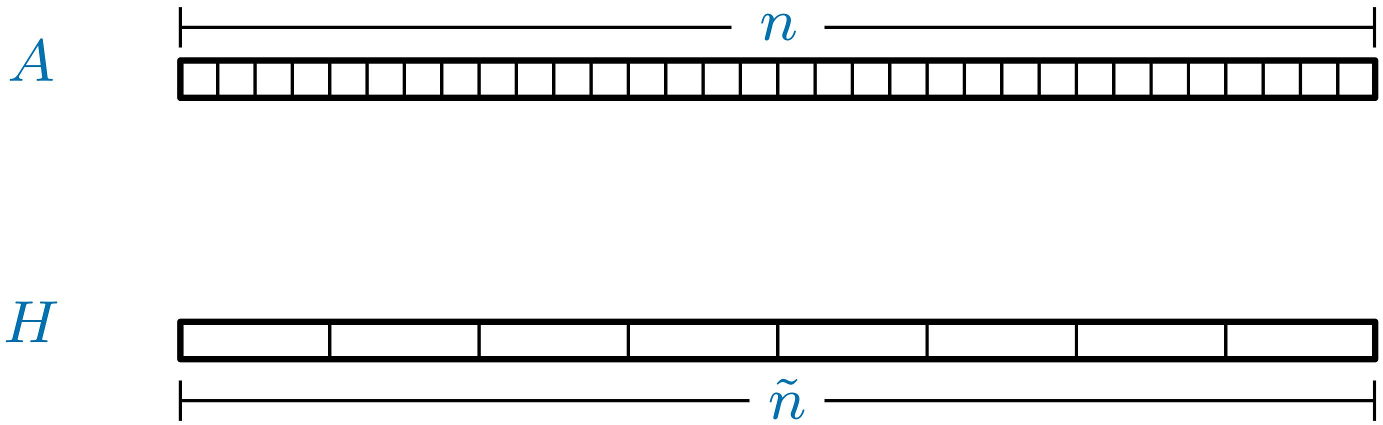

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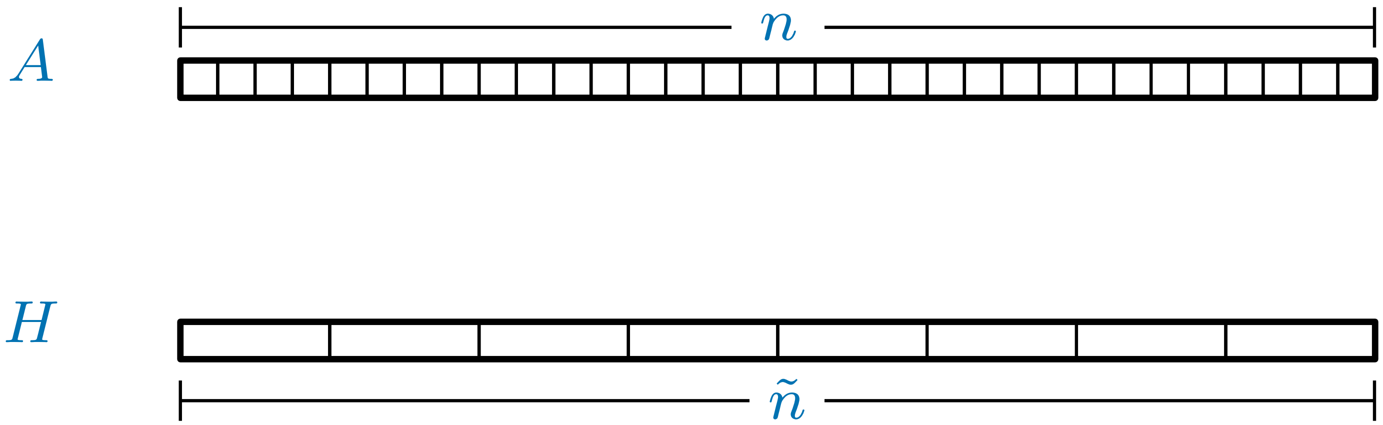
$$\boxed{\tilde{n} = \frac{n}{\log n}}$$



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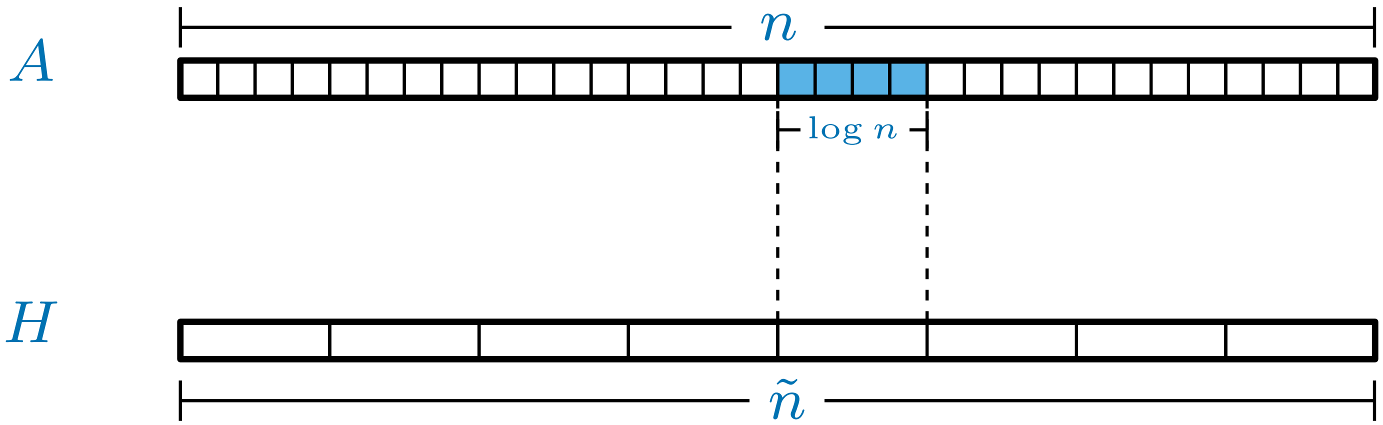

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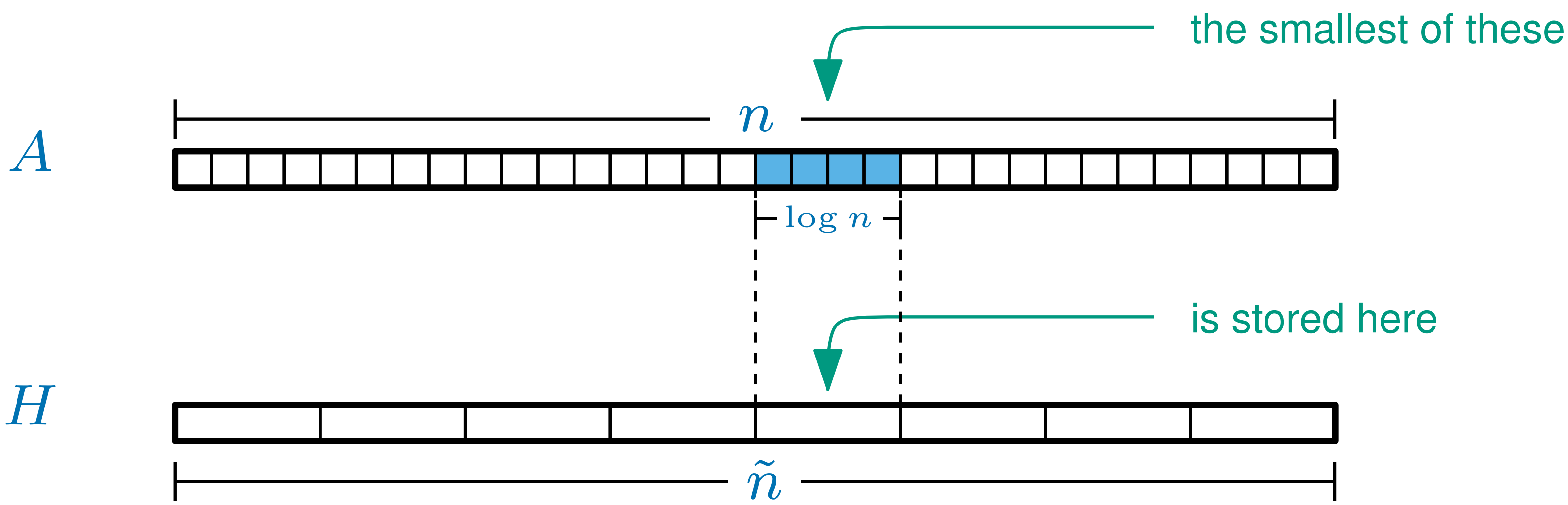

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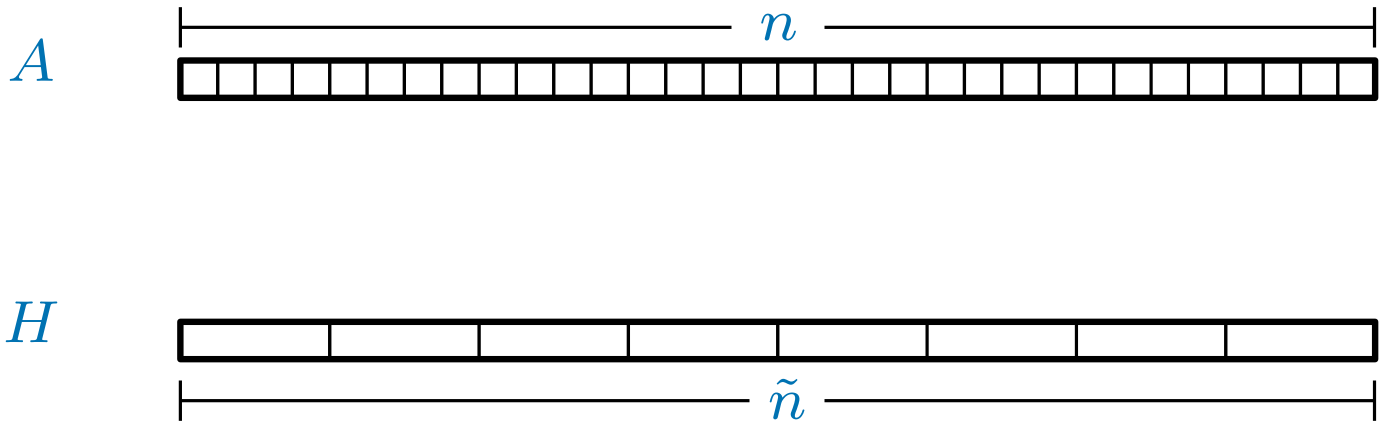

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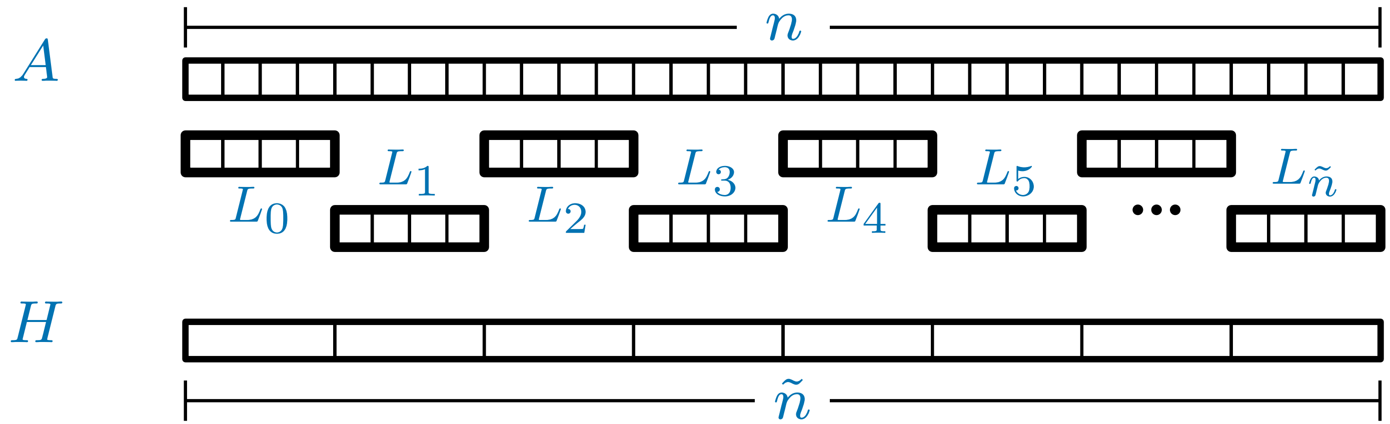
and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



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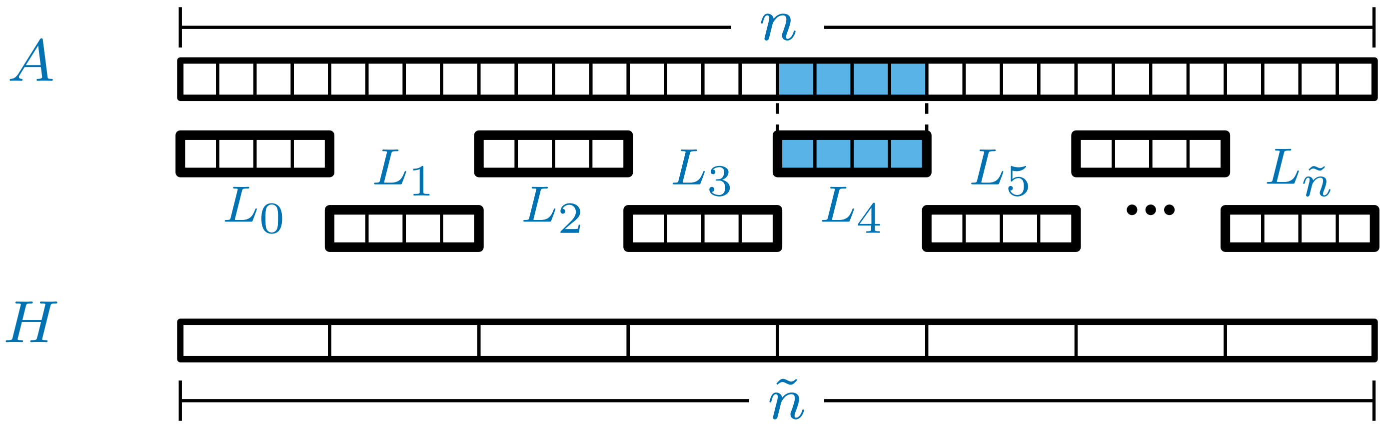

