Advanced Algorithms – COMS31900

Lowest Common Ancestor

Raphaël Clifford

Slides by Benjamin Sach

Advanced Algorithms – COMS31900

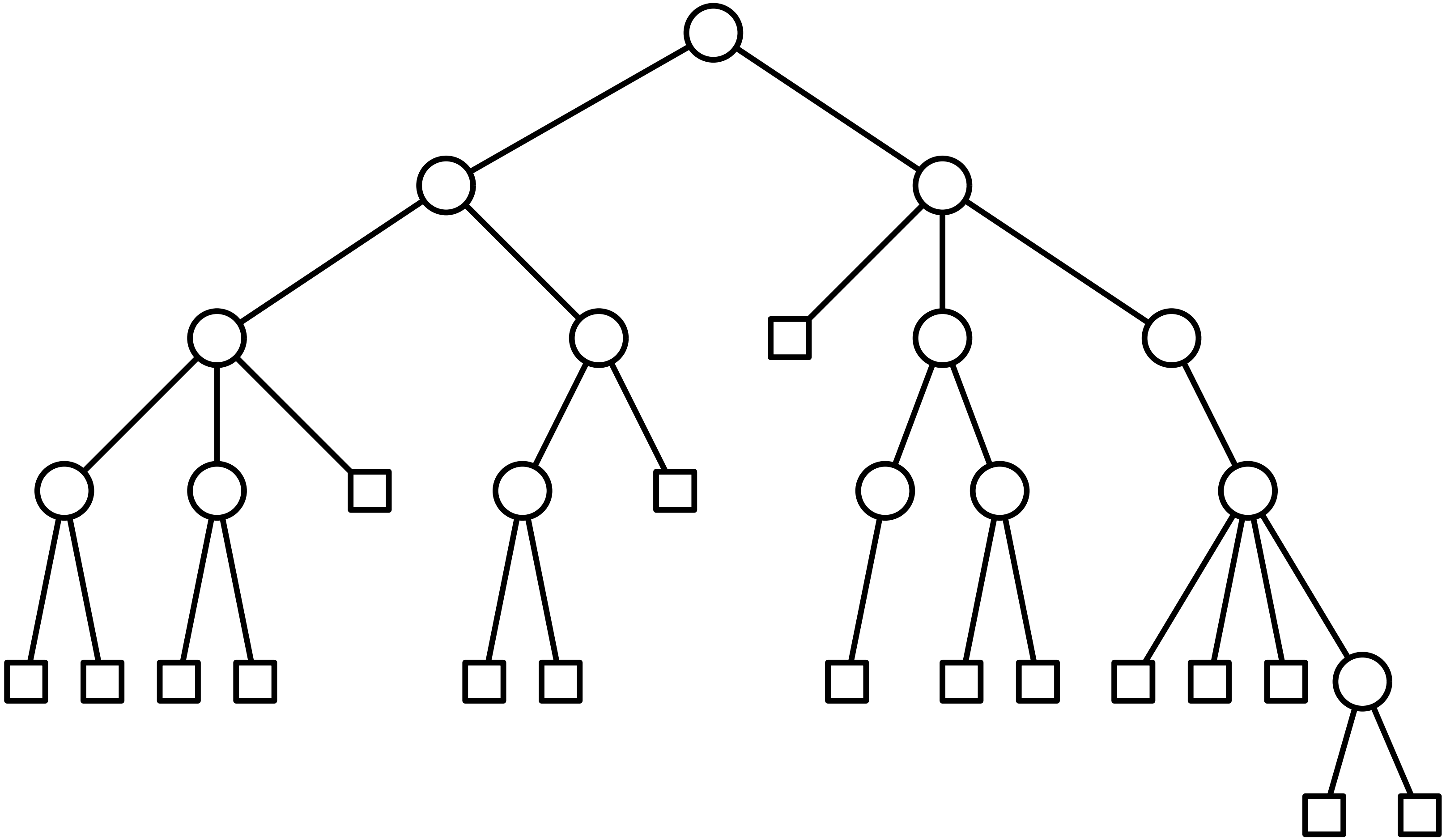
Lowest Common Ancestor

(with a bit on on Range Minimum Queries)

Raphaël Clifford

Lowest common ancestor

Preprocess a tree T (with n nodes) to answer lowest common ancestor queries. . .



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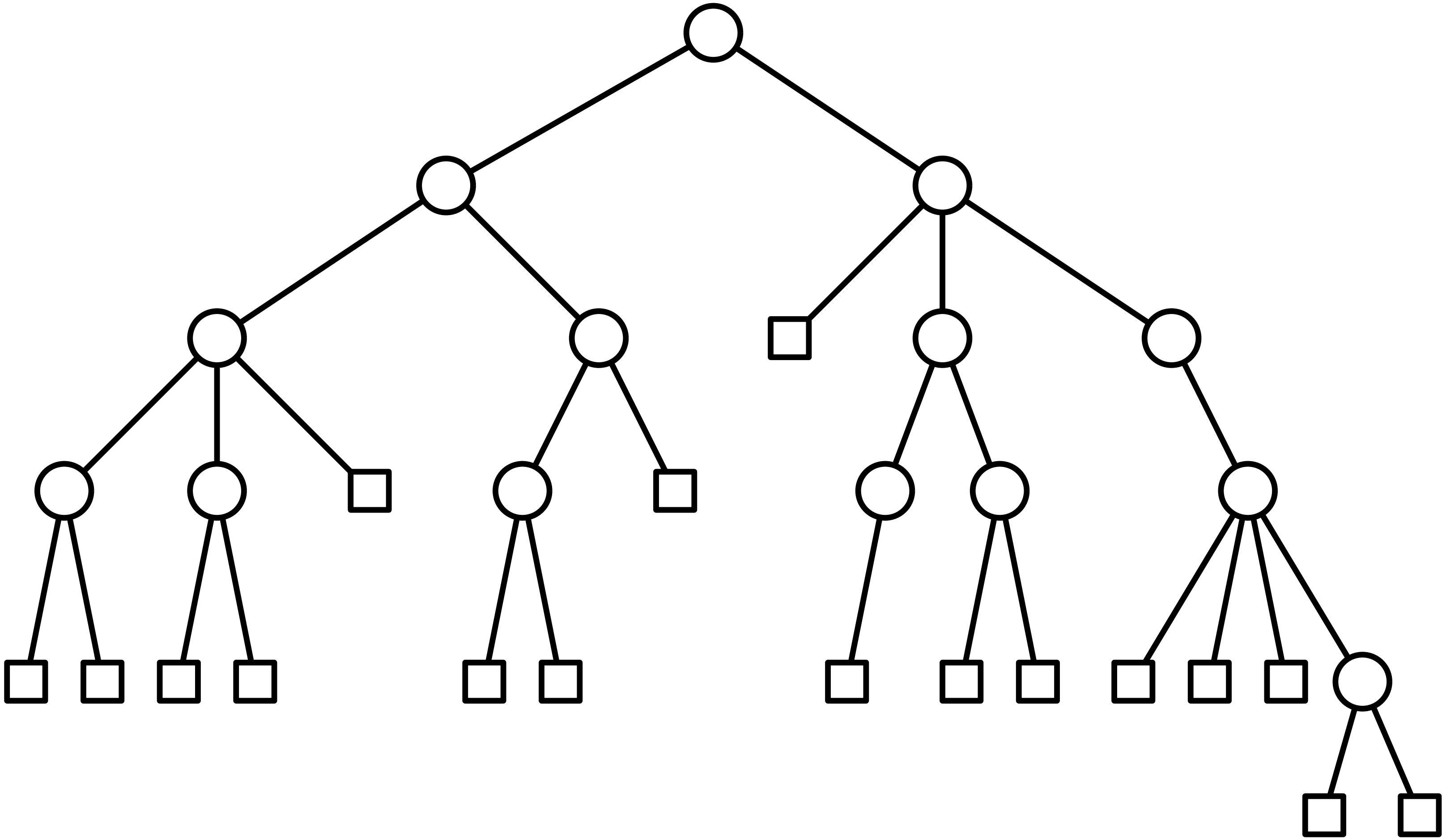

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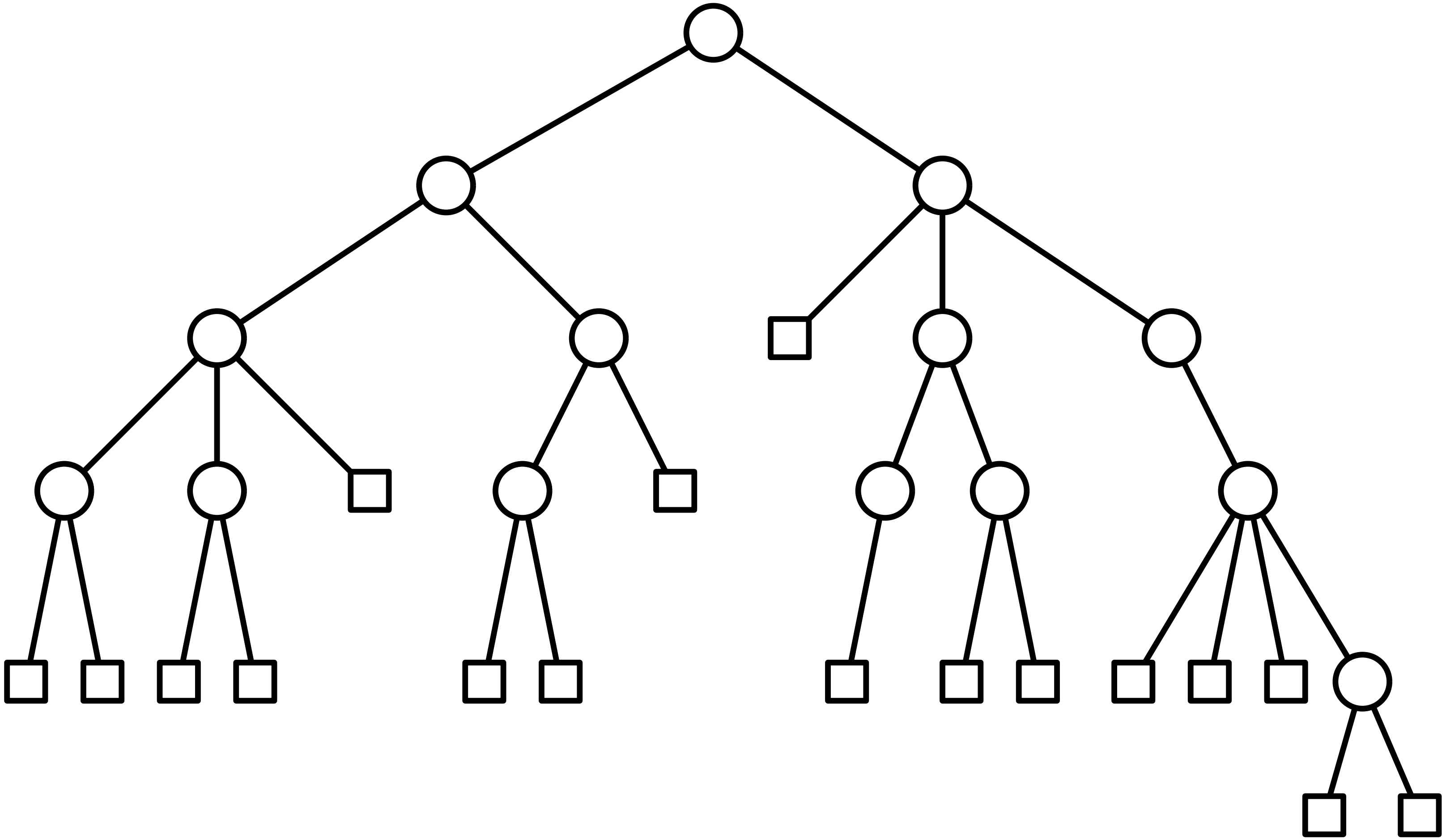

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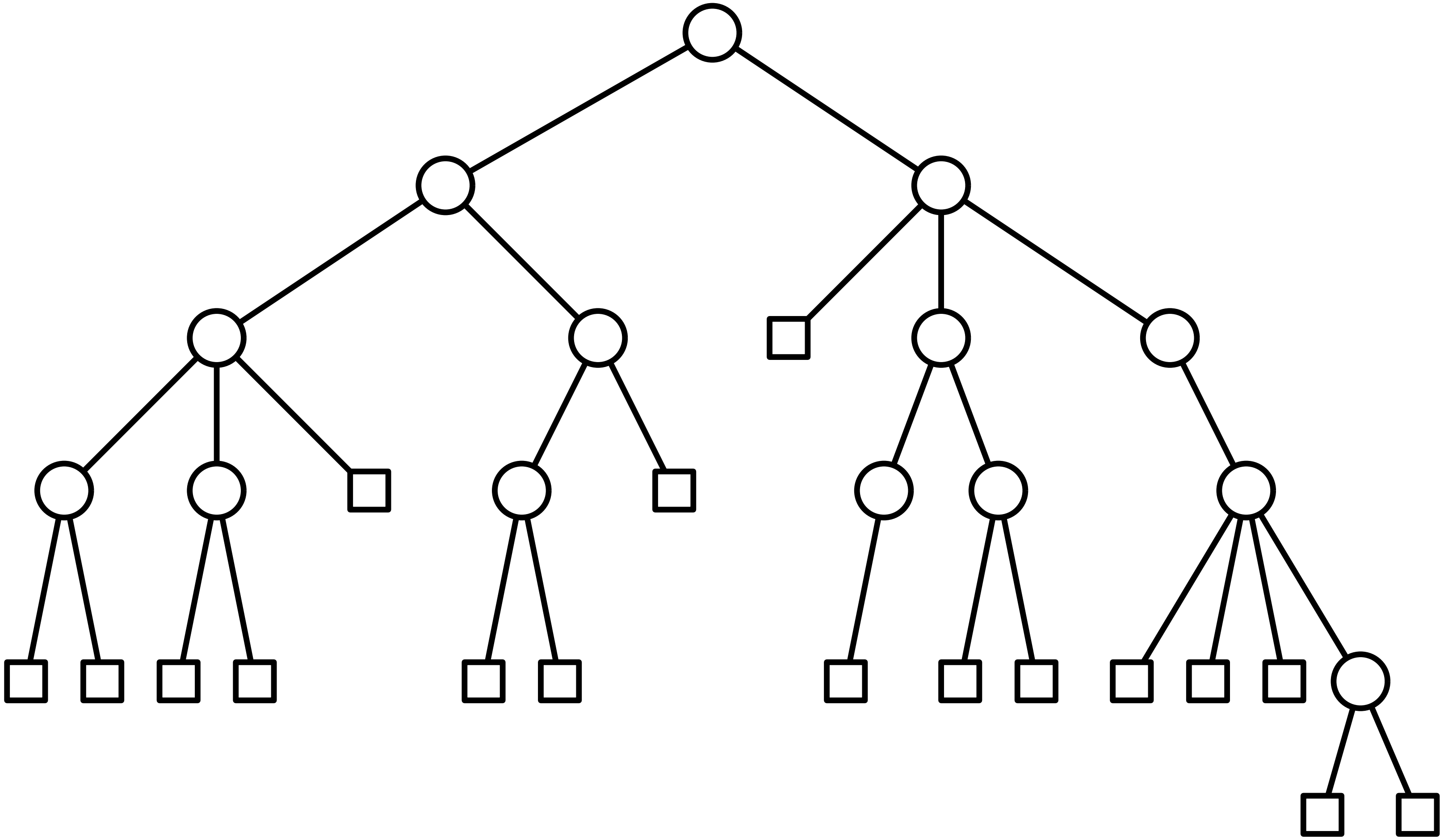