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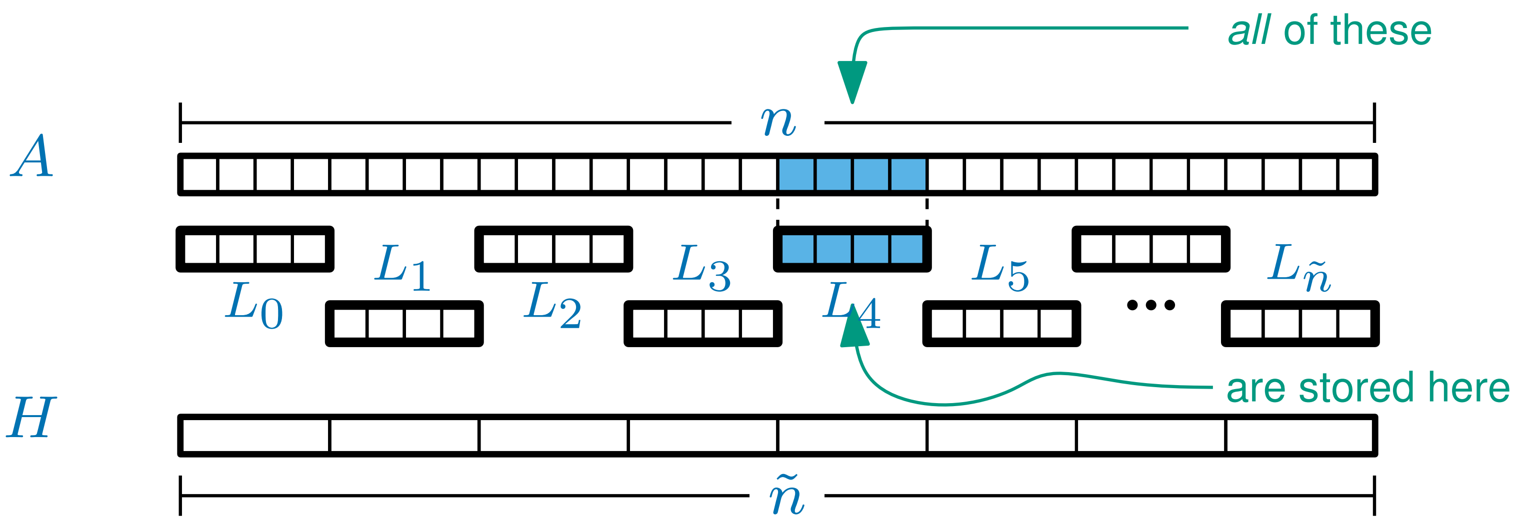

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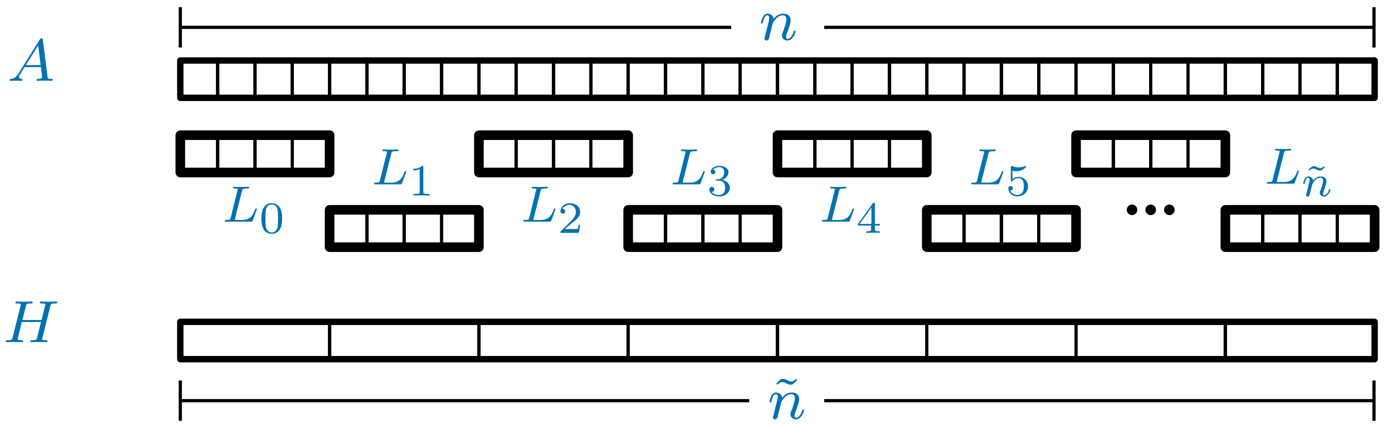

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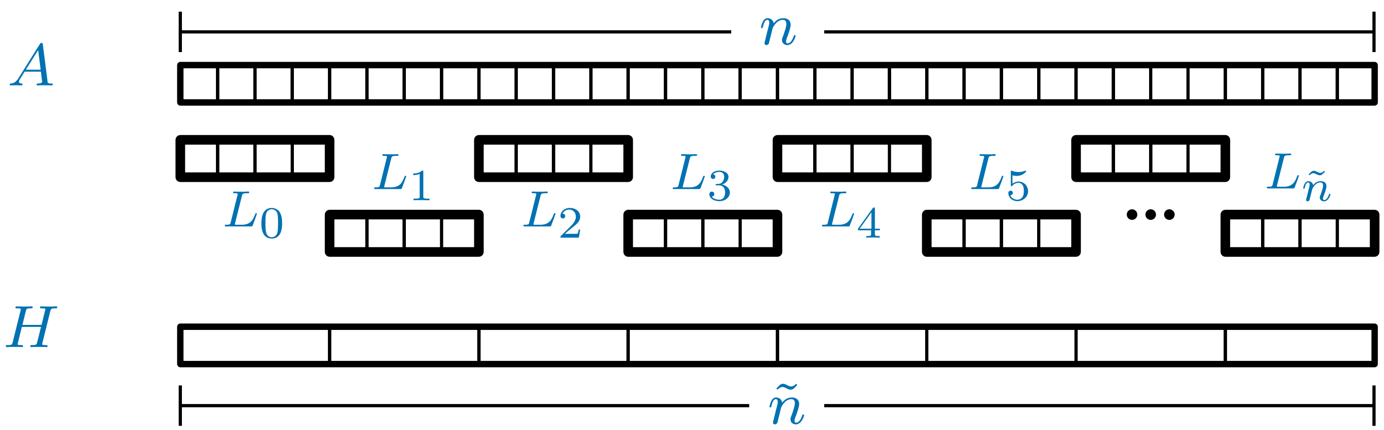

Preprocess the array H (which has length $\tilde{n} = \frac{n}{\log n}$) to answer RMQs. . .

Low-resolution RMQ

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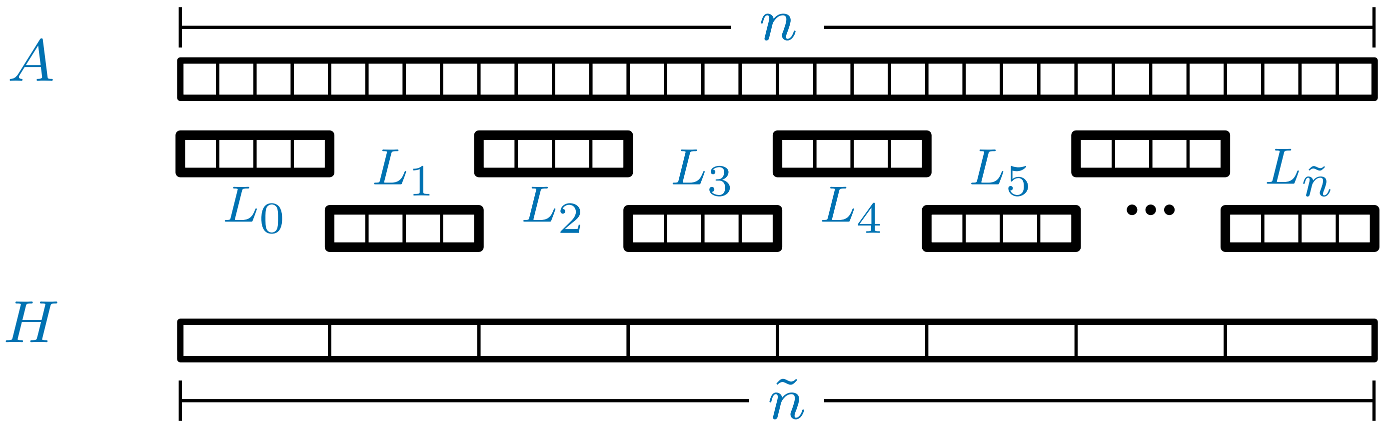

Preprocess the array H (which has length $\tilde{n} = \frac{n}{\log n}$) to answer RMQs...

using **Solution 2**

Low-resolution RMQ

$$\boxed{\tilde{n} = \frac{n}{\log n}}$$

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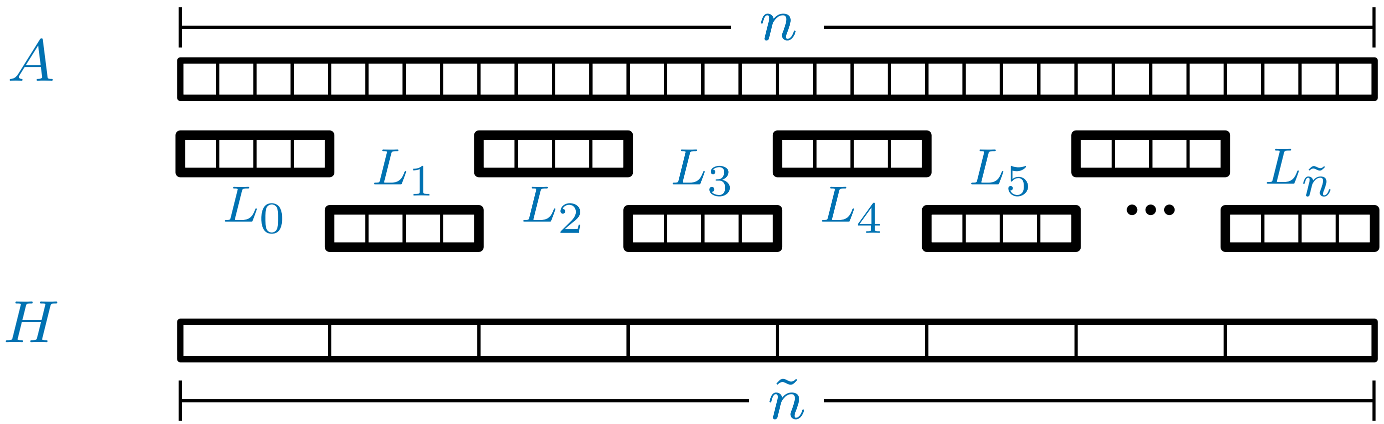

Preprocess the array H (which has length $\tilde{n} = \frac{n}{\log n}$) to answer RMQs. . .

using **Solution 2**

Recall. . .

Solution 2 on A

$O(n \log n)$ space

$O(n \log n)$ prep time

$O(1)$ query time

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

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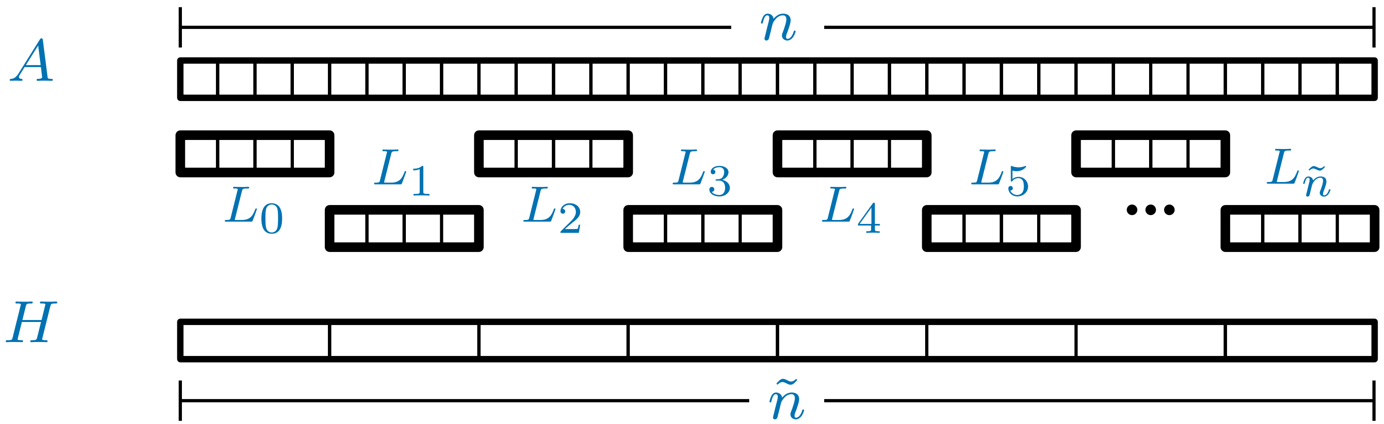

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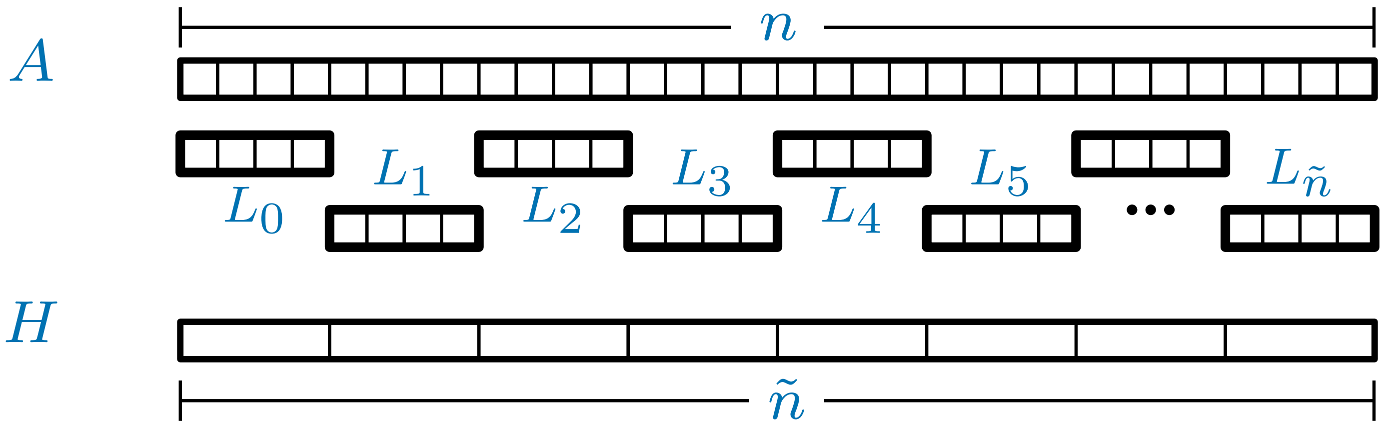

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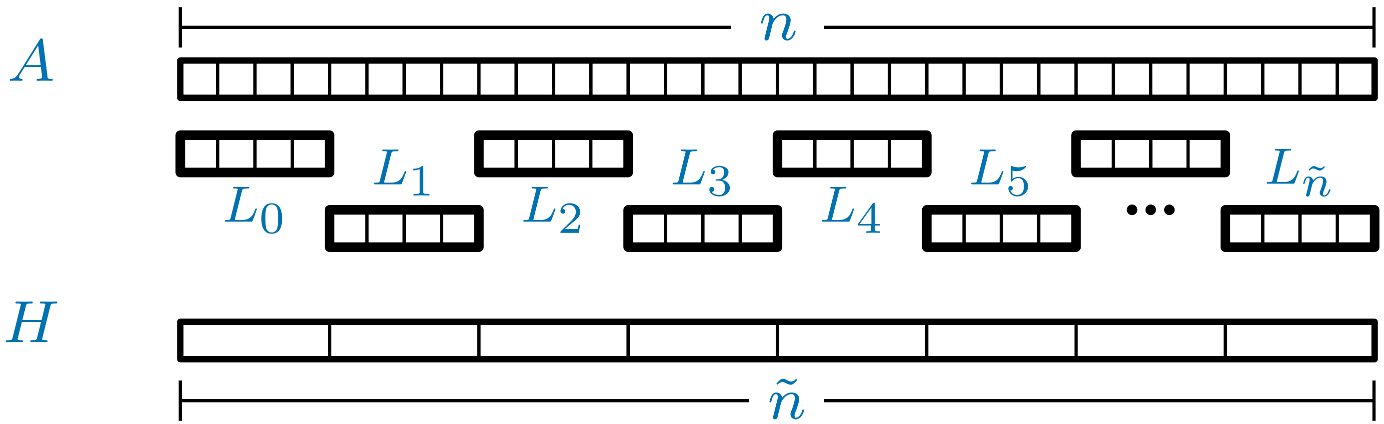

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Solution 2 on H

$O(\tilde{n} \log \tilde{n})$ space $= O\left(\left(\frac{n}{\log n}\right) \log \left(\frac{n}{\log n}\right)\right)$

$O(\tilde{n} \log \tilde{n})$ prep time

$O(1)$ query time

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

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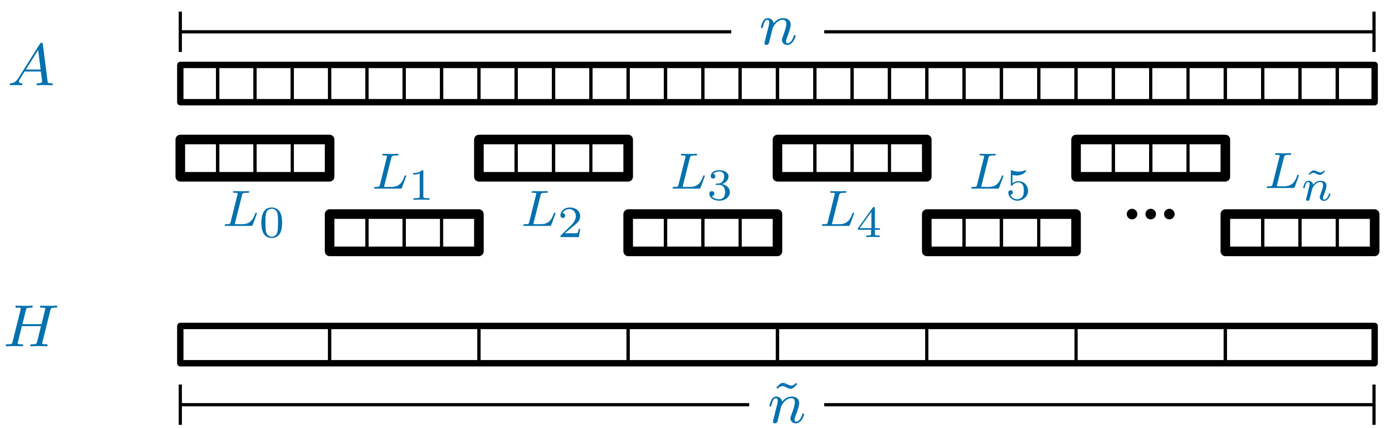

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Solution 2 on A

$O(n \log n)$ space

$O(n \log n)$ prep time

$O(1)$ query time

Solution 2 on H

$O(\tilde{n} \log \tilde{n})$ space $= O\left(\left(\frac{n}{\log n}\right) \log\left(\frac{n}{\log n}\right)\right) = O(n)$

$O(\tilde{n} \log \tilde{n})$ prep time

$O(1)$ query time

Low-resolution RMQ

$$\boxed{\tilde{n} = \frac{n}{\log n}}$$

Key Idea replace A with a smaller, *'low resolution'* array H

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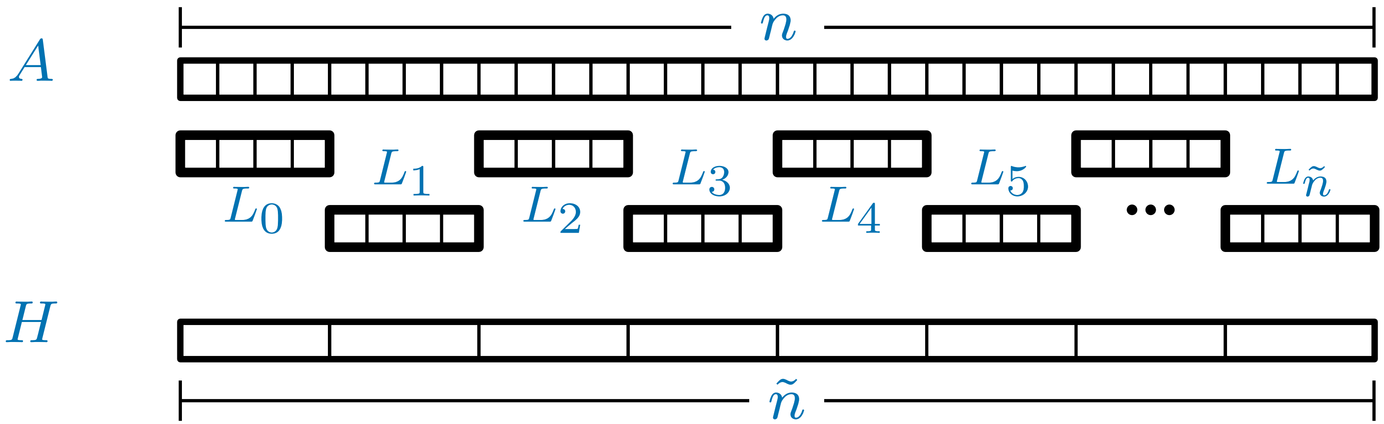

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Low-resolution RMQ

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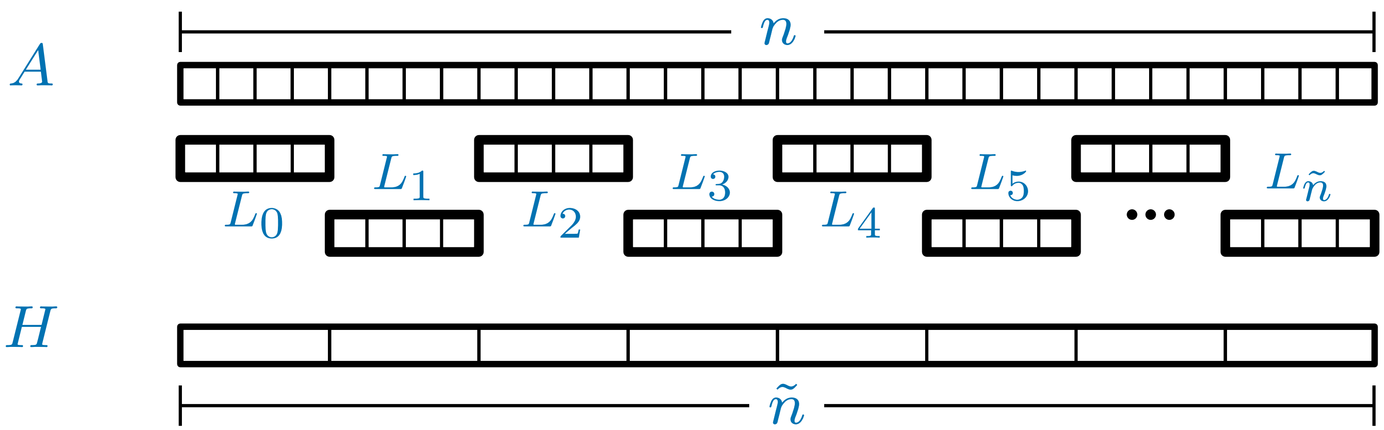

Preprocess the array H (which has length $\tilde{n} = \frac{n}{\log n}$) to answer RMQs...

using **Solution 2** in $O(n)$ space/prep time

Recall...

Solution 2 on A

$O(n \log n)$ space

$O(n \log n)$ prep time

$O(1)$ query time

Solution 2 on H

$O(\tilde{n} \log \tilde{n})$ space $= O\left(\left(\frac{n}{\log n}\right) \log \left(\frac{n}{\log n}\right)\right) = O(n)$

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Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

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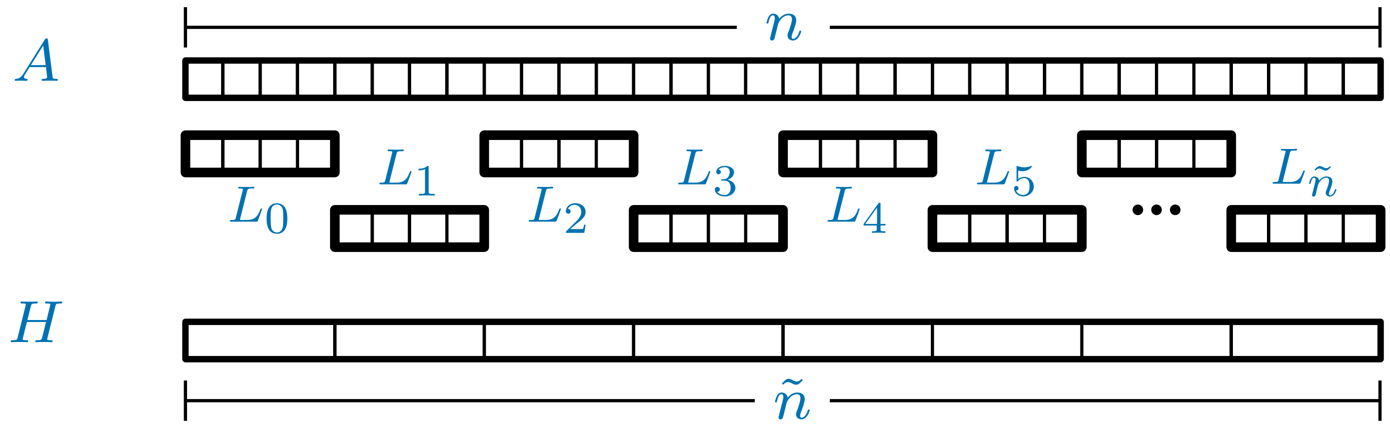

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Low-resolution RMQ

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Key Idea replace A with a smaller, *'low resolution'* array H

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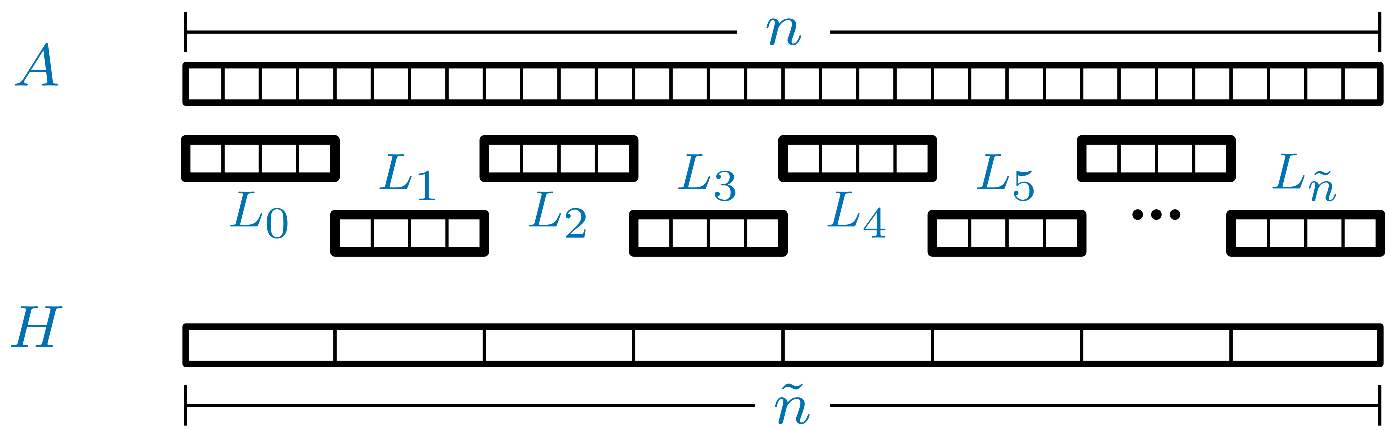


Preprocess the array H (which has length $\tilde{n} = \frac{n}{\log n}$) to answer RMQs...

using **Solution 2** in $O(n)$ space/prep time

Preprocess each array L_i (which has length $\log n$) to answer RMQs...