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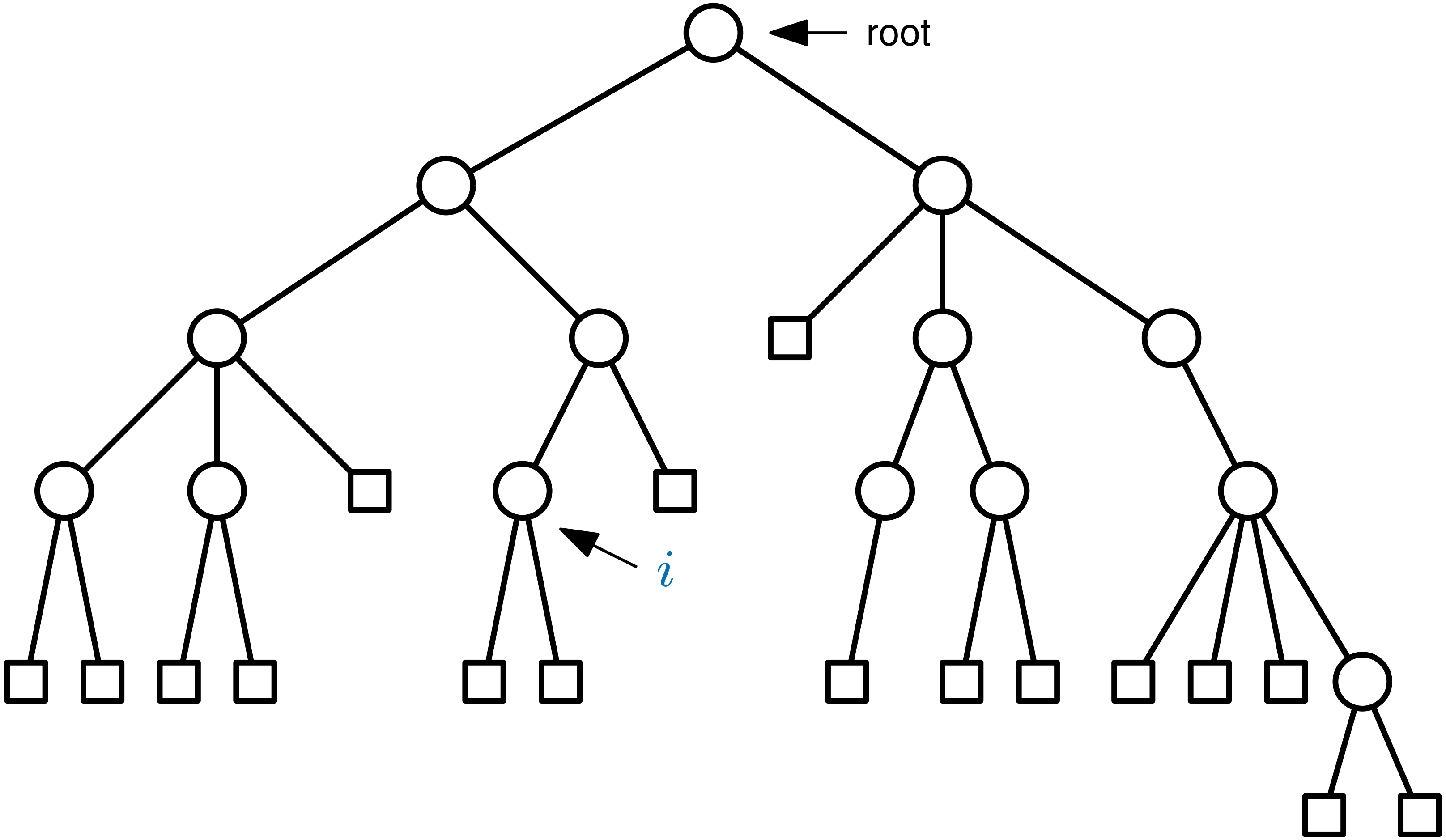

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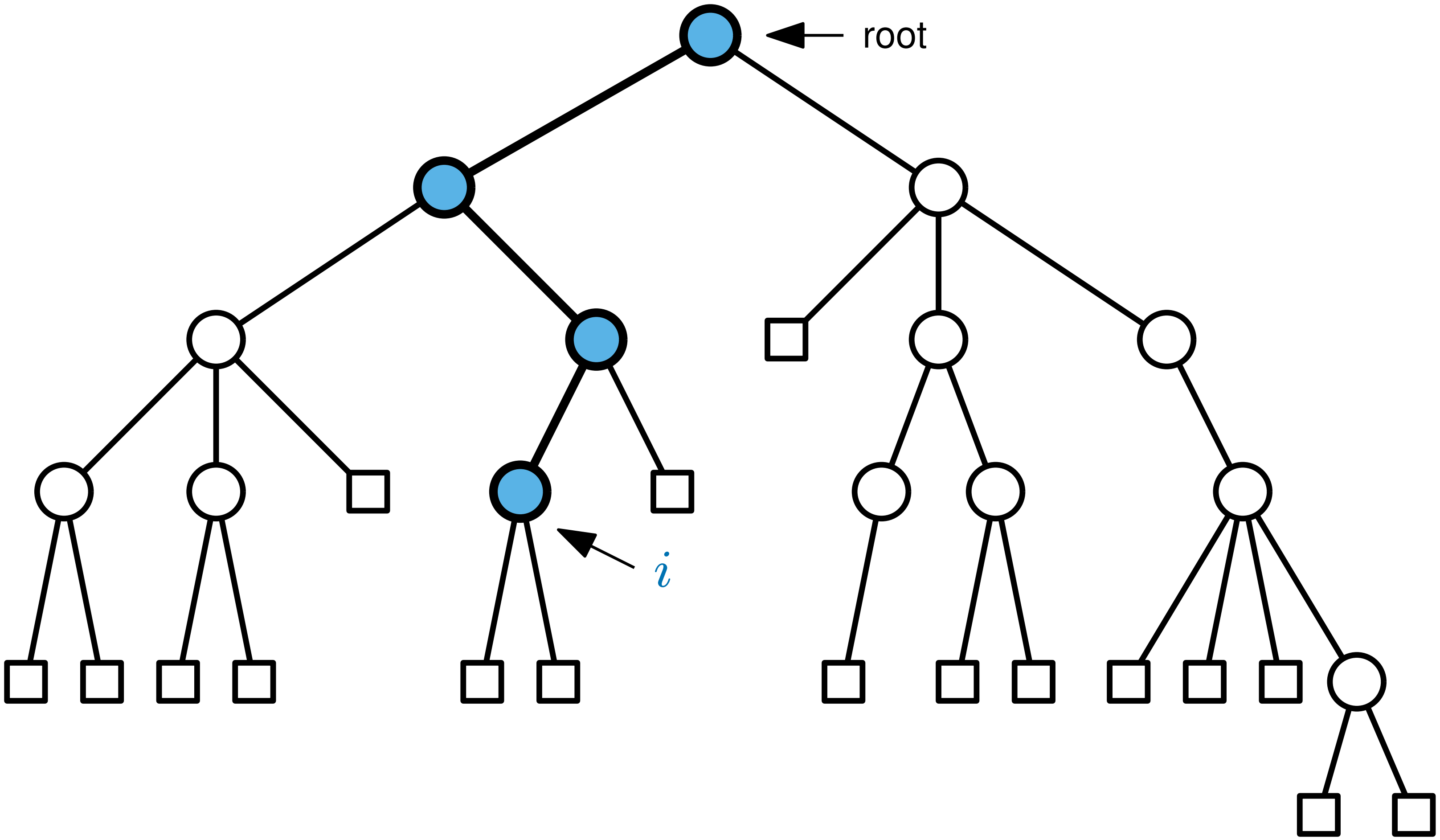

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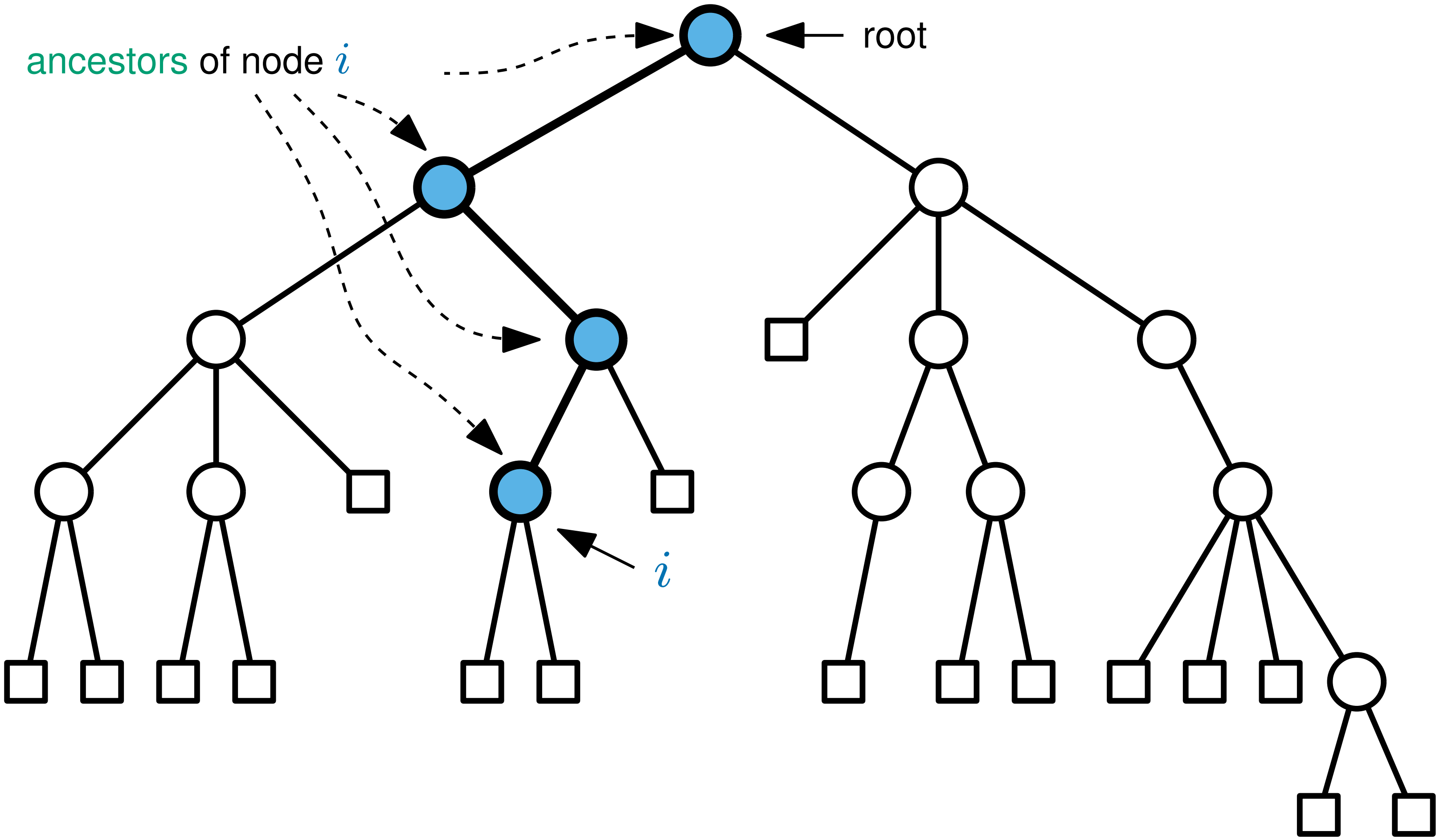

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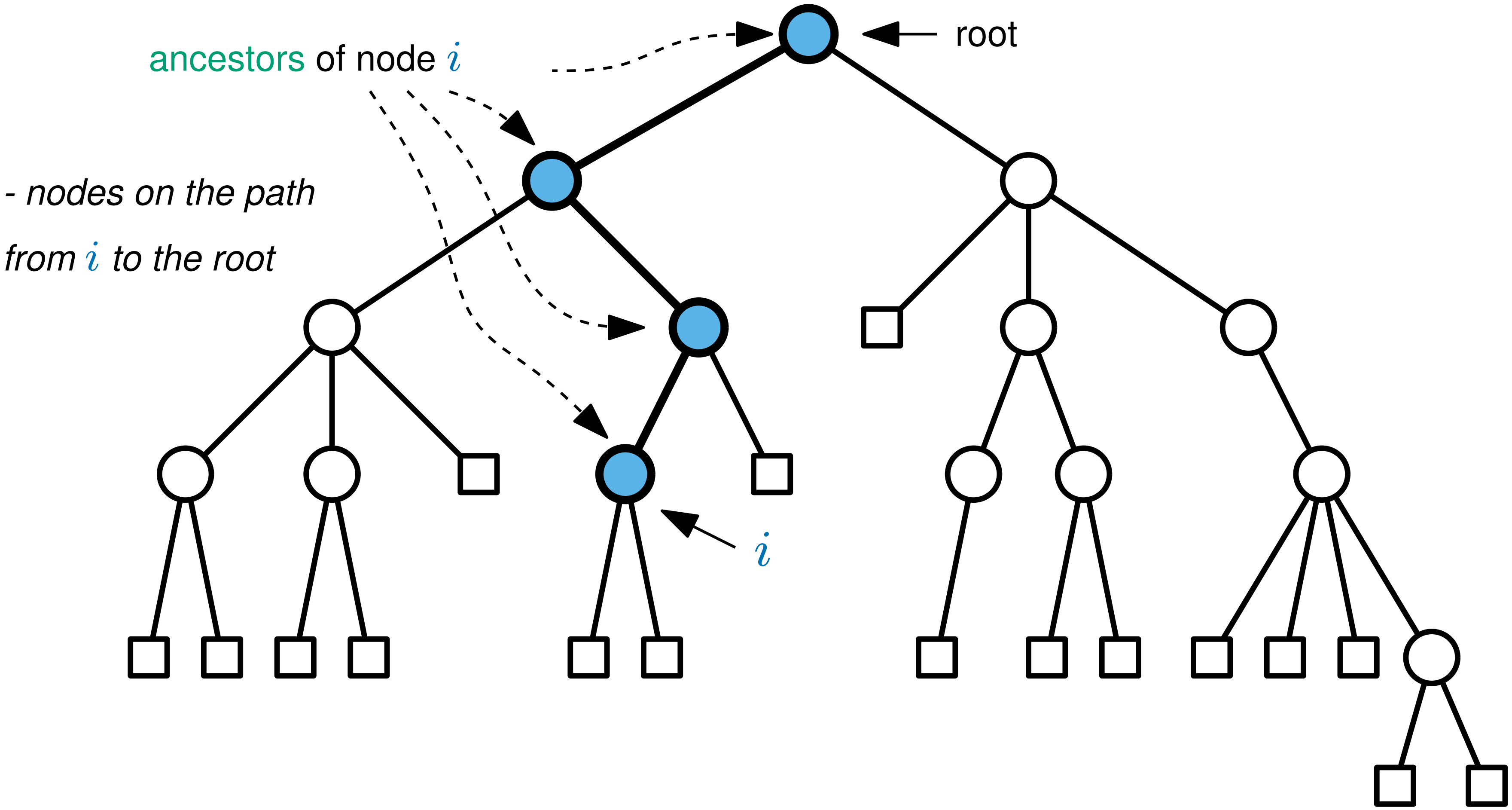