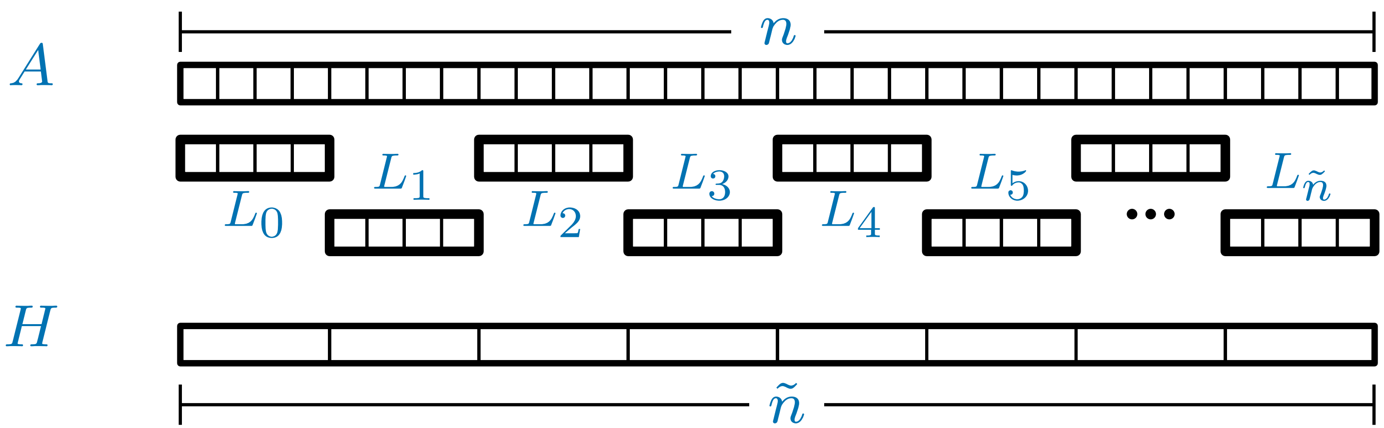
using **Solution 2**

Low-resolution RMQ

$$\boxed{\tilde{n} = \frac{n}{\log n}}$$

Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



Preprocess the array H (which has length $\tilde{n} = \frac{n}{\log n}$) to answer RMQs. . .

using **Solution 2** in $O(n)$ space/prep time

Preprocess each array L_i (which has length $\log n$) to answer RMQs. . .

using **Solution 2**

Solution 2 on L_i

$O((\log n) \log\log n))$ space/prep time $\quad$ $O(1)$ query time

Low-resolution RMQ

$\tilde{n} = \frac{n}{\log n}$

Key Idea replace A with a smaller, *'low resolution'* array H

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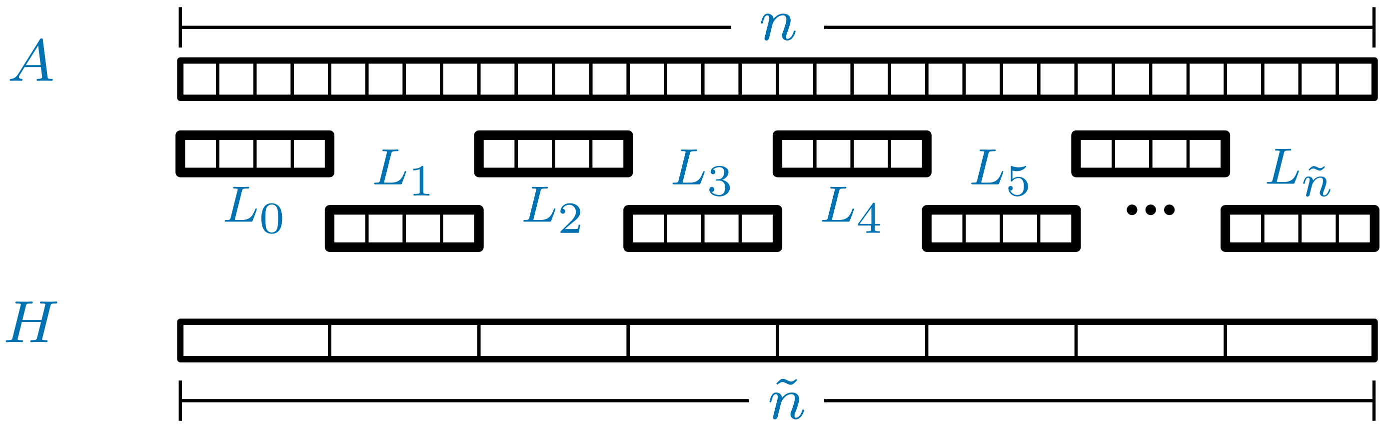

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Preprocess each array L_i (which has length $\log n$) to answer RMQs. . .

using **Solution 2** in $O(\log n \log \log n)$ space/prep time

Solution 2 on L_i

$O((\log n) \log \log n))$ space/prep time $\quad$ $O(1)$ query time

Low-resolution RMQ

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Key Idea replace A with a smaller, *'low resolution'* array H

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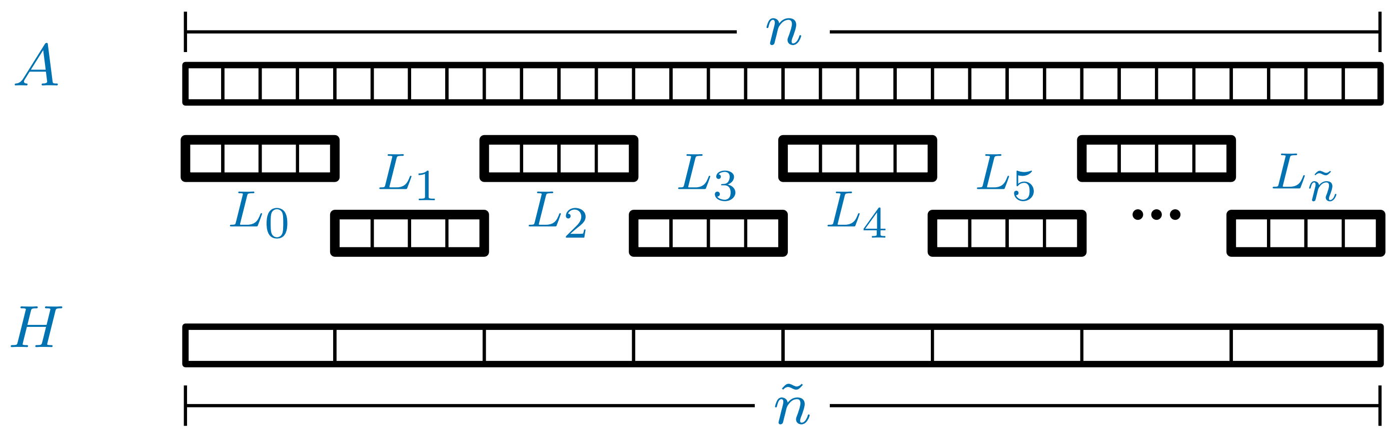

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using **Solution 2** in $O(n)$ space/prep time

Preprocess each array L_i (which has length $\log n$) to answer RMQs. . .

using **Solution 2** in $O(\log n \log \log n)$ space/prep time

Total space = $O(n) + O(\tilde{n} \log n \log \log n)$

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

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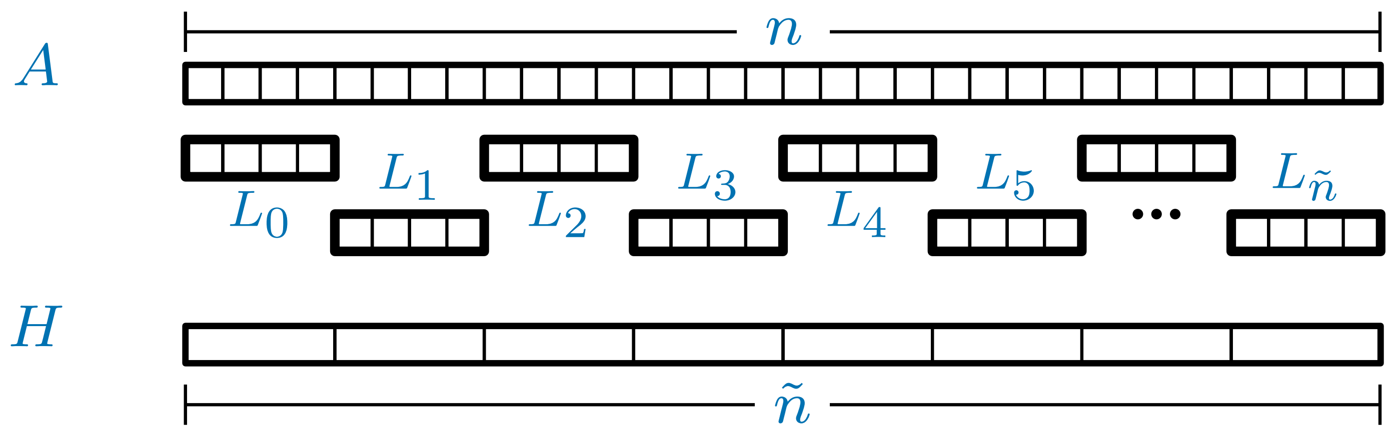

Preprocess the array H (which has length $\tilde{n} = \frac{n}{\log n}$) to answer RMQs...

using **Solution 2** in $O(n)$ space/prep time

Preprocess each array L_i (which has length $\log n$) to answer RMQs...

using **Solution 2** in $O(\log n \log \log n)$ space/prep time

Total space = $O(n) + O(\tilde{n} \log n \log \log n)$

space for RMQ structure for H

space for RMQ structures for all the L_i arrays

Low-resolution RMQ

$$\boxed{\tilde{n} = \frac{n}{\log n}}$$

Key Idea replace A with a smaller, *'low resolution'* array H

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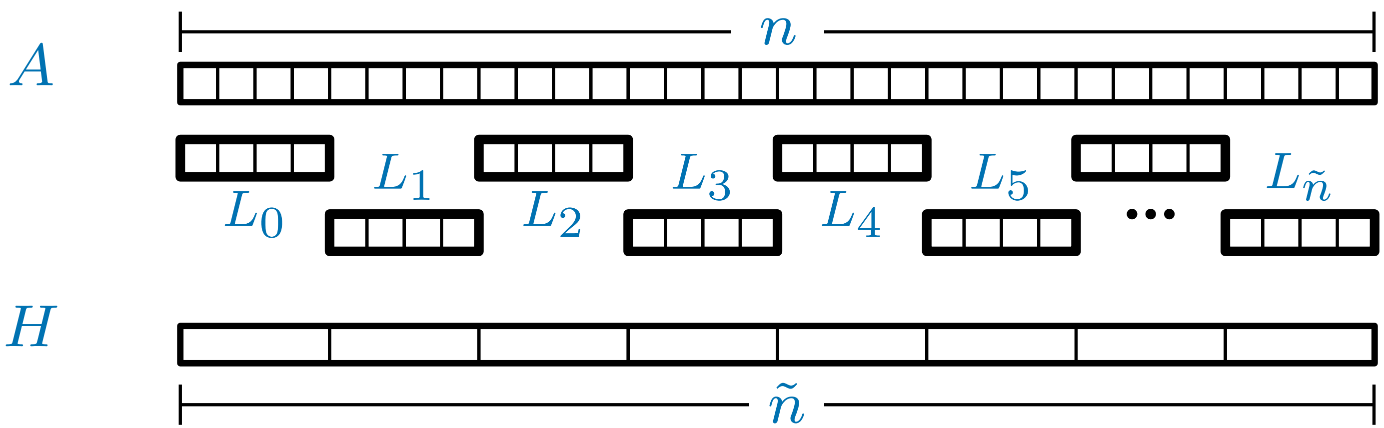

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Preprocess each array L_i (which has length $\log n$) to answer RMQs. . .