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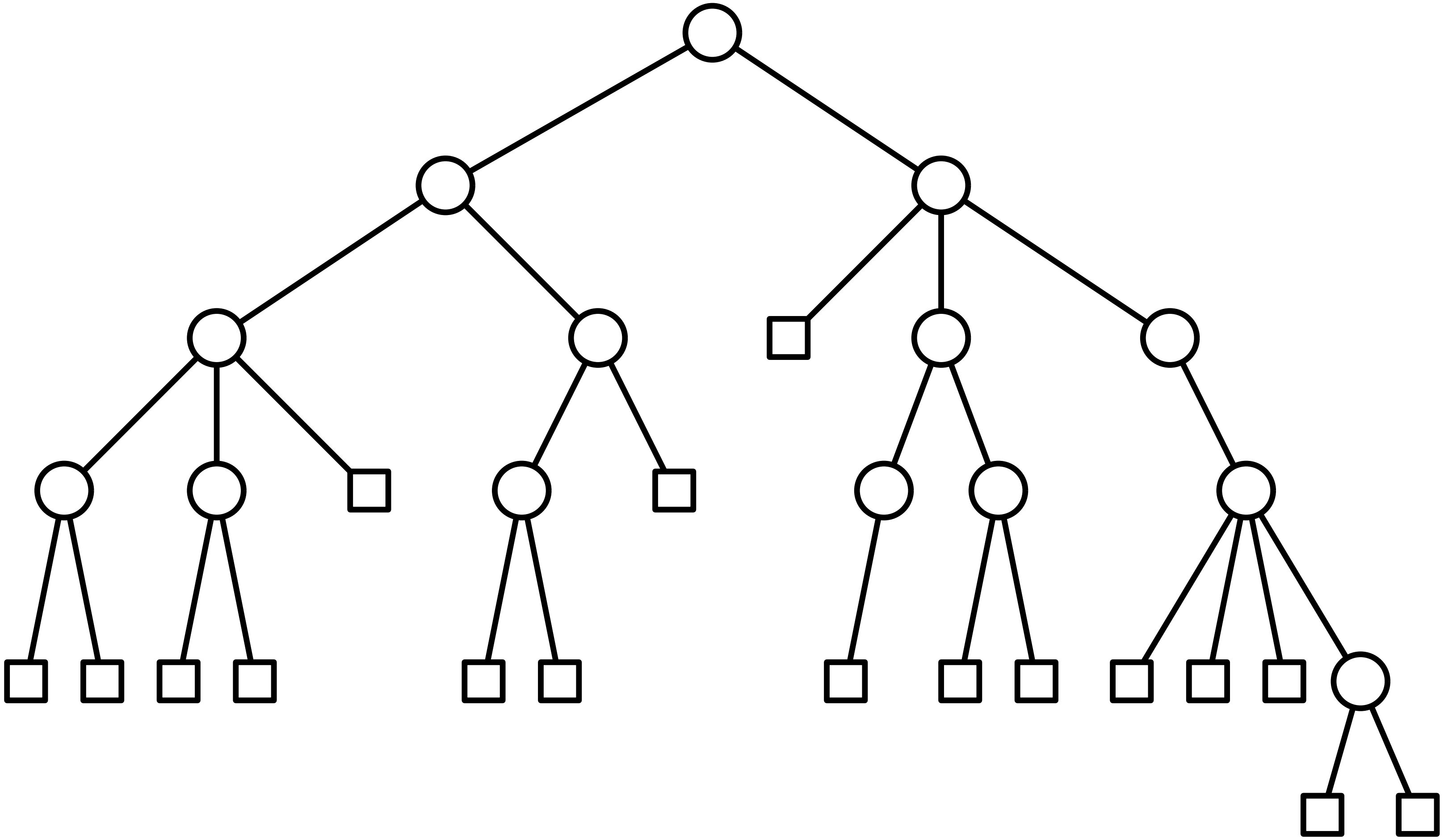

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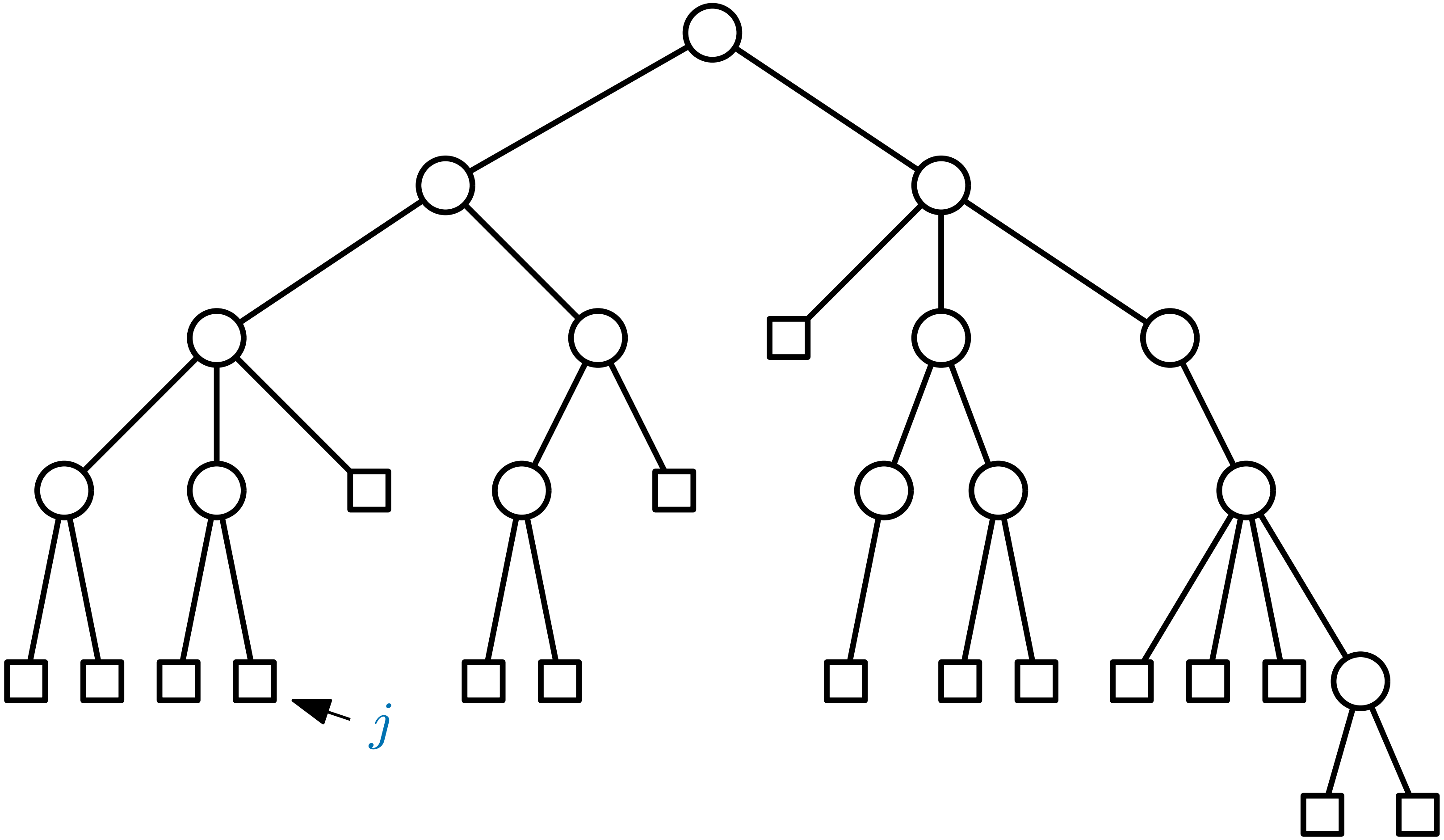

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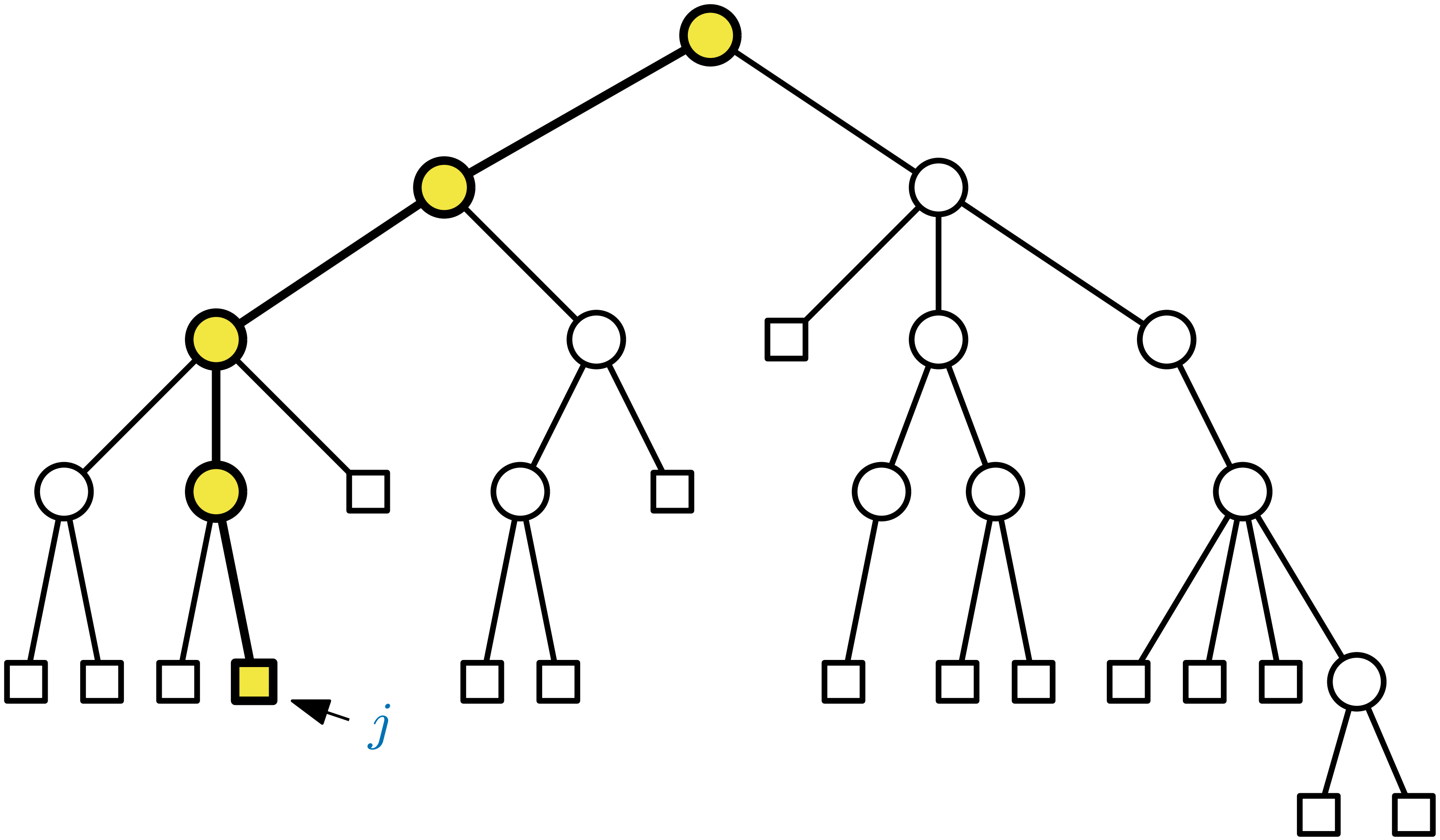

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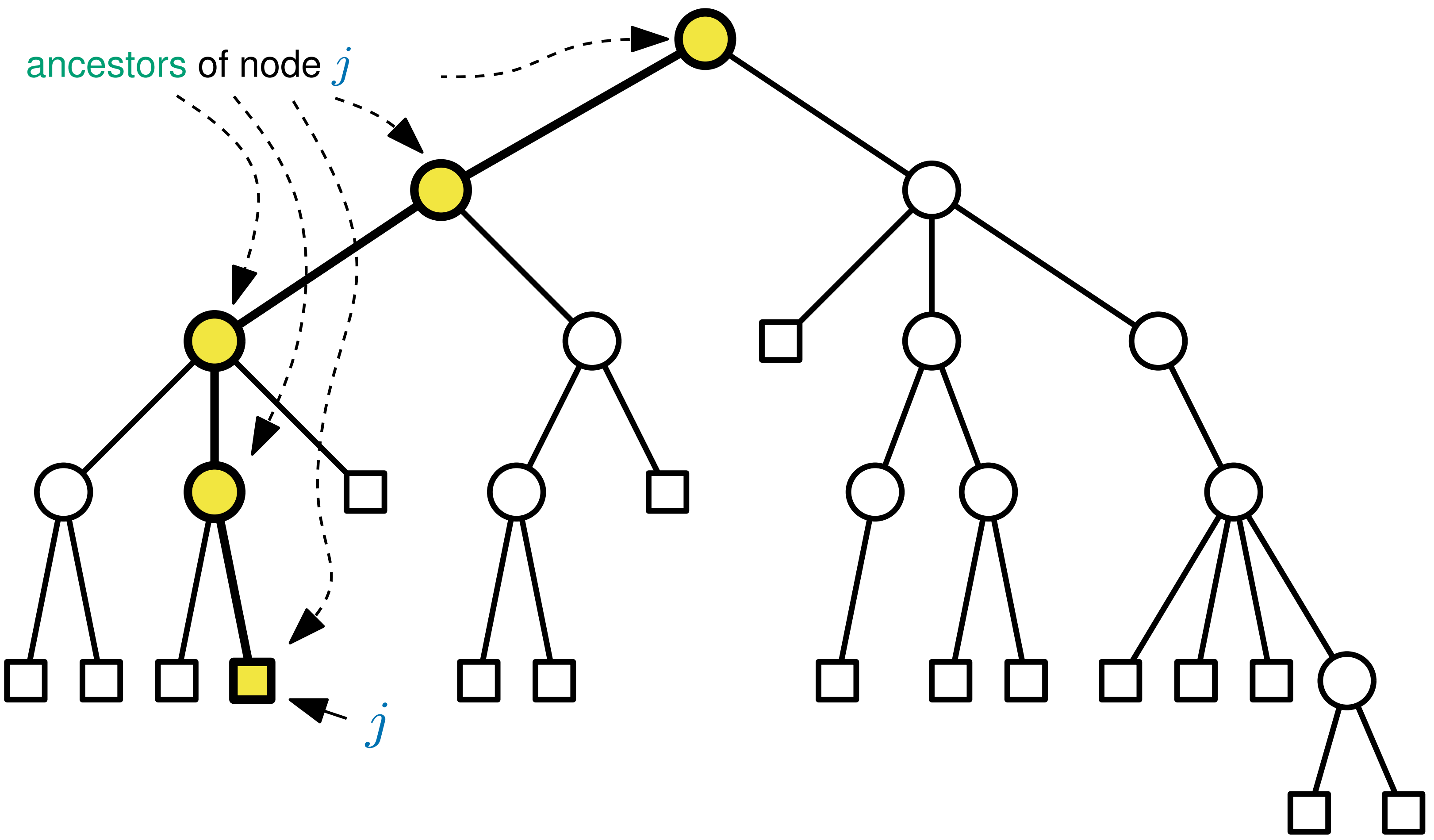

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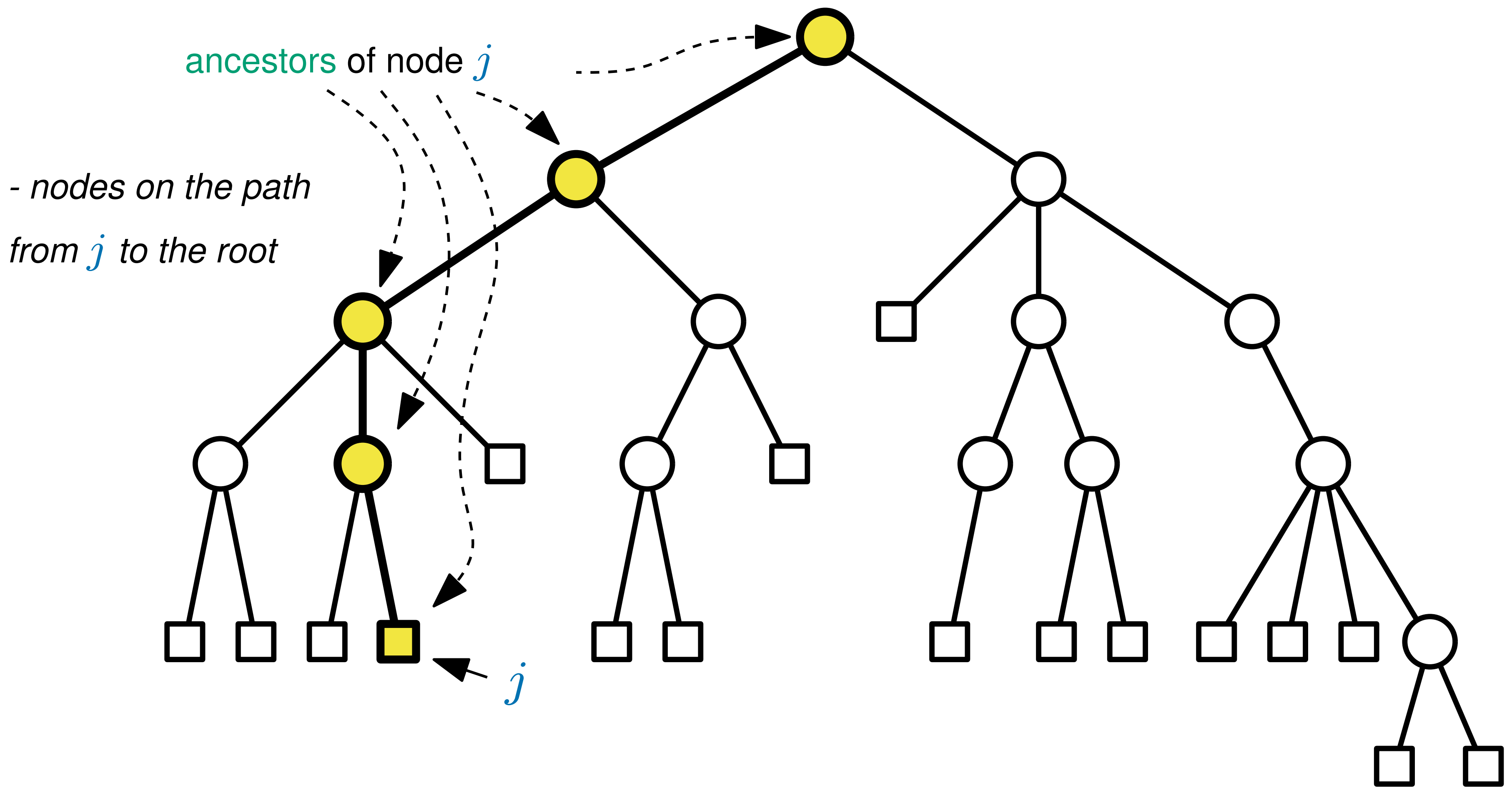

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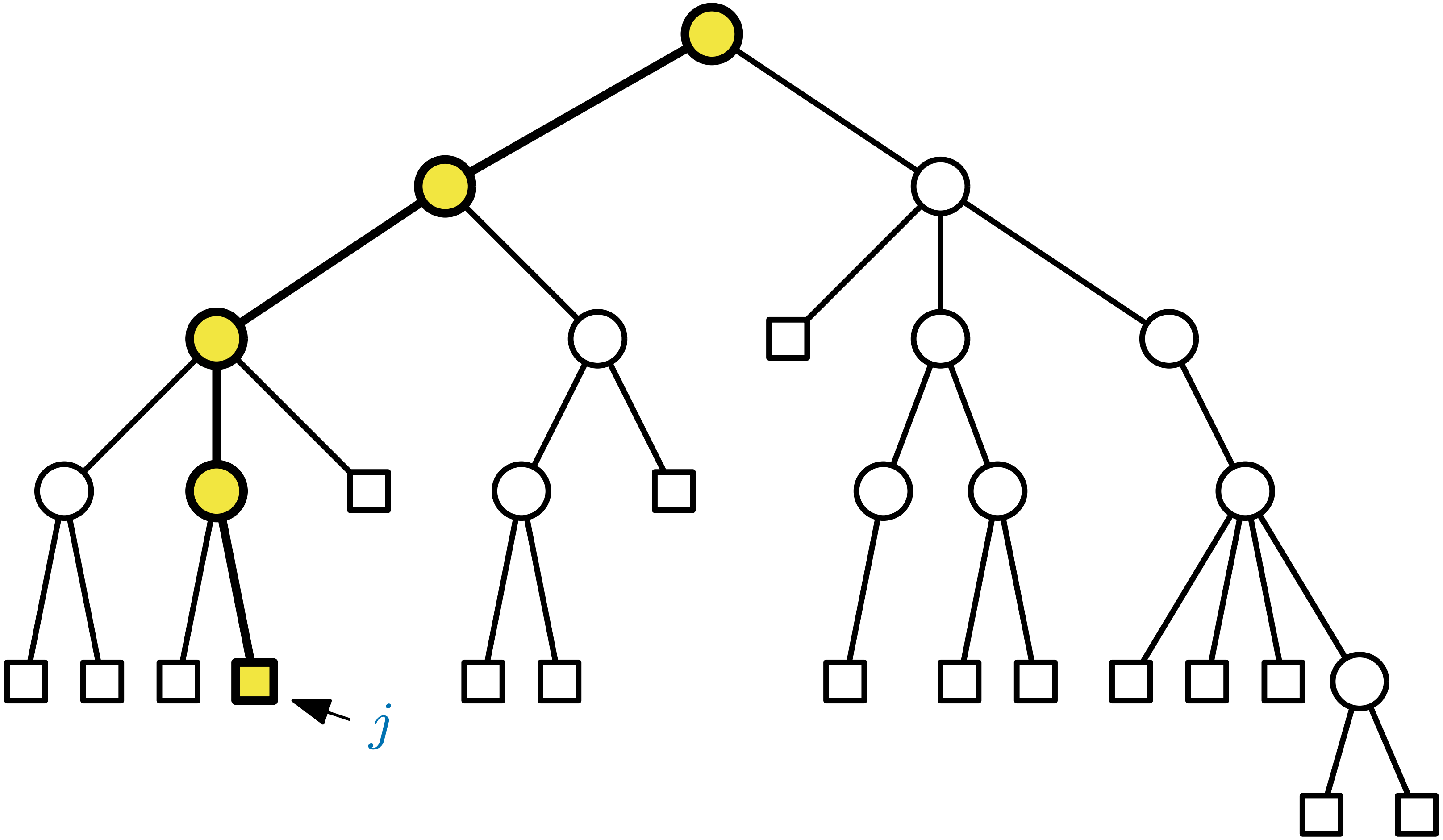

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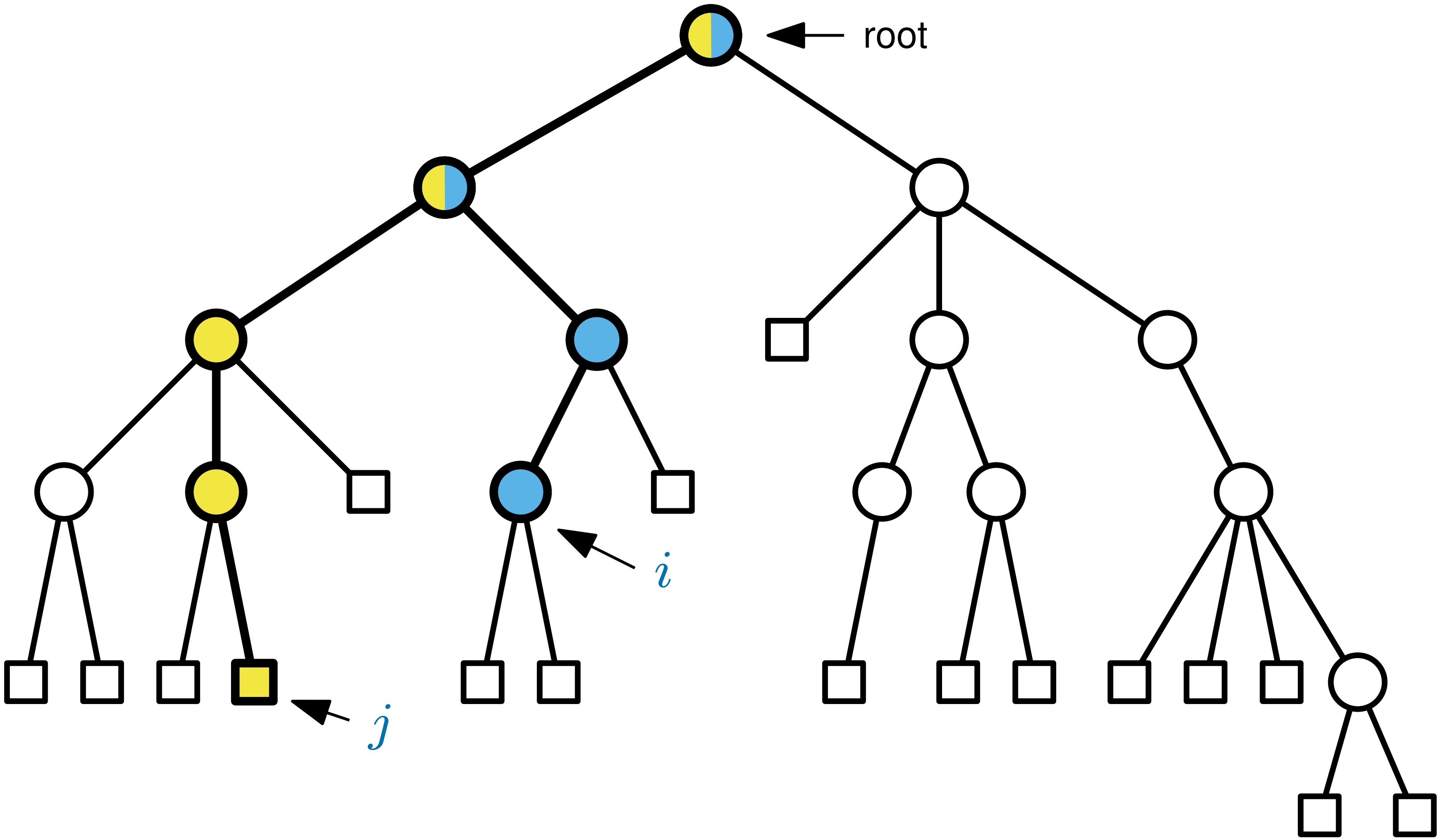

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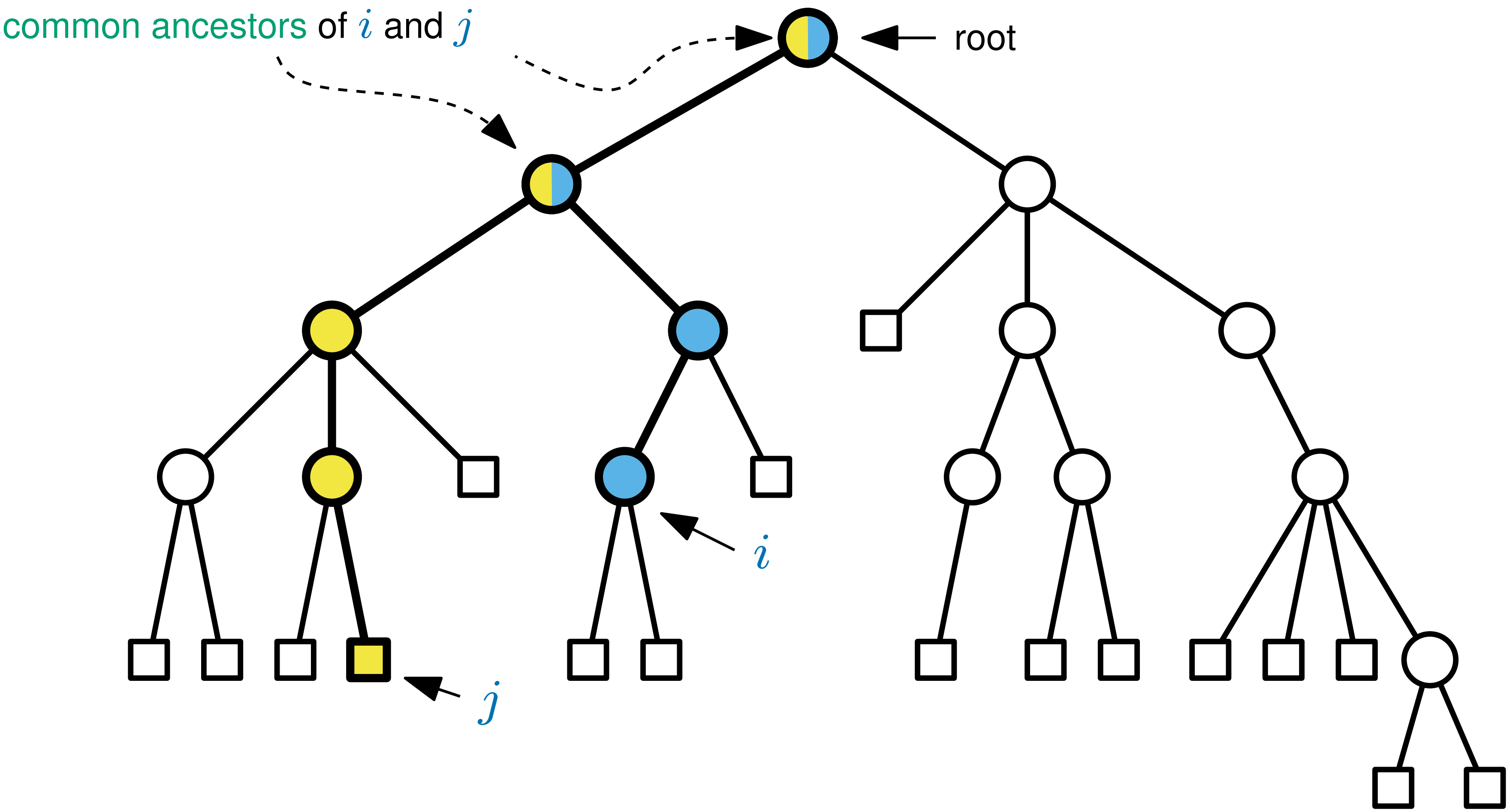