using **Solution 2** in $O(\log n \log \log n)$ space/prep time

Total space = $O(n) + O(\tilde{n} \log n \log \log n) = O(n \log \log n)$

space for RMQ structure for H

space for RMQ structures for all the L_i arrays

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

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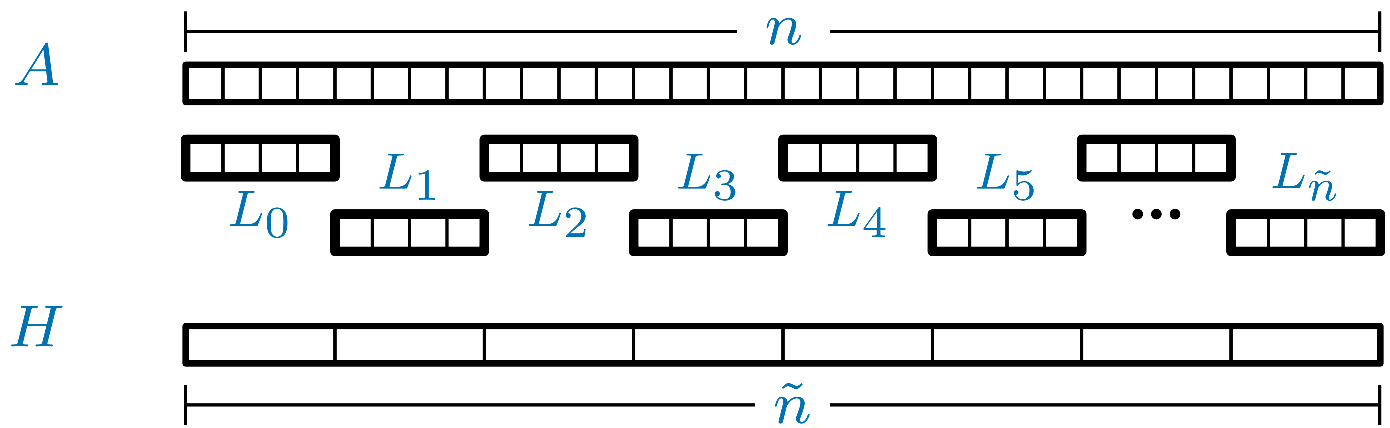

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using **Solution 2** in $O(n)$ space/prep time

Preprocess each array L_i (which has length $\log n$) to answer RMQs...

using **Solution 2** in $O(\log n \log\log n)$ space/prep time

Total space = $O(n) + O(\tilde{n} \log n \log\log n) = O(n \log\log n)$

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

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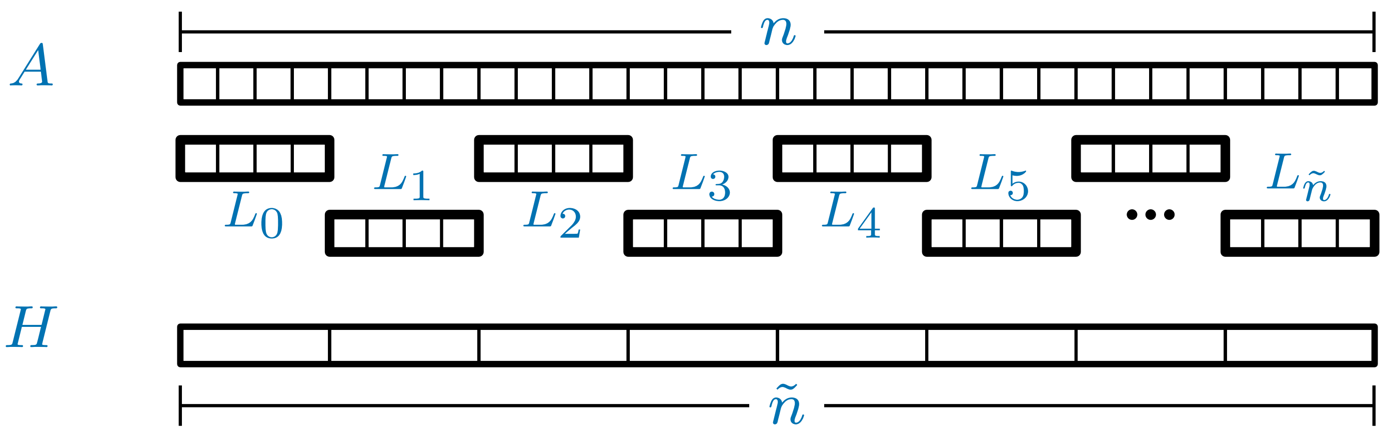

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using **Solution 2** in $O(n)$ space/prep time

Preprocess each array L_i (which has length $\log n$) to answer RMQs...

using **Solution 2** in $O(\log n \log \log n)$ space/prep time

Total space = $O(n) + O(\tilde{n} \log n \log \log n) \ = O(n \log \log n)$

Total prep. time = $O(n \log \log n)$

Low-resolution RMQ

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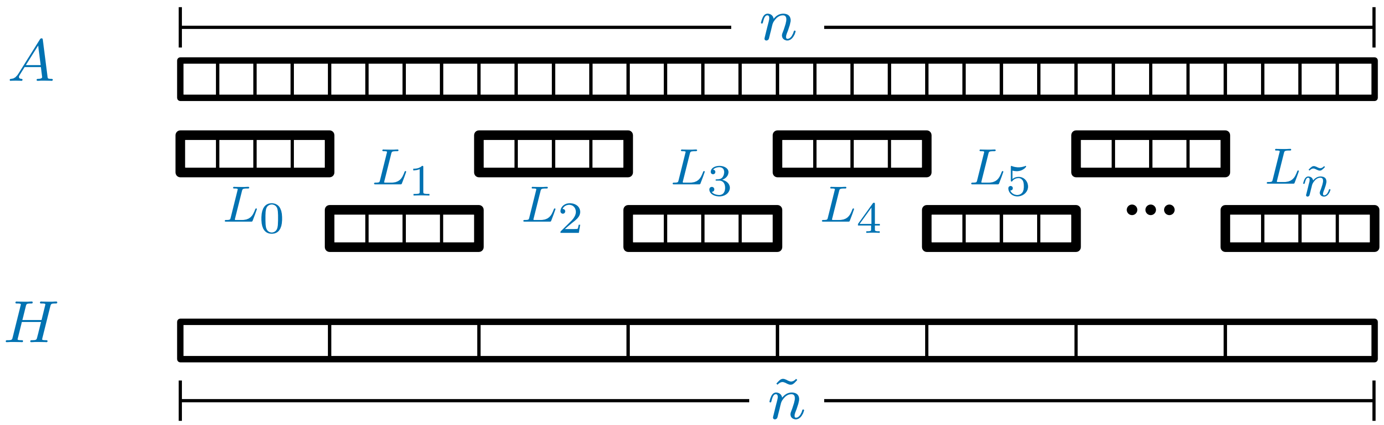

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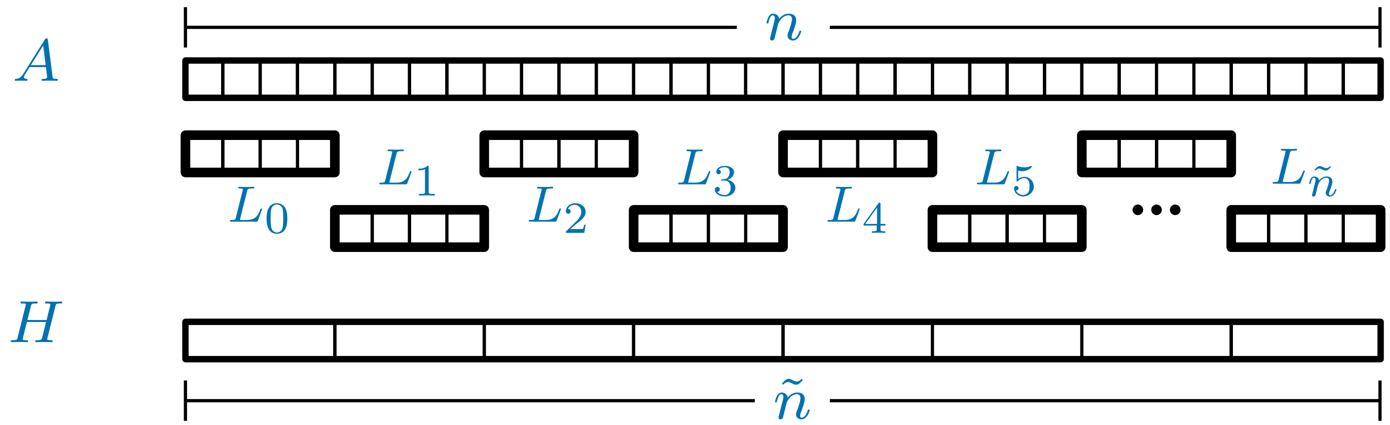

How do we answer a query in A?

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

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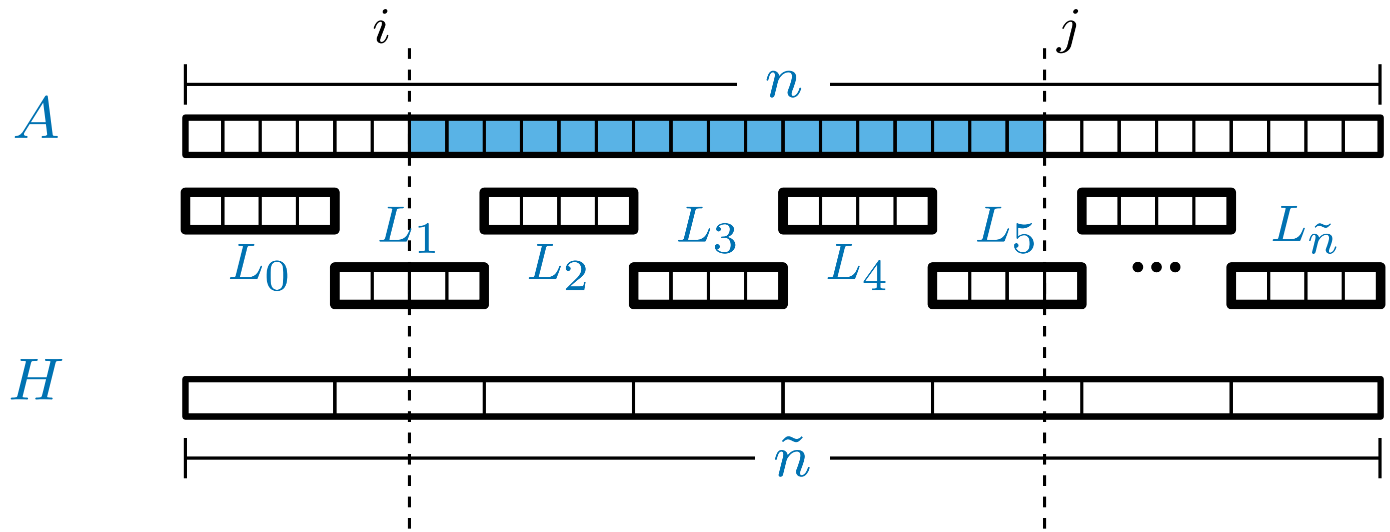

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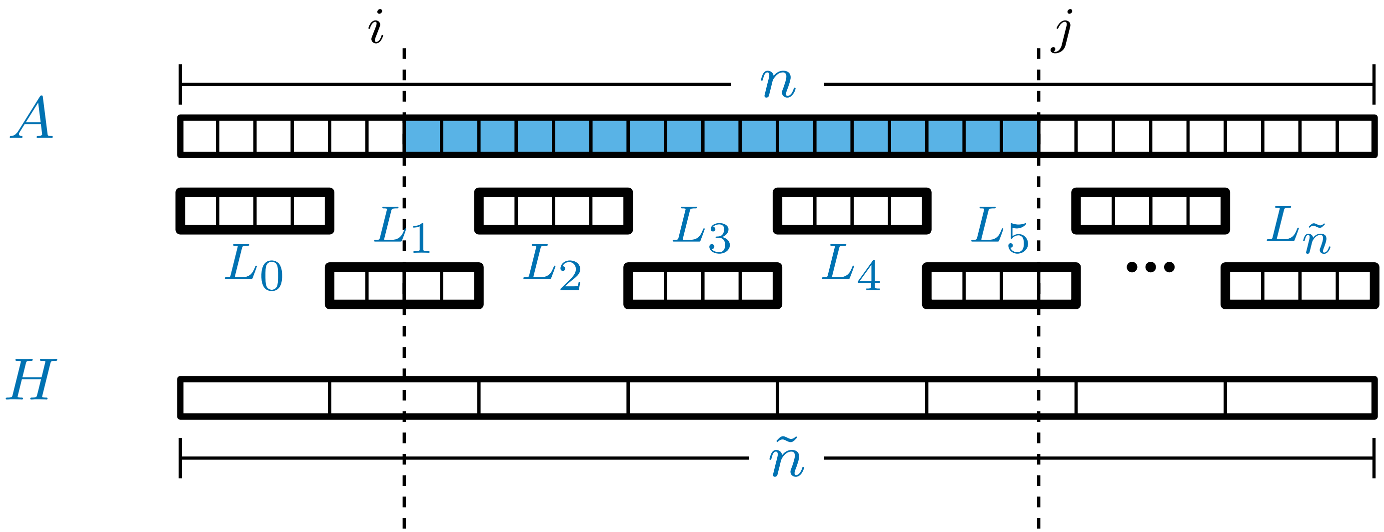

How do we answer a query in A?