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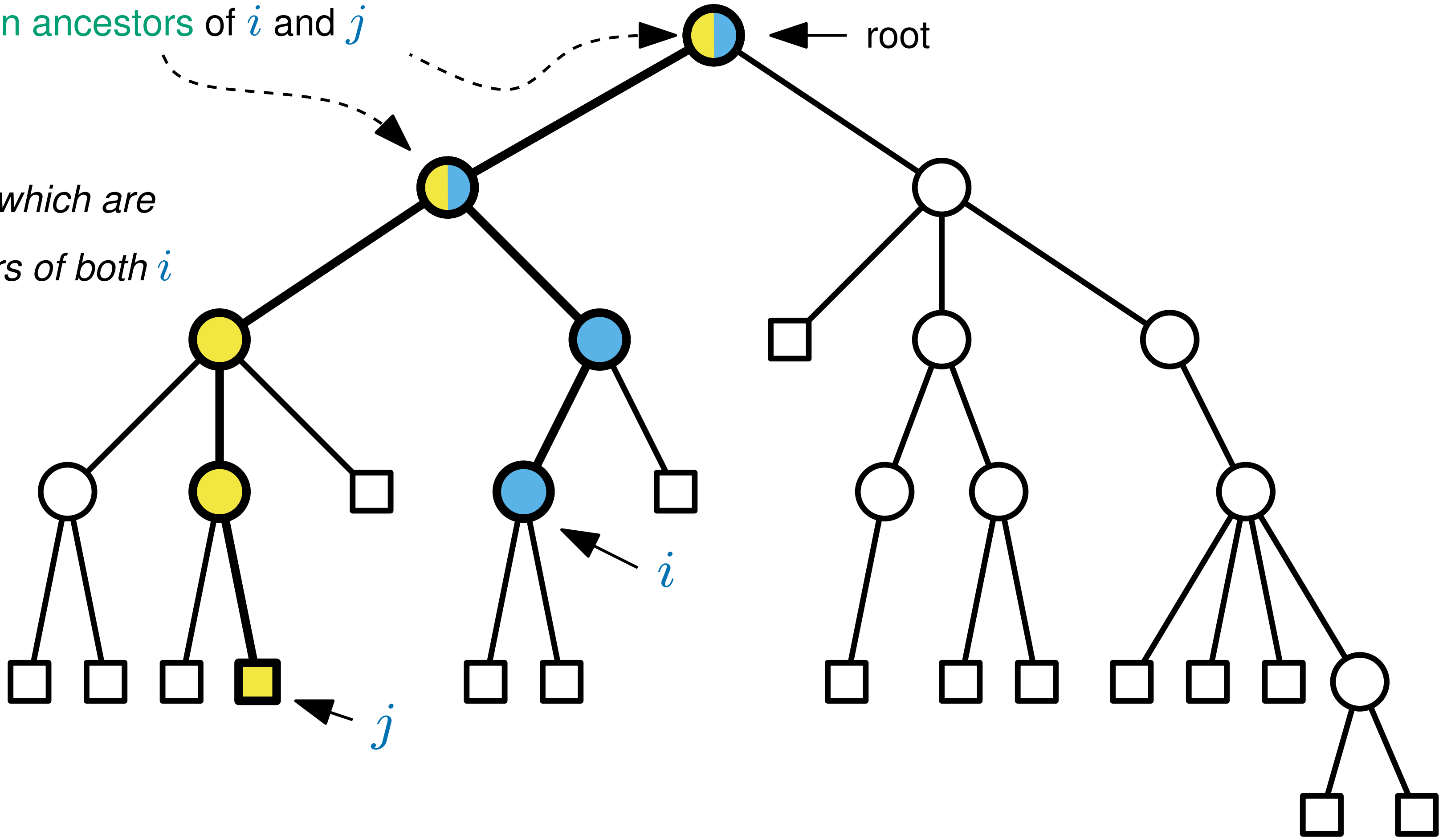

common ancestors of i and j

- nodes which are ancestors of both i and j

root

i

j

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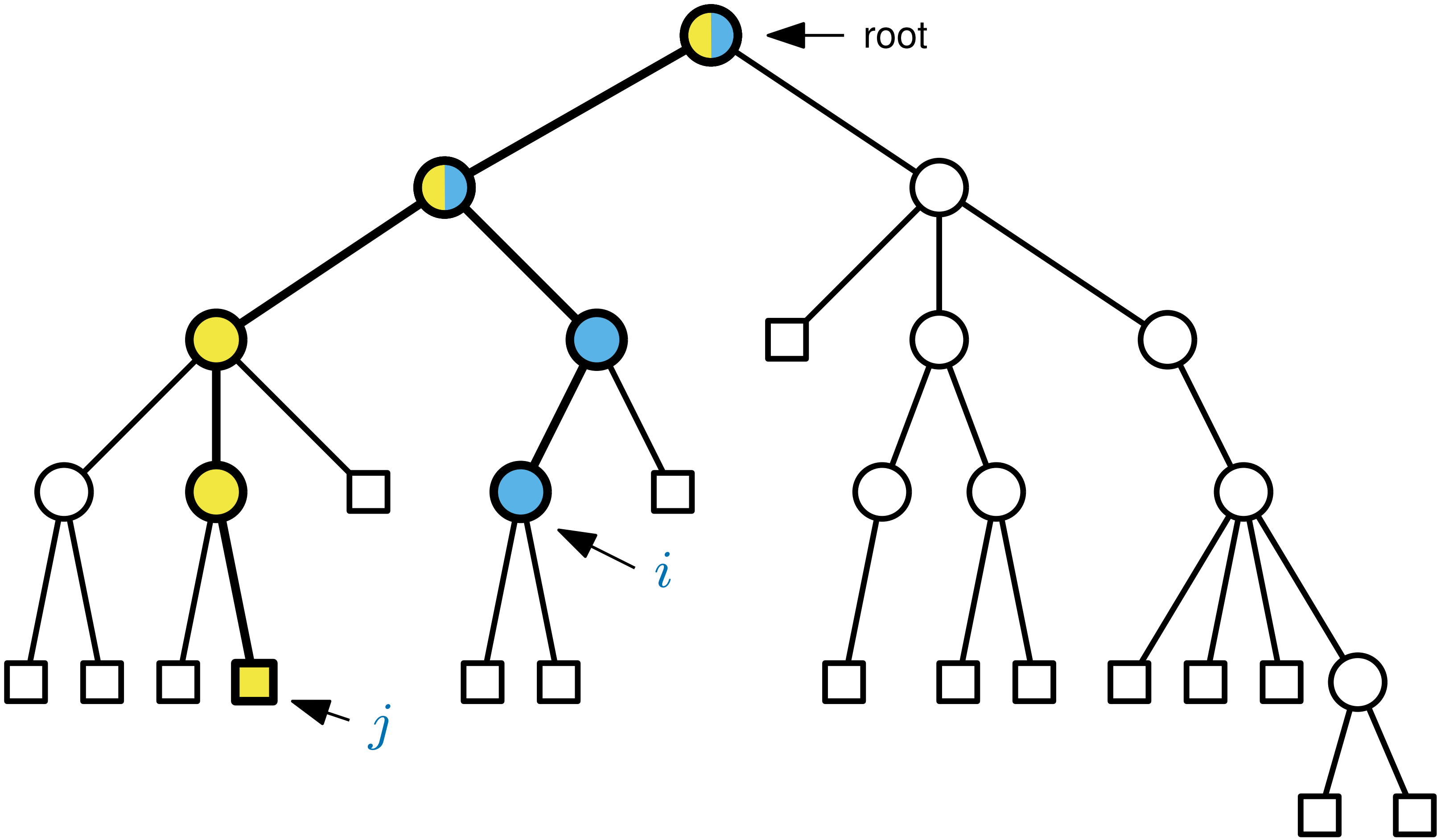

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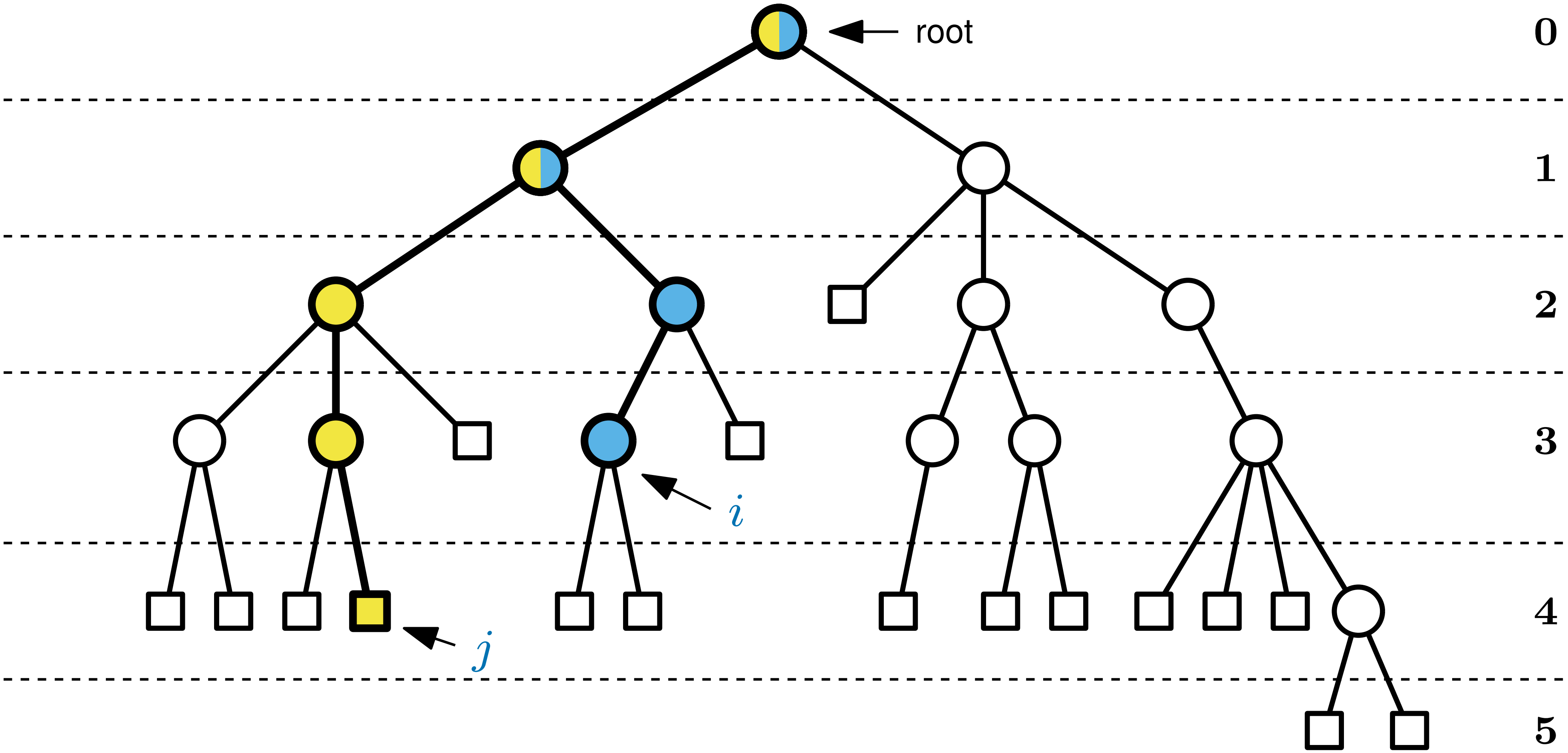

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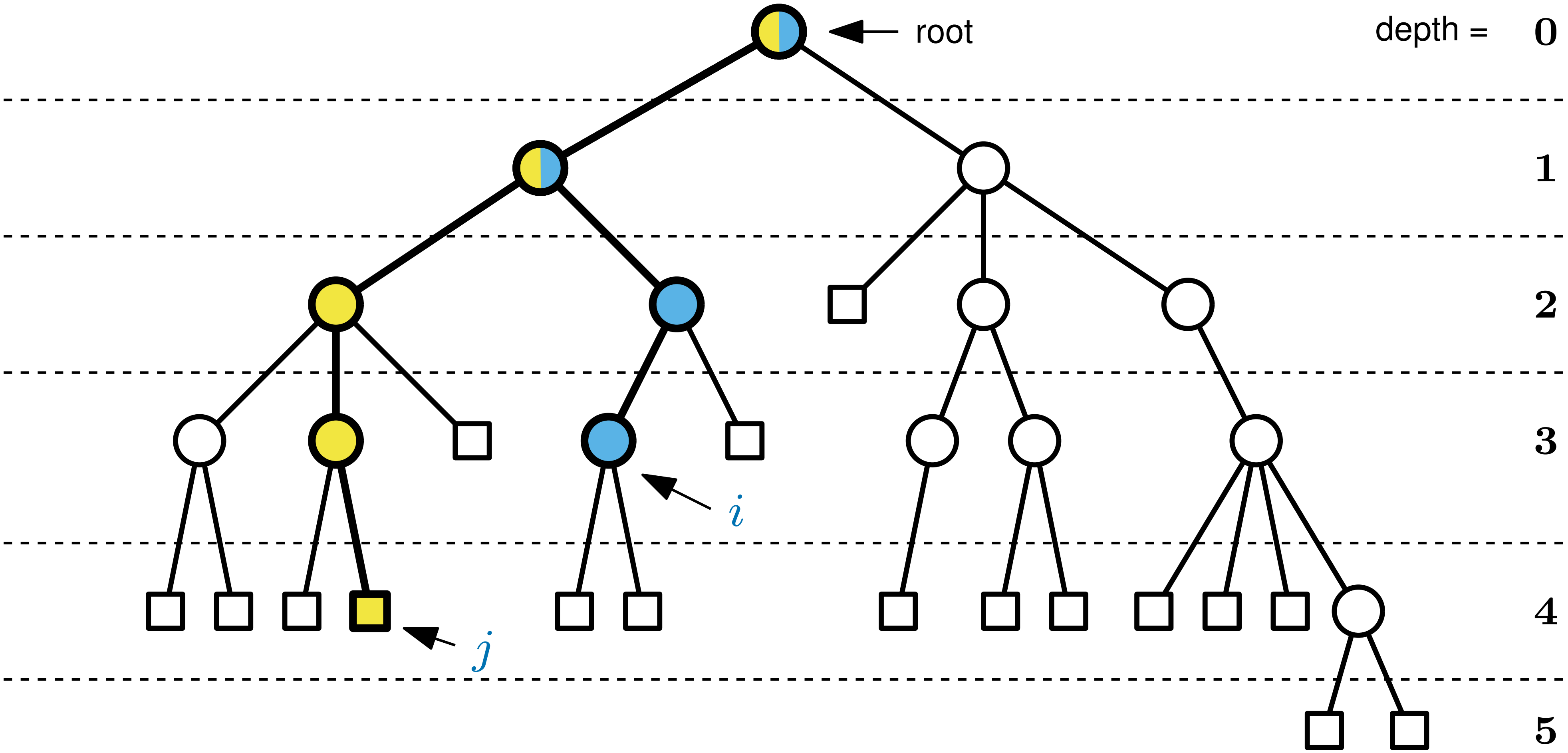

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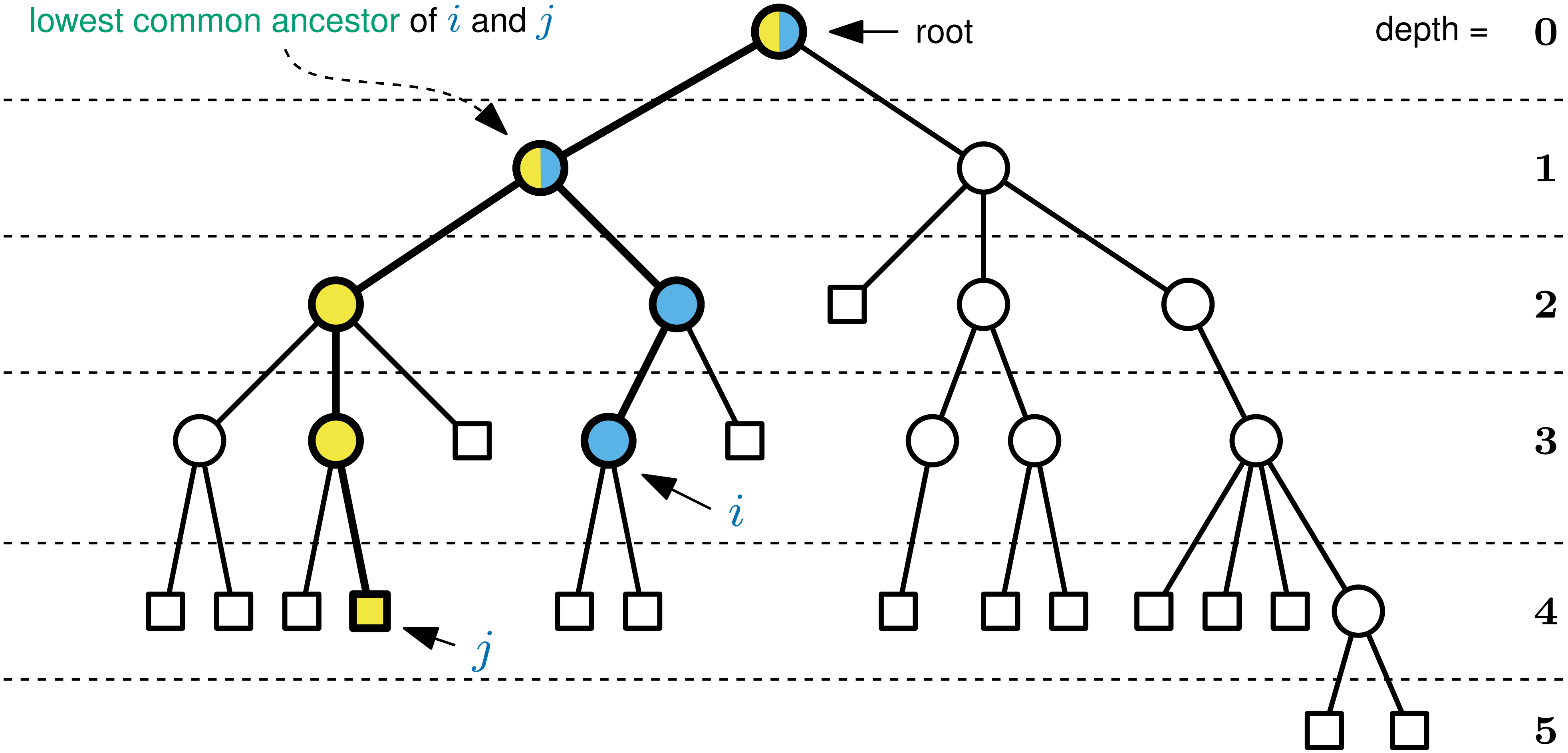