Do at most one query in H...

and one query in at most two different L_i

then take the smallest

Low-resolution RMQ

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Key Idea replace A with a smaller, *'low resolution'* array H

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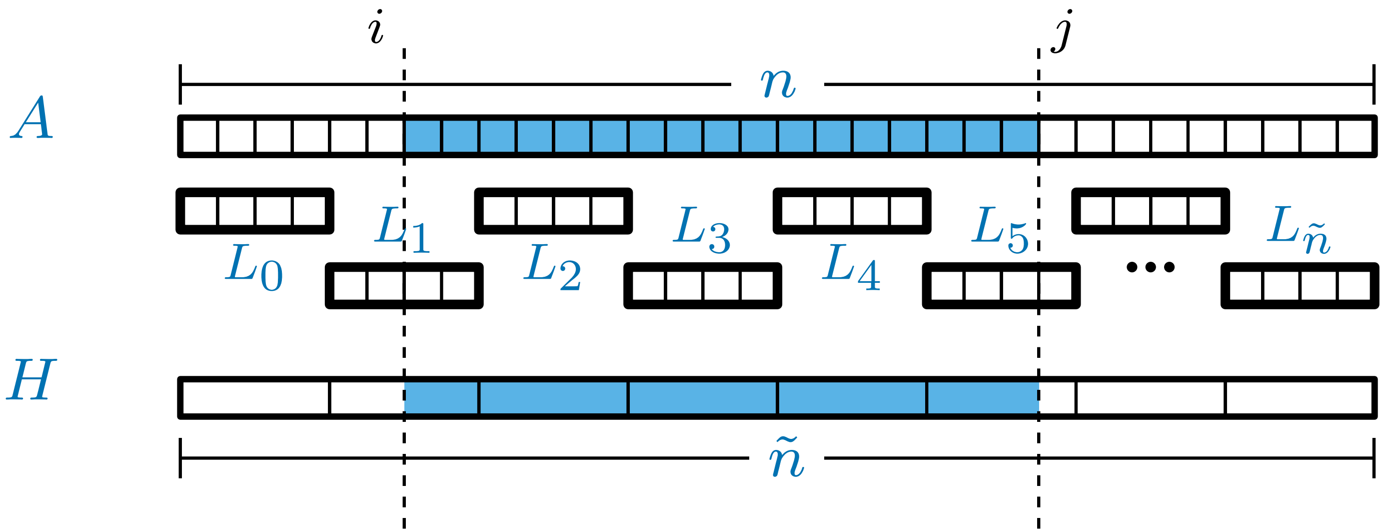

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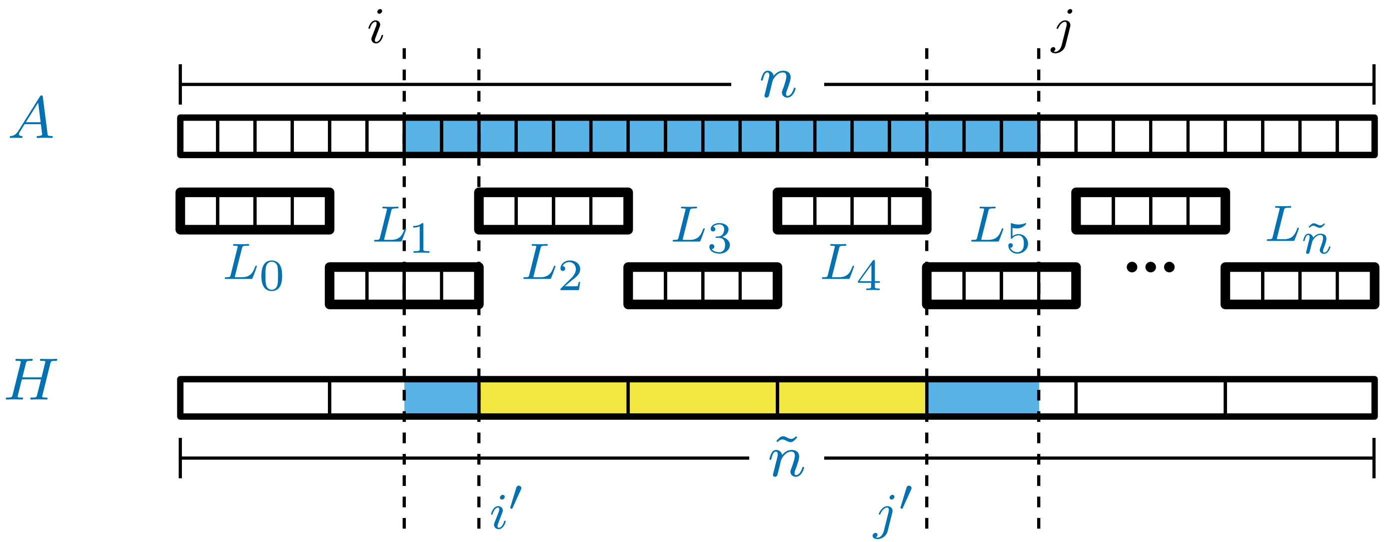

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Low-resolution RMQ

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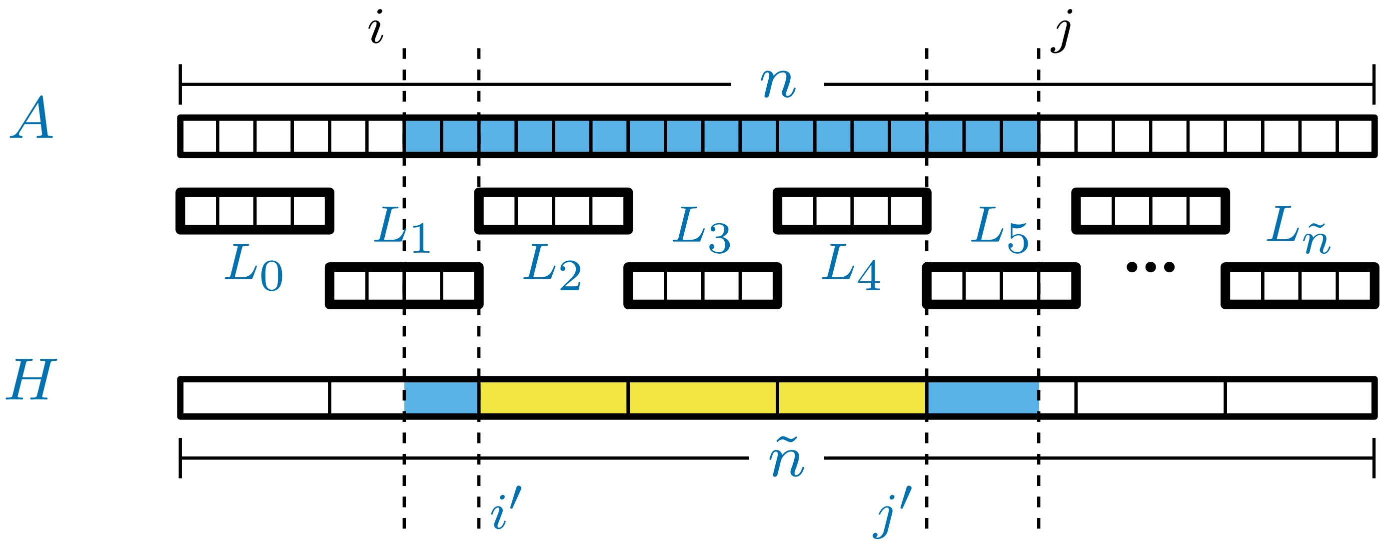

$$i' = \left\lceil \frac{i}{\log n} \right\rceil \quad j' = \left\lfloor \frac{j}{\log n} \right\rfloor$$

How do we answer a query in A?

Do at most one query in $H\ldots$

and one query in at most two different L_i

then take the smallest

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

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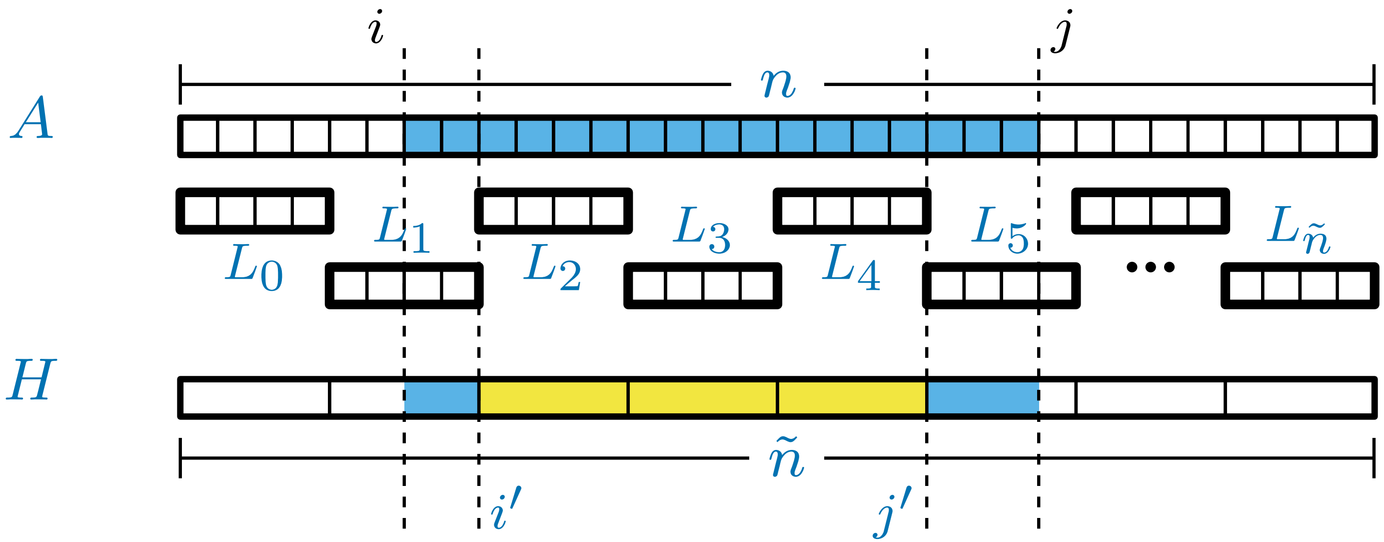

How do we answer a query in A?

$$i' = \left\lceil \frac{i}{\log n} \right\rceil \quad j' = \left\lfloor \frac{j}{\log n} \right\rfloor$$

indices into H

Do at most one query in H...

and one query in at most two different L_i

then take the smallest

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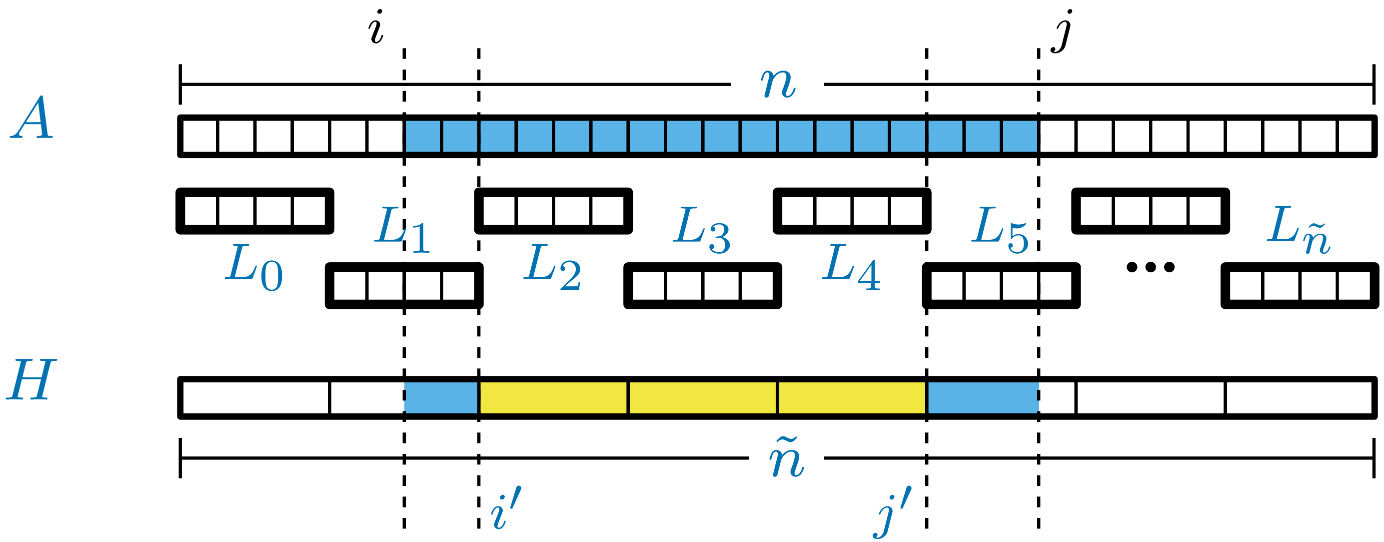
Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



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Low-resolution RMQ

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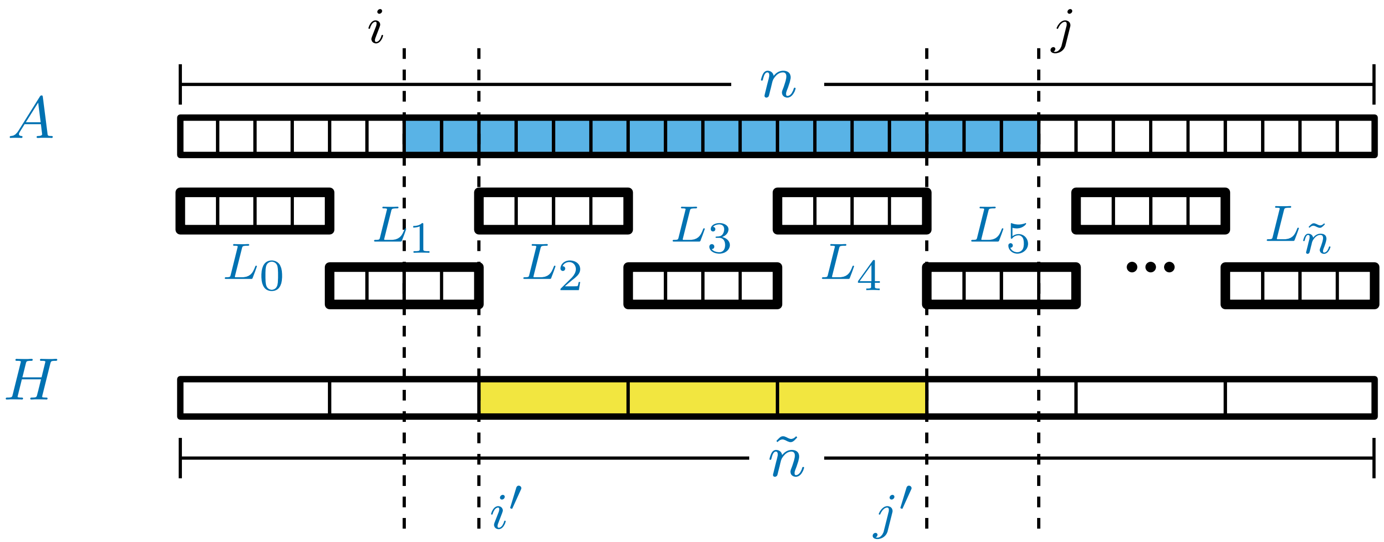
Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



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How do we answer a query in A?

Do at most one query in $H \ldots$

and one query in at most two different L_i

then take the smallest

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

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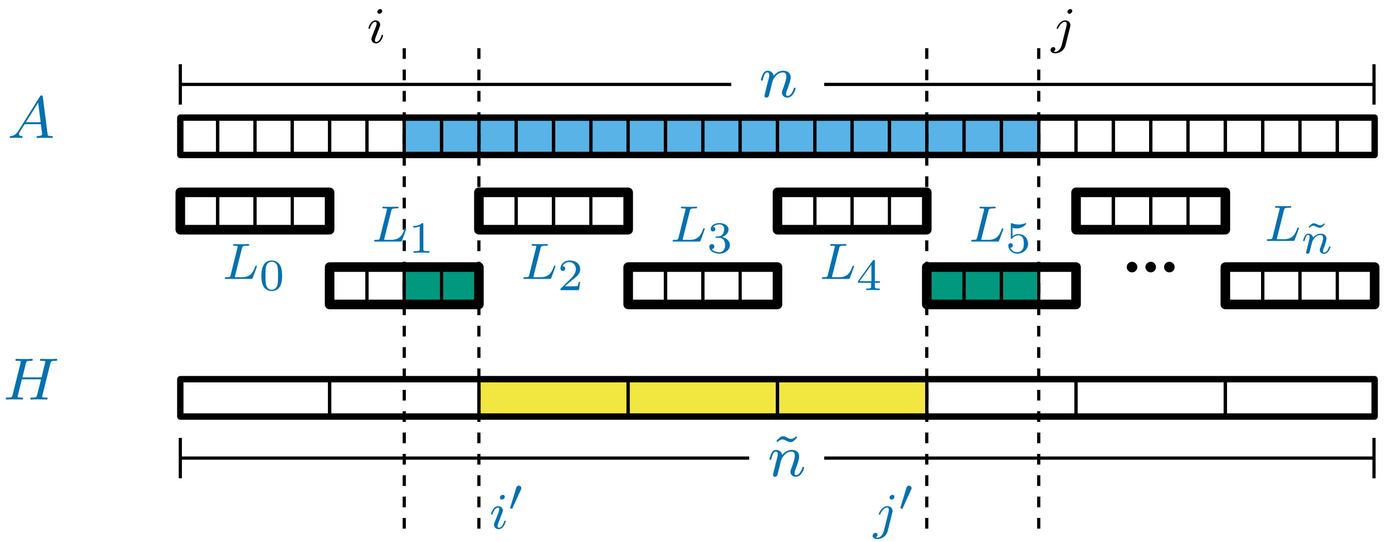

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How do we answer a query in A?

Do at most one query in H...

and one query in at most two different L_i

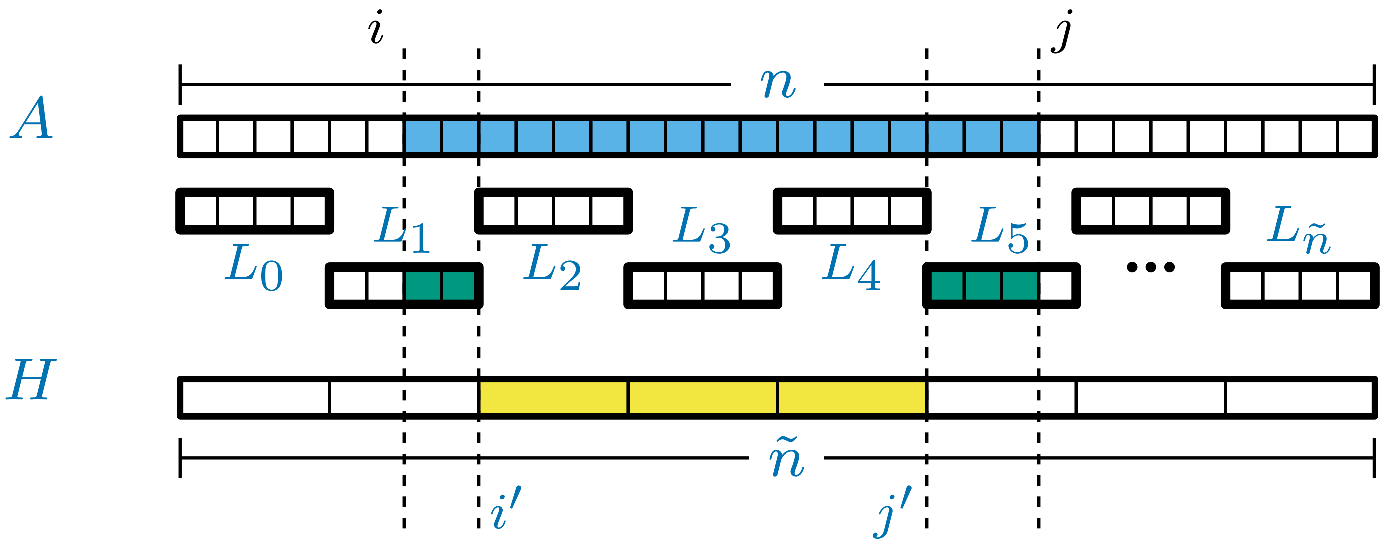
then take the smallest

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



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How do we answer a query in A?

Do at most one query in H...

and one query in at most two different L_i (here we query L_1 and L_5)

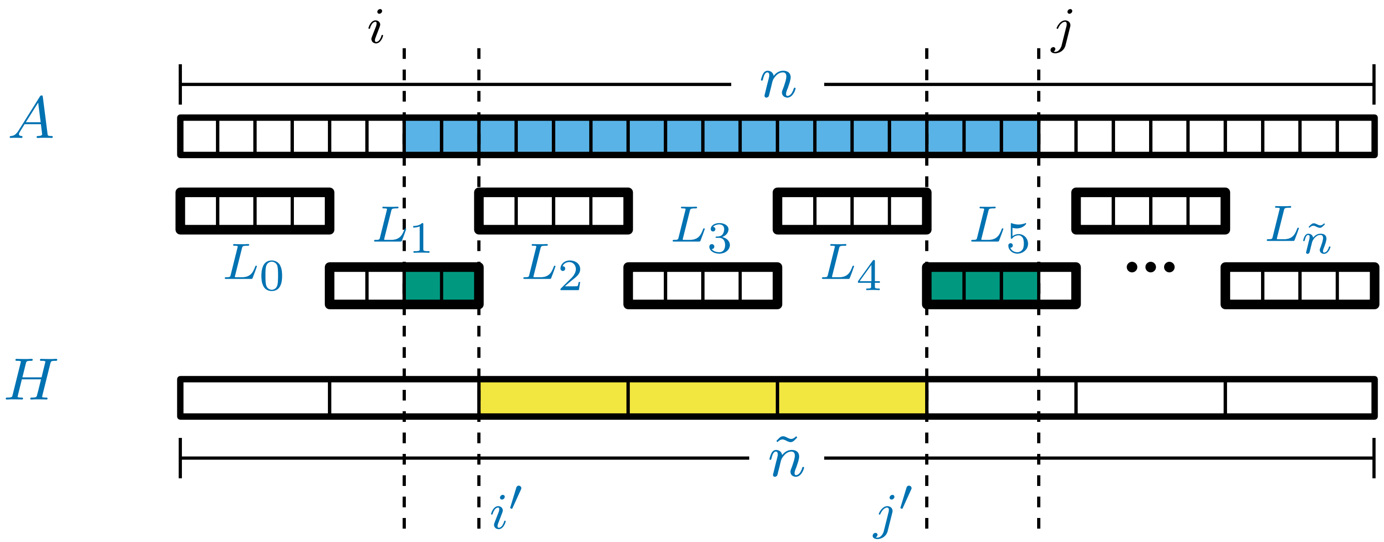
then take the smallest

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



$$i' = \left\lceil \frac{i}{\log n} \right\rceil \quad j' = \left\lfloor \frac{j}{\log n} \right\rfloor$$

How do we answer a query in A?

Do at most one query in $H \ldots$

and one query in at most two different L_i (here we query L_1 and L_5)

then take the smallest

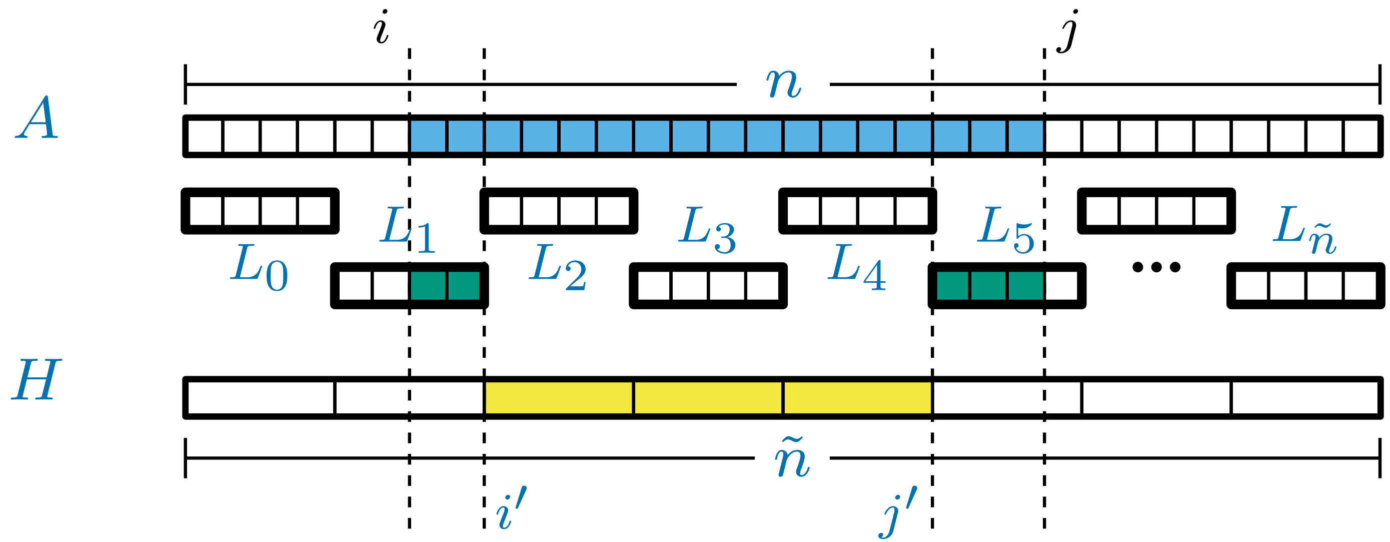
This takes $O(1)$ *total query time*

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



How do we answer a query in A?

$$i' = \left\lceil \frac{i}{\log n} \right\rceil \quad j' = \left\lfloor \frac{j}{\log n} \right\rfloor$$

Do at most one query in H...

and one query in at most two different L_i (here we query L_1 and L_5)

then take the smallest

This takes $O(1)$ total query time

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

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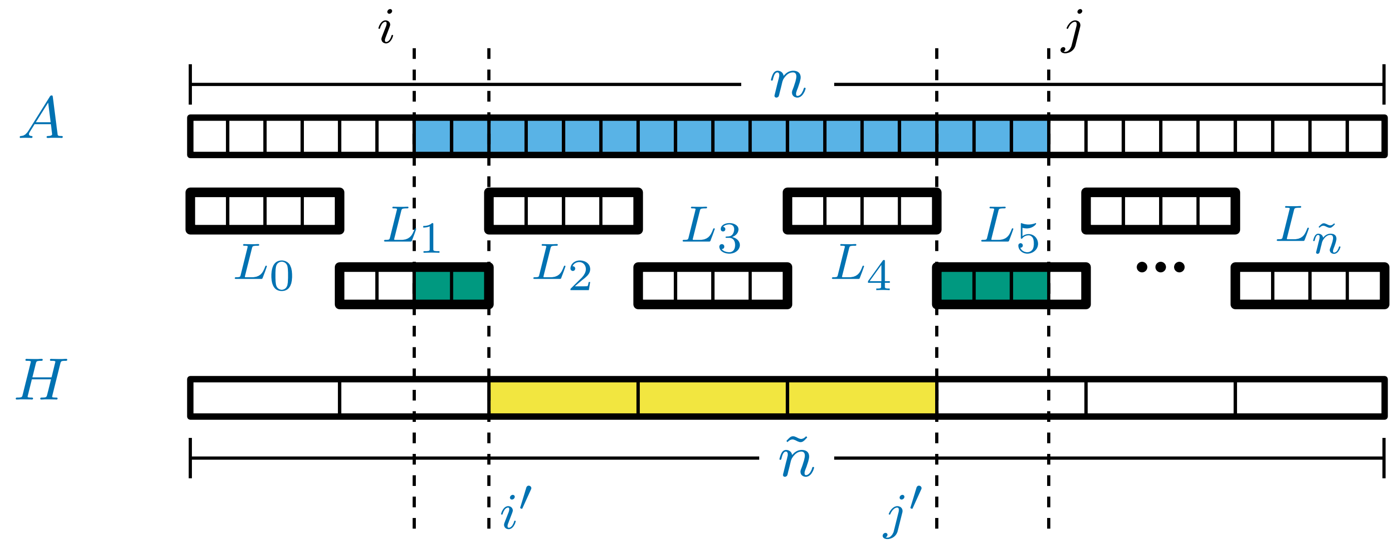