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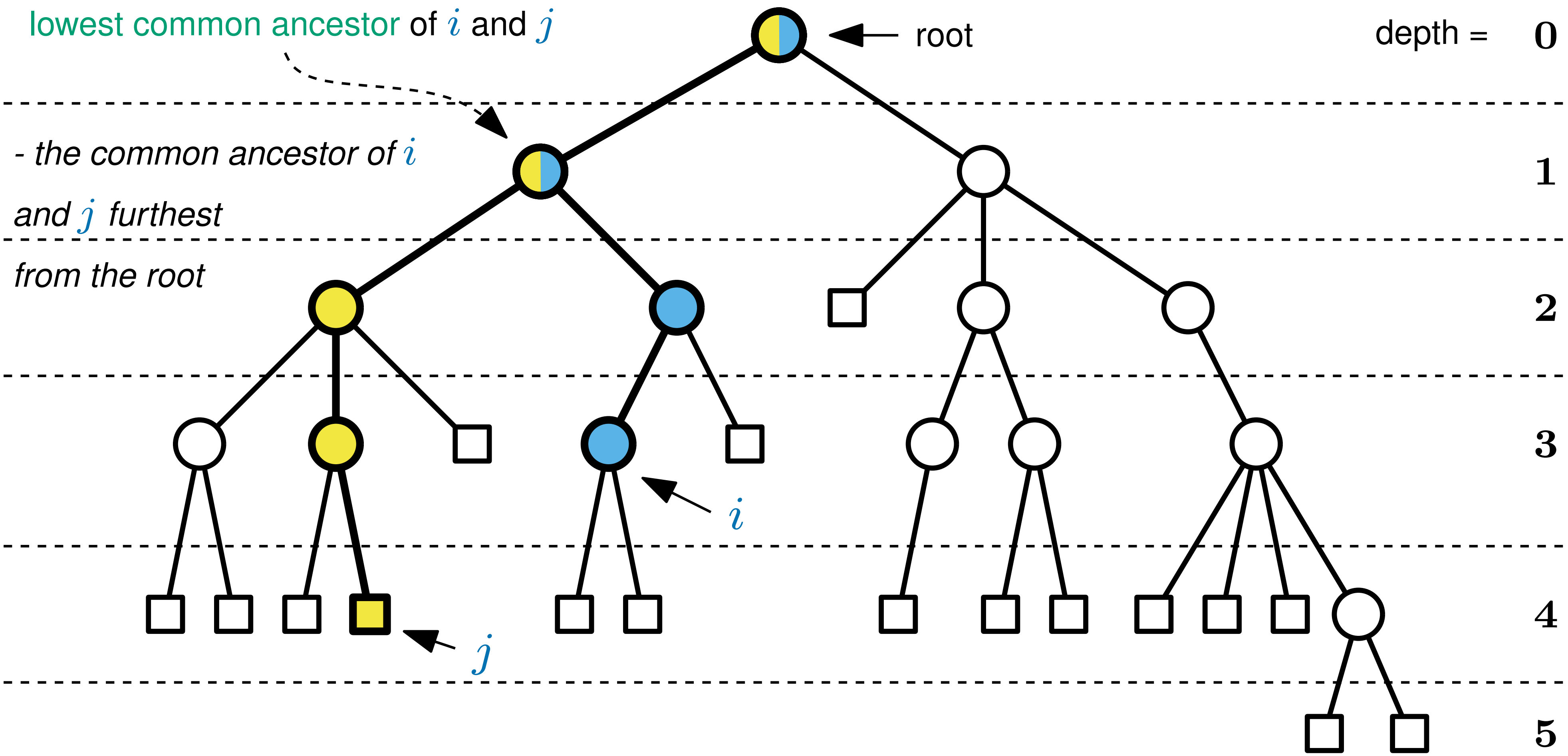
lowest common ancestor of i and j
- the common ancestor of i
and j furthest
from the root
root
depth =
0
1
2
3
4
5
i
j

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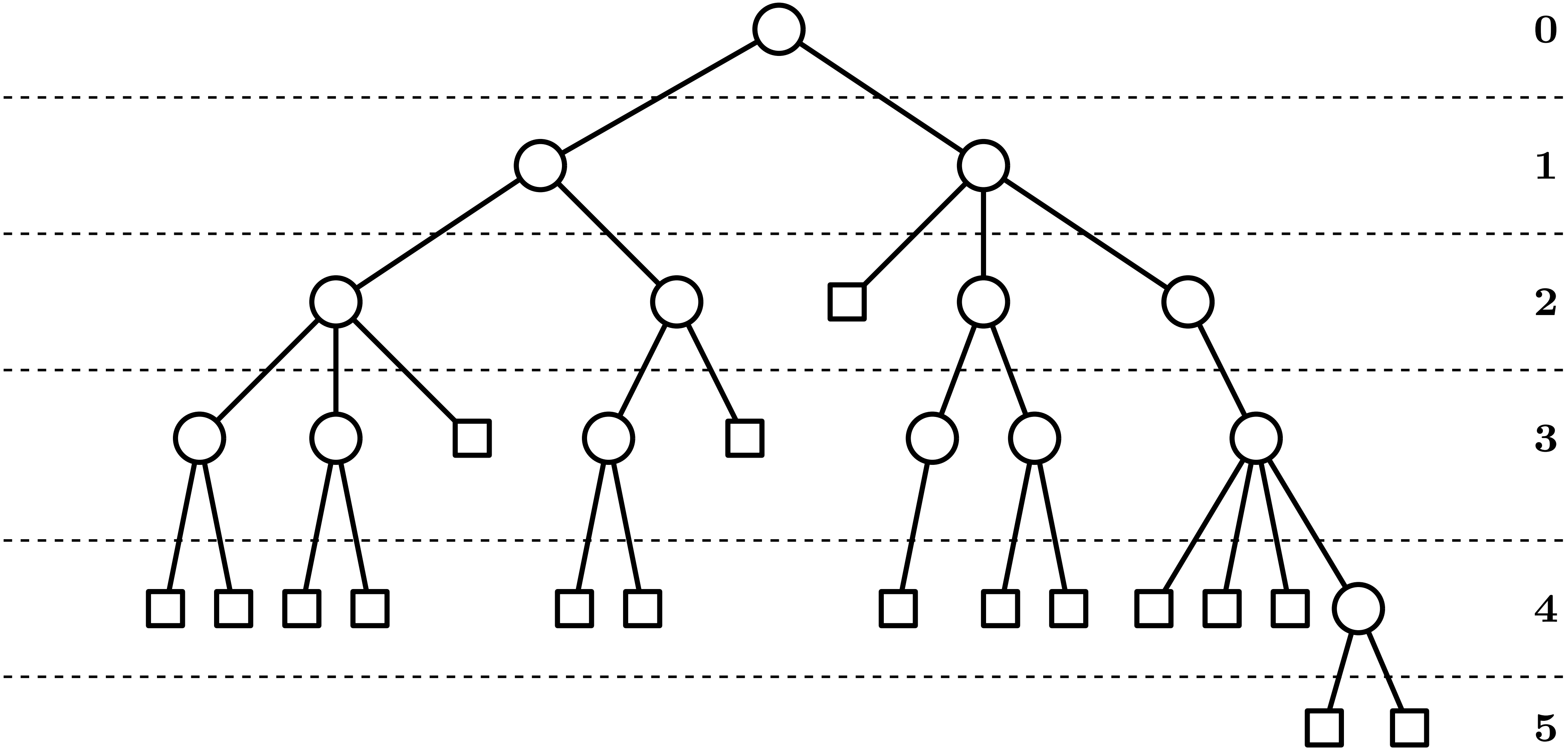

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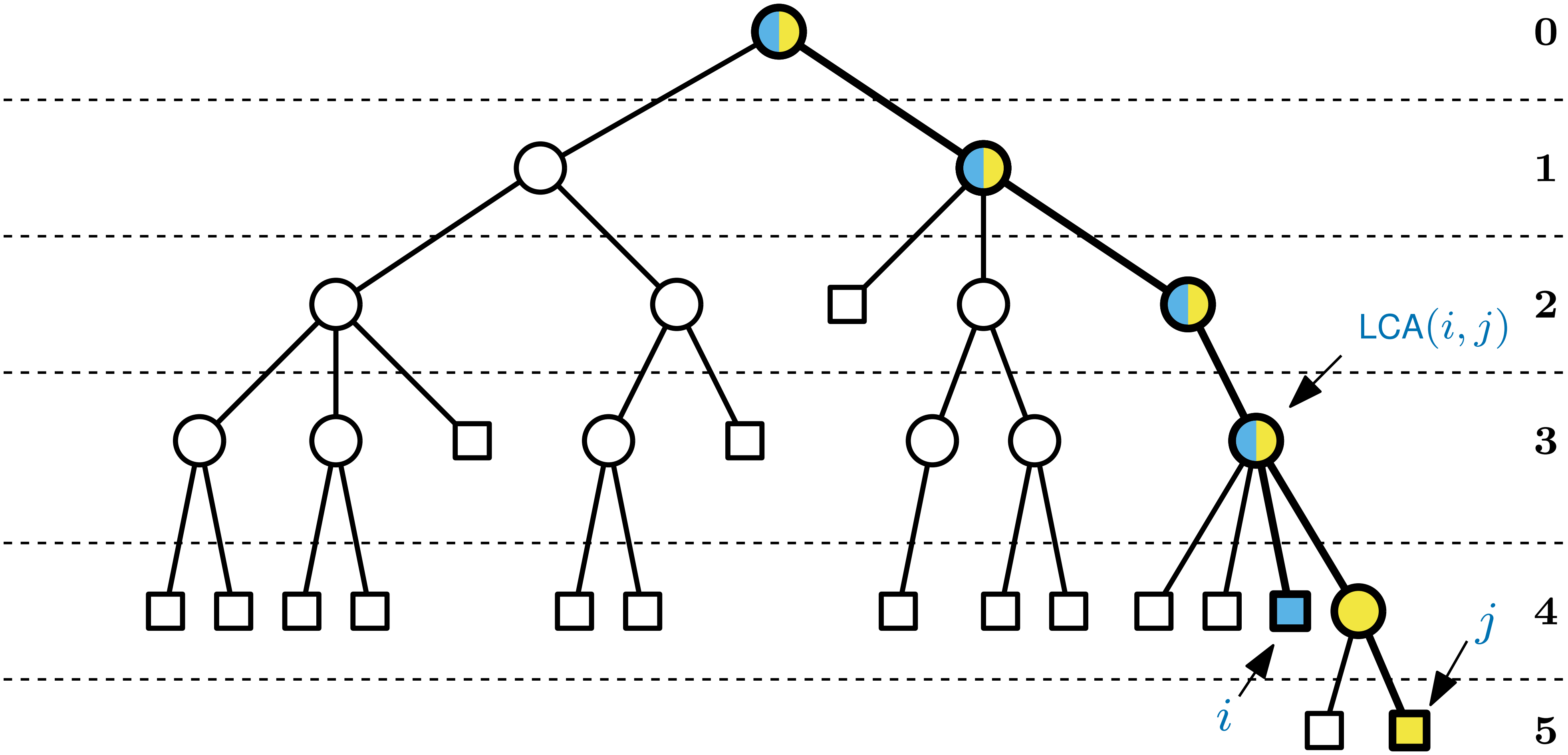

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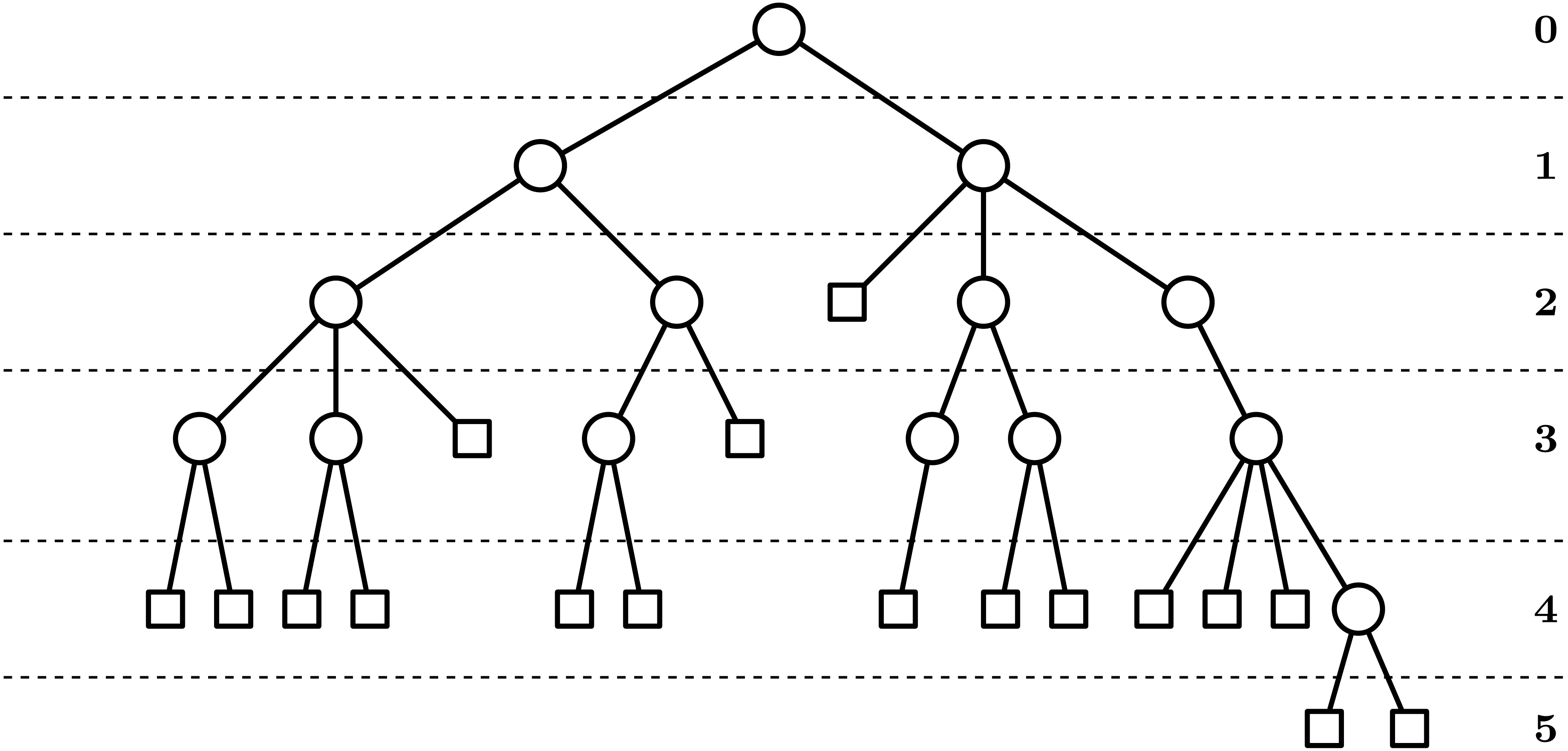

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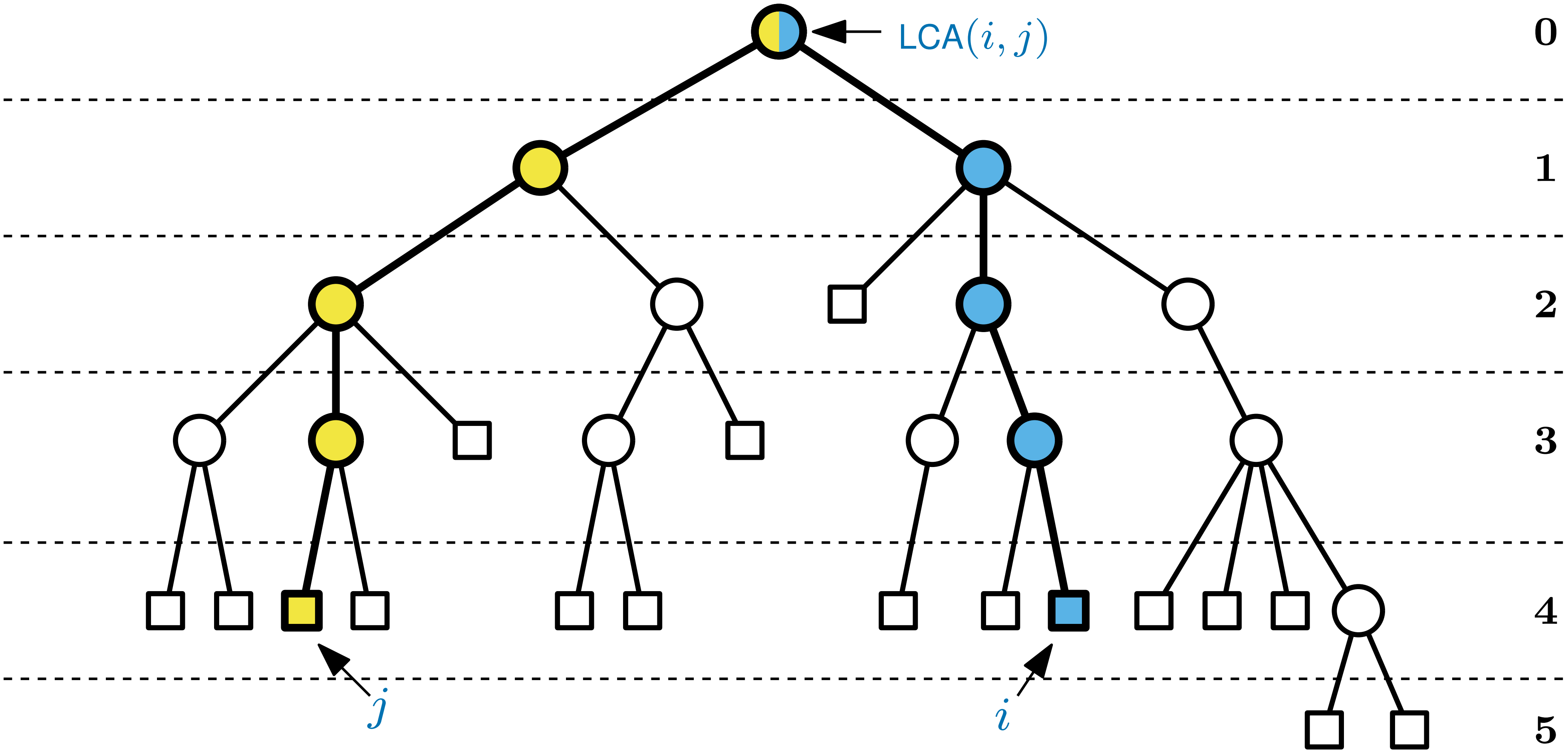

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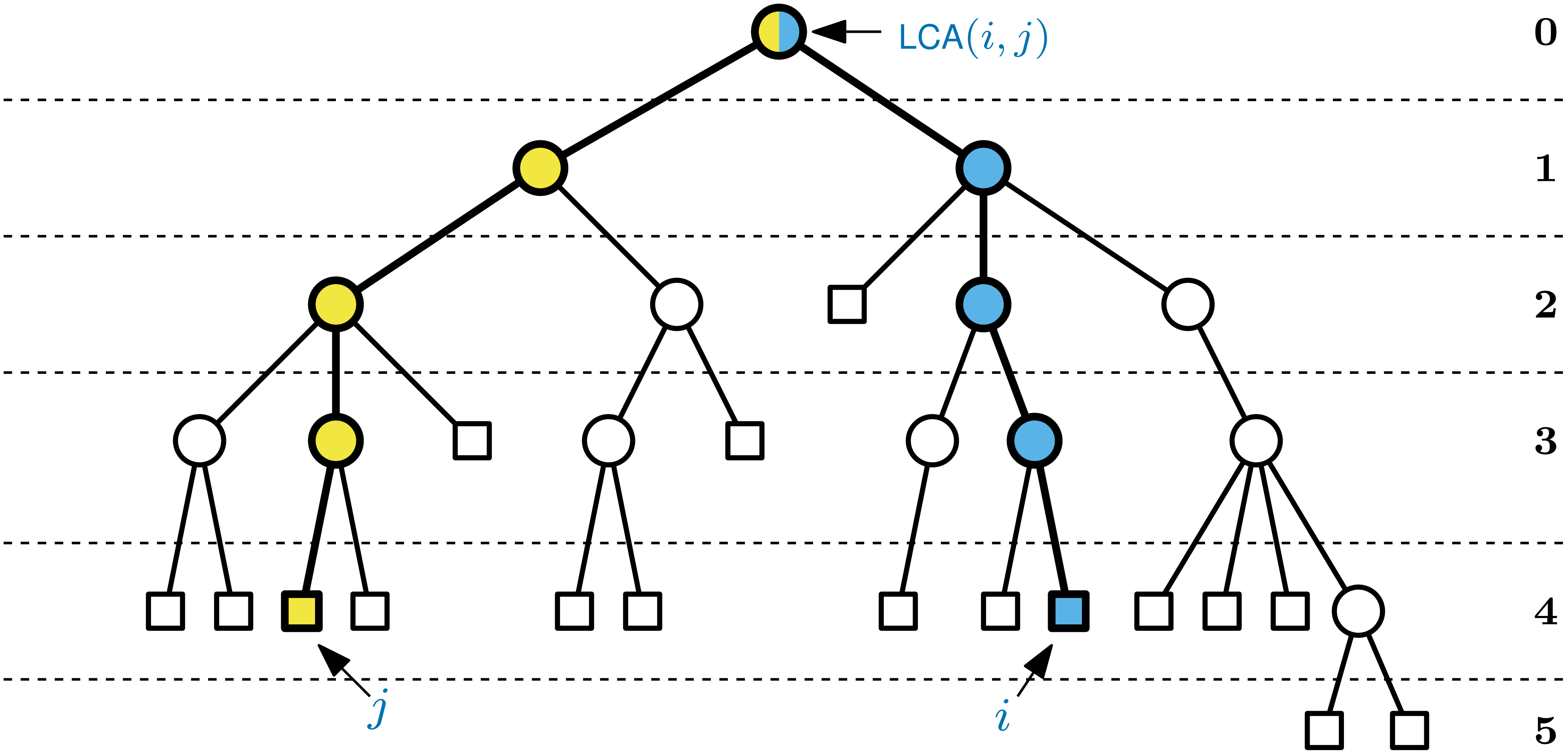

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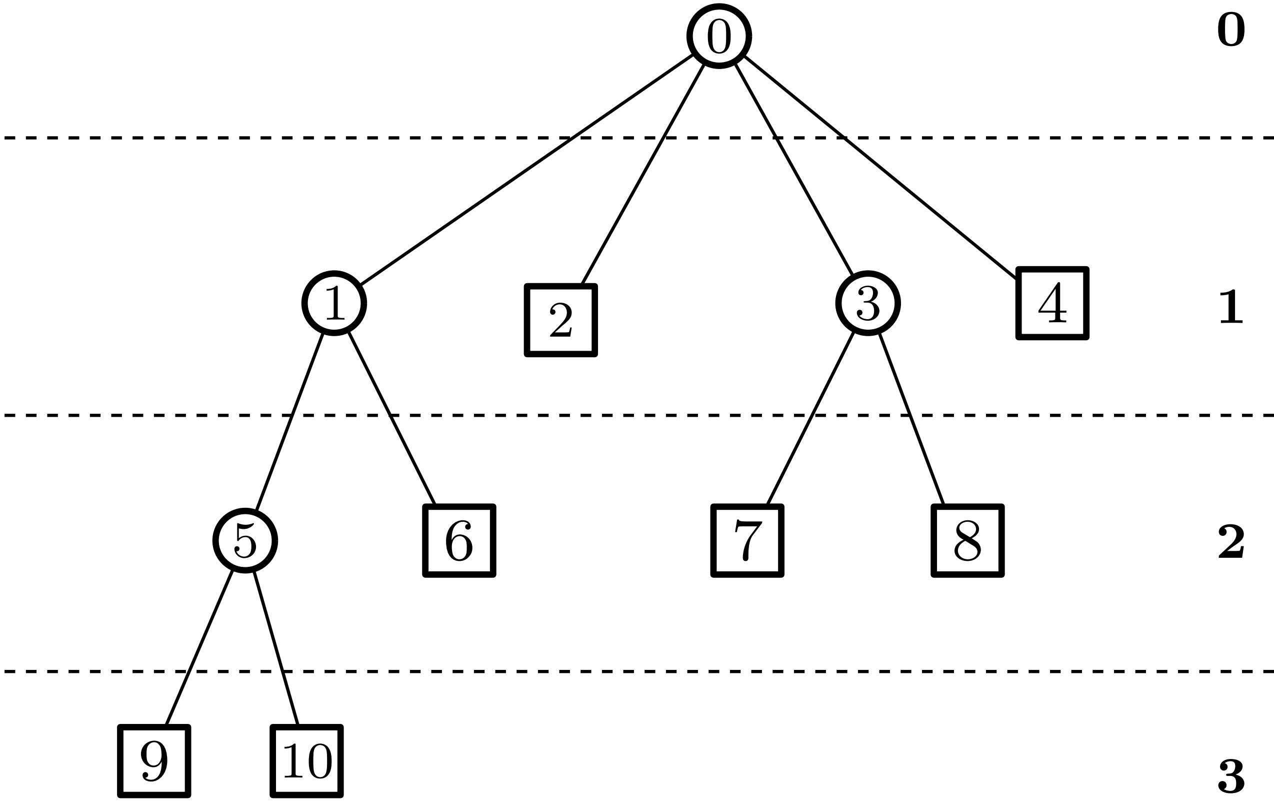
- Ideally, we would like $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time

Solving LCAs using RMQs



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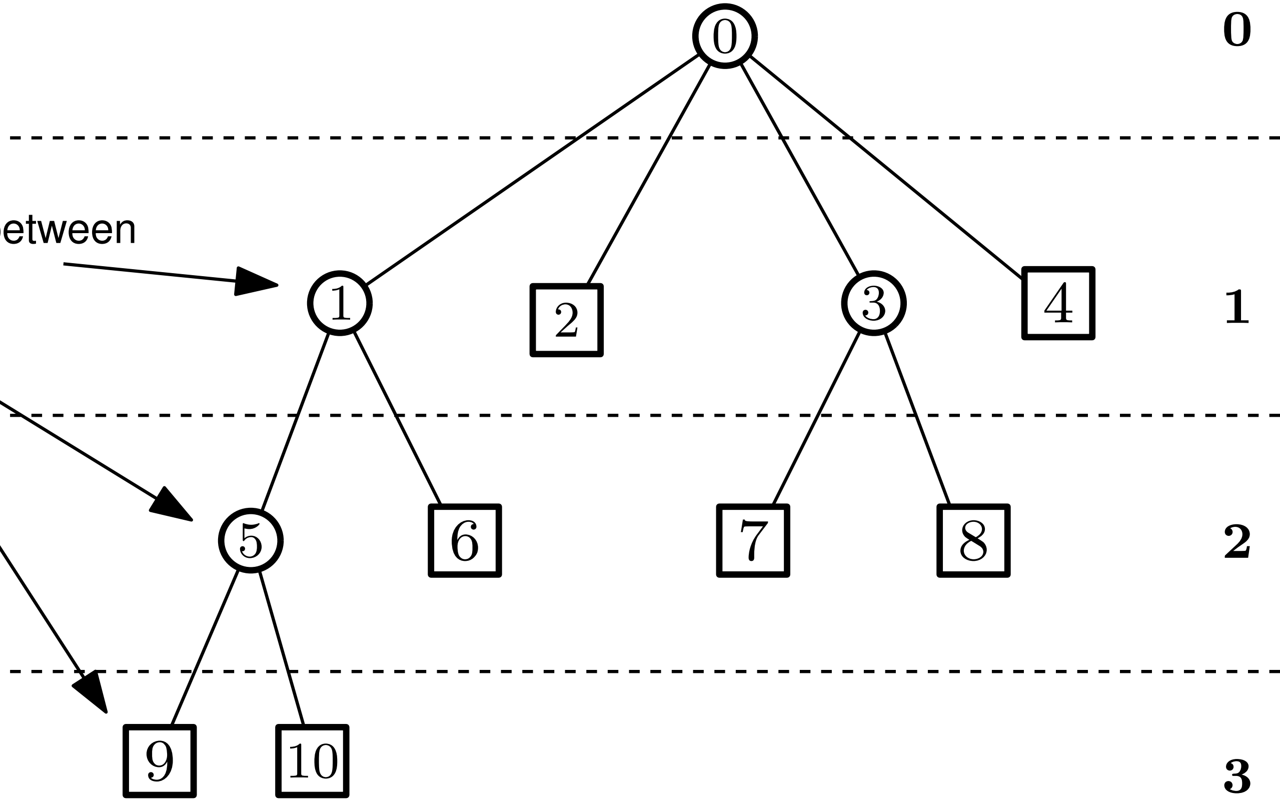

the nodes are numbered between

0 and $(n-1)$

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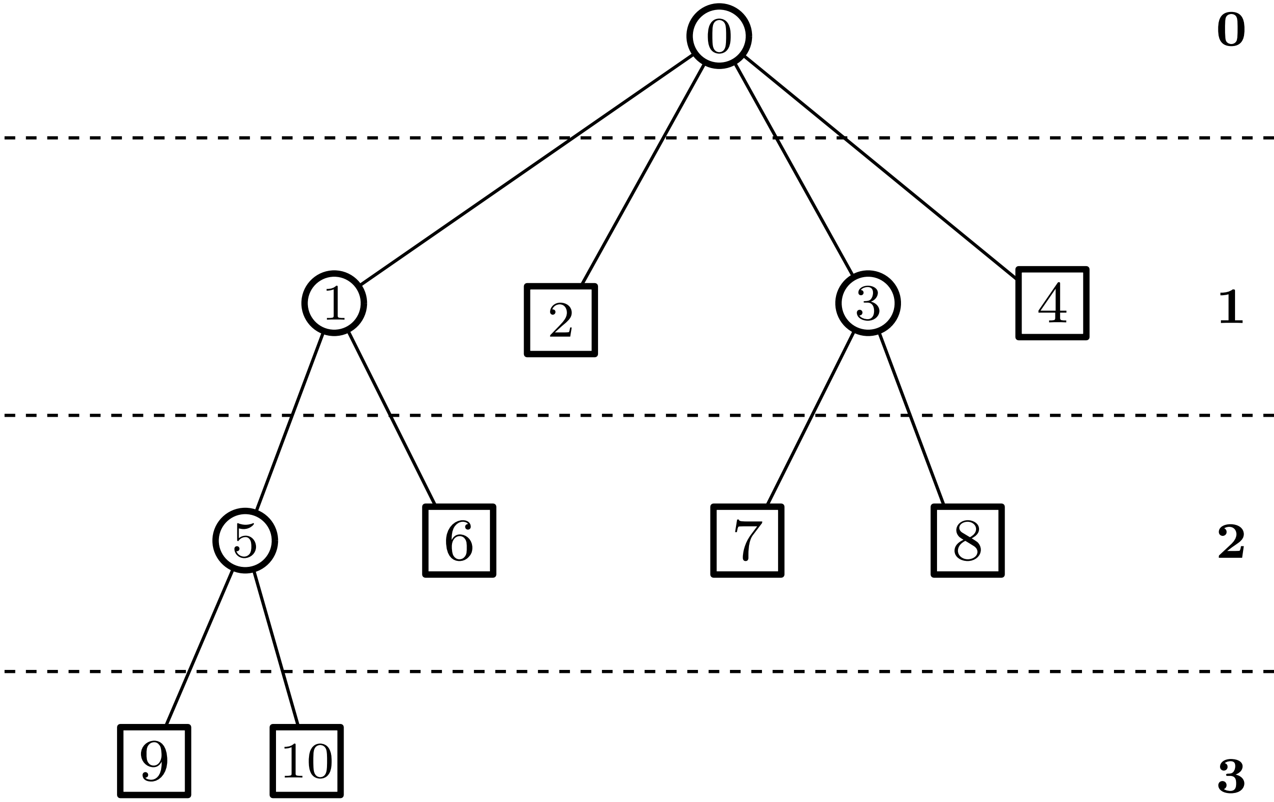