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Solution 3

$O(n \log \log n)$ space $O(n \log \log n)$ prep time $O(1)$ query time

Low-resolution RMQ

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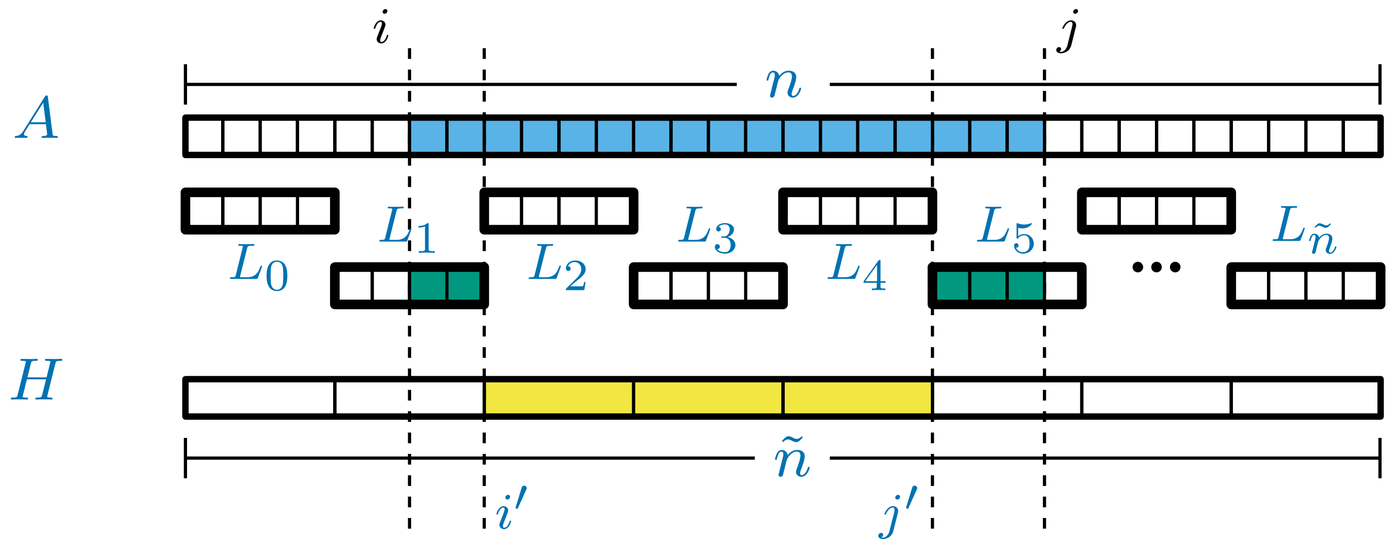

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Solution 4

$O(n \log \log \log n)$ space $\quad O(n \log \log \log n)$ prep time $\quad O(1)$ query time

Low-resolution RMQ

$$\tilde{n} = \frac{n}{\log n}$$

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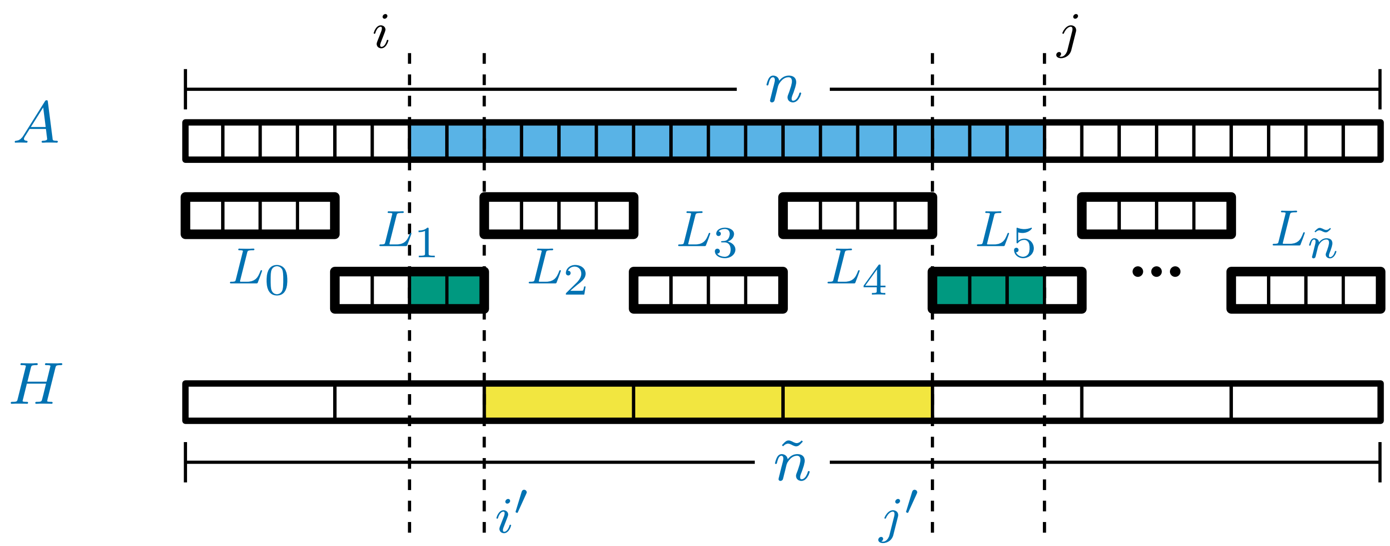

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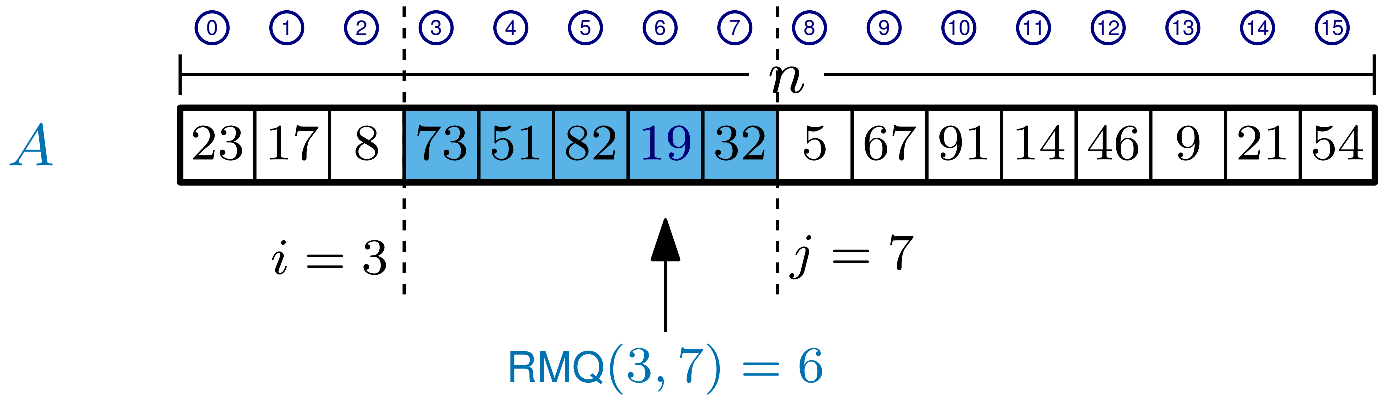
This takes $O(1)$ total query time

Solution 4 *how?*

$O(n \log \log \log n)$ space $O(n \log \log \log n)$ prep time $O(1)$ query time

Range minimum query summary

Preprocess an integer array A (length n) to answer range minimum queries...



After preprocessing, a **range minimum query** is given by RMQ(i, j)

the output is the location of the smallest element in $A[i, j]$

Solution 1

$O(n)$ space

$O(n)$ prep time

$O(\log n)$ query time

Solution 3

$O(n \log \log n)$ space

$O(n \log \log n)$ prep time

$O(1)$ query time

Solution 2

$O(n \log n)$ space

$O(n \log n)$ prep time

$O(1)$ query time

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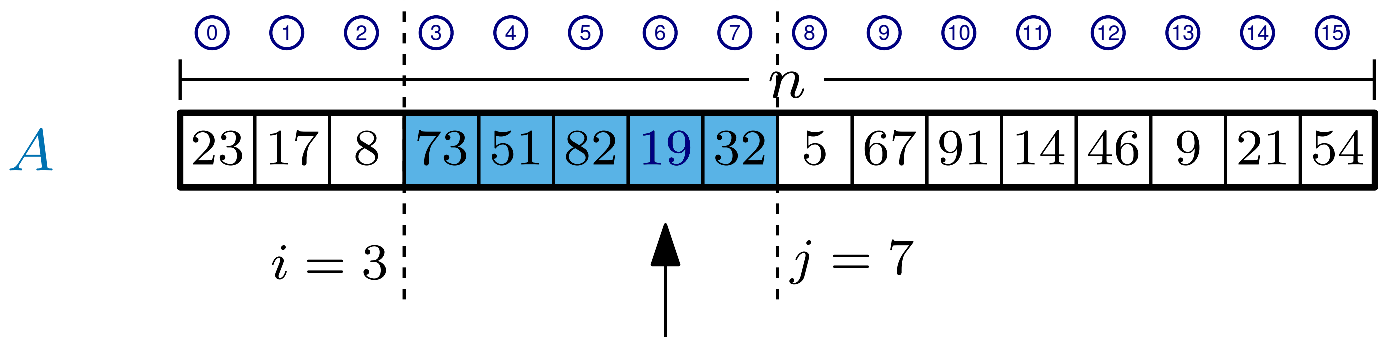

$\text{RMQ}(3, 7) = 6$

After preprocessing, a **range minimum query** is given by $\text{RMQ}(i, j)$

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$O(n \log \log n)$ space

$O(n \log \log n)$ prep time

$O(1)$ query time

Solution 2

$O(n \log n)$ space

$O(n \log n)$ prep time

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Can we do $O(n)$ space and $O(1)$ query time?

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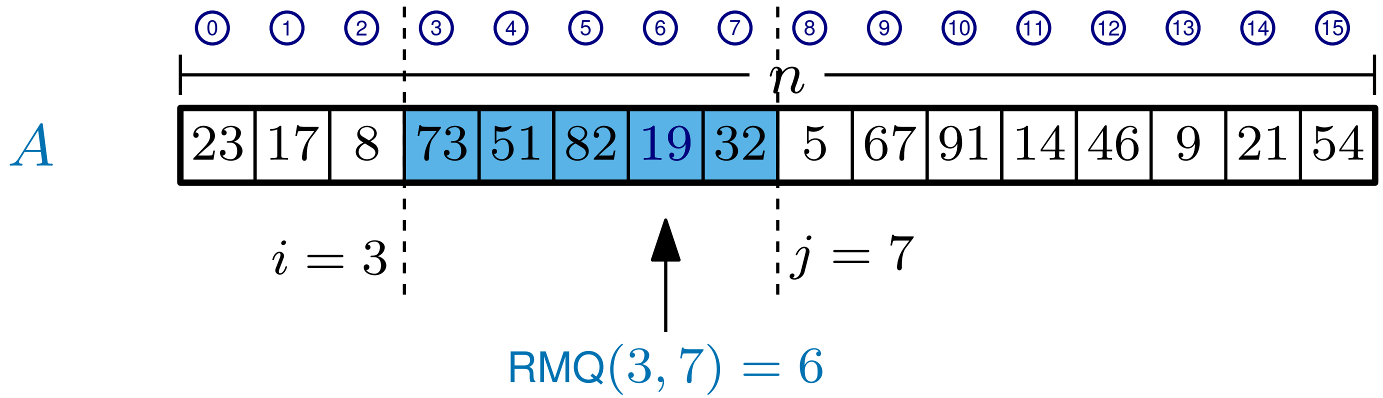

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$O(n \log n)$ prep time

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Can we do $O(n)$ space and $O(1)$ query time? *Yes...*

Range minimum query summary

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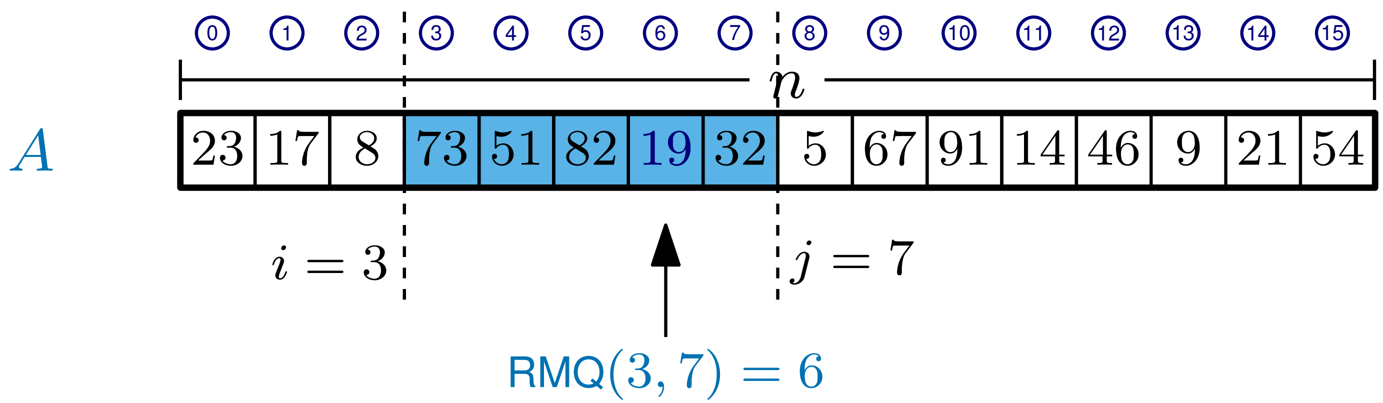

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$O(n \log \log n)$ prep time

$O(1)$ query time

Solution 2

$O(n \log n)$ space

$O(n \log n)$ prep time

$O(1)$ query time

Can we do $O(n)$ space and $O(1)$ query time? *Yes...* *but not until next lecture*