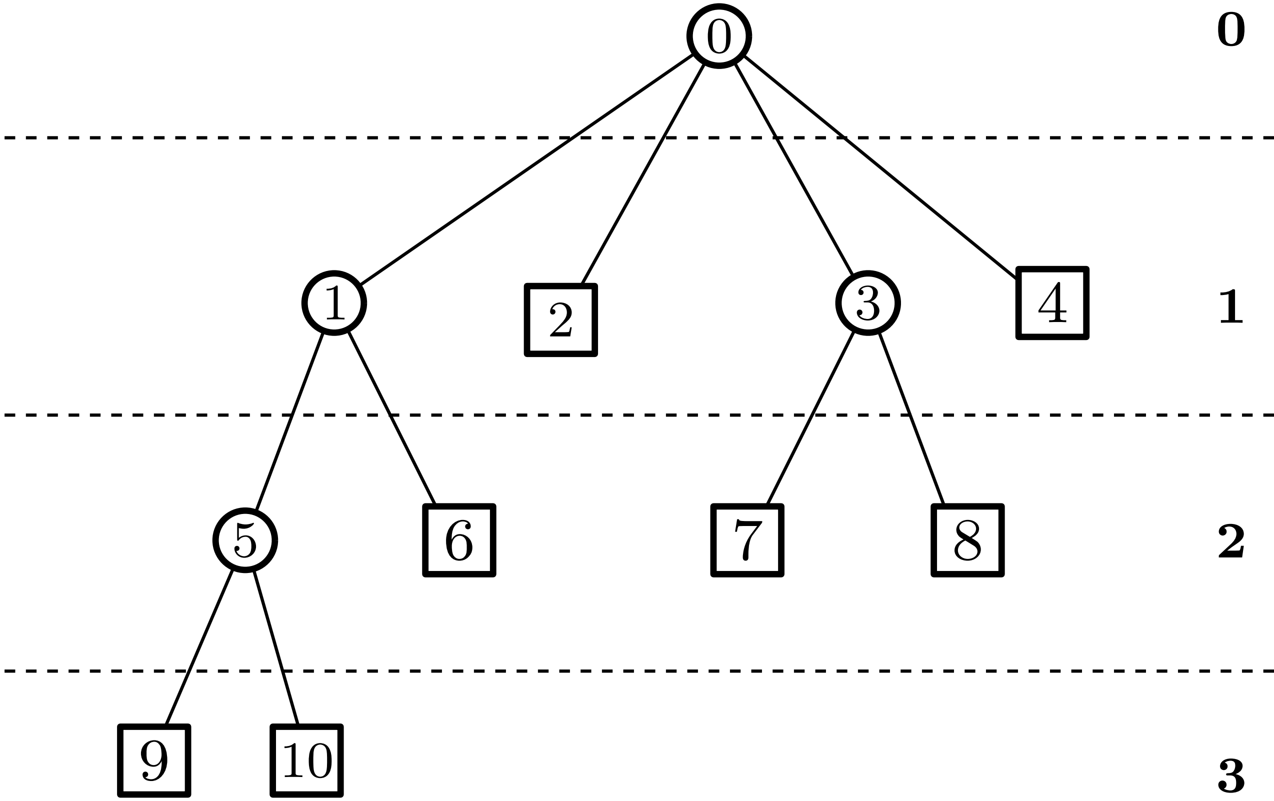
Solving LCAs using RMQs

Compute an Euler tour of T...

(a depth first search with repeats)

Write down every node you visit

...and its depth



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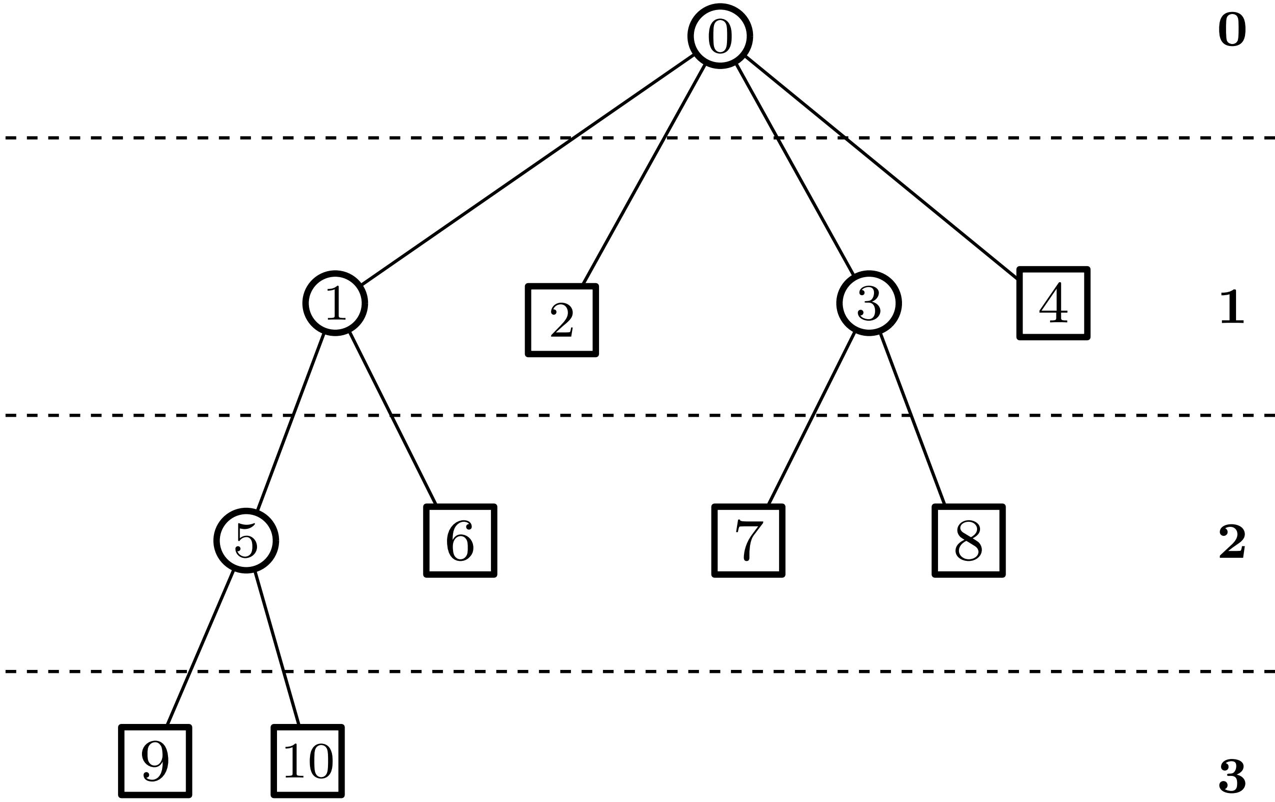

(node) N $\boxed{0}$

(depth) D $\boxed{\mathbf{0}}$

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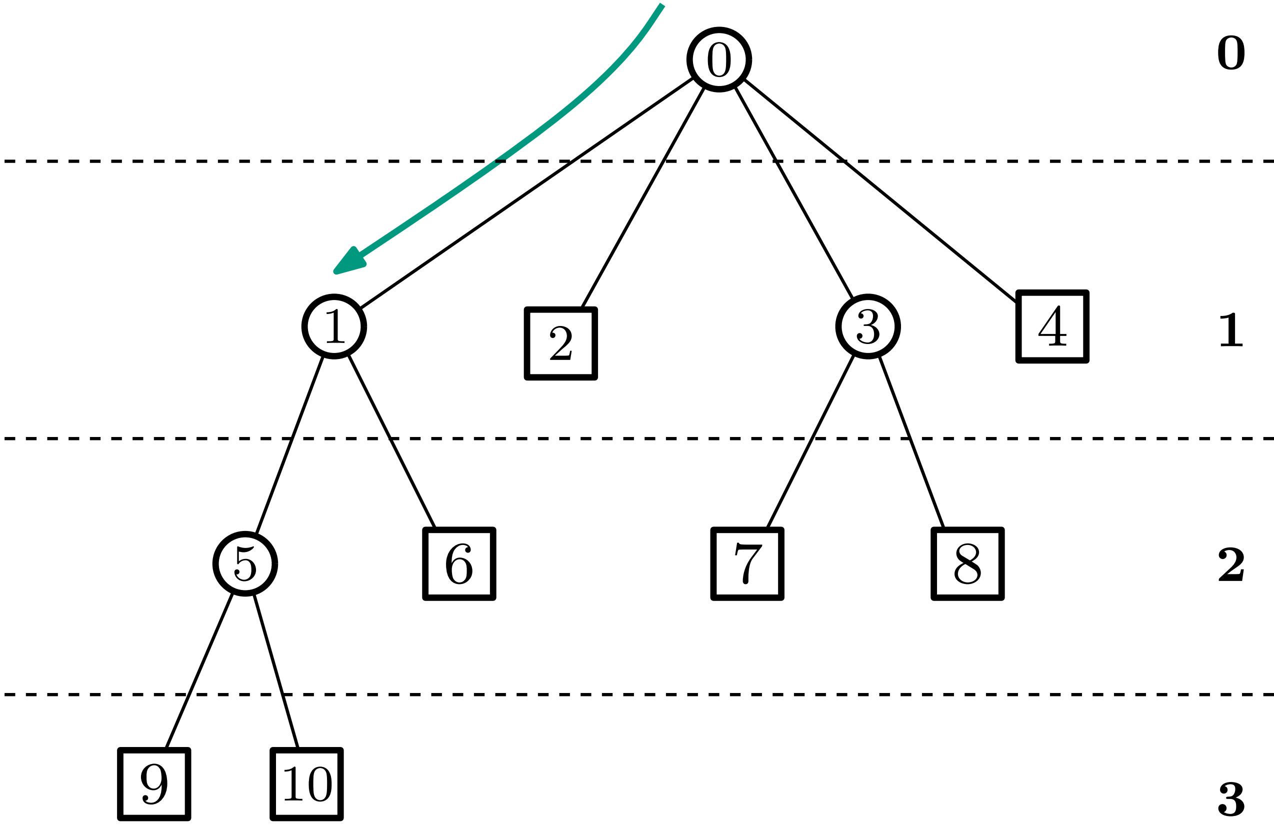

(node) N	0	1
(depth) D	**0**	**1**

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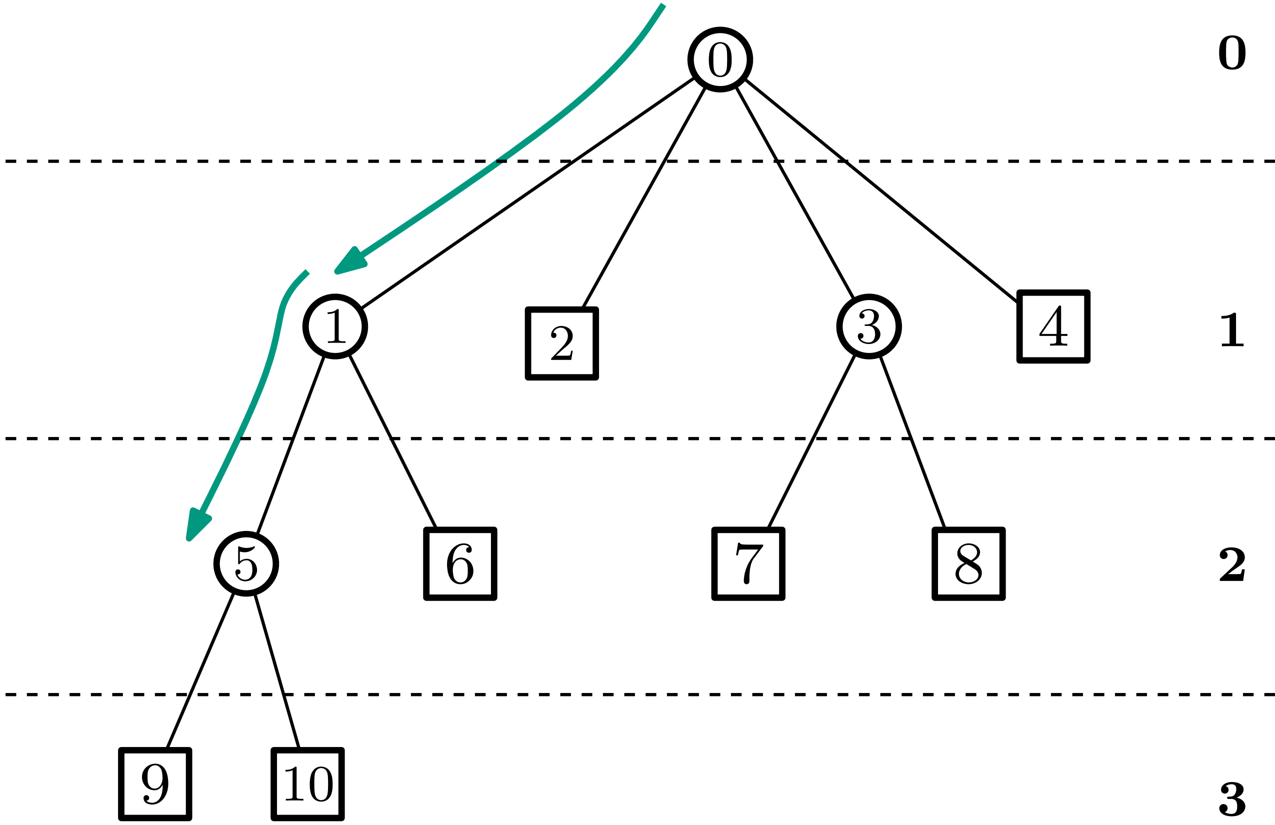

(node) N	0	1	5
(depth) D	**0**	**1**	**2**

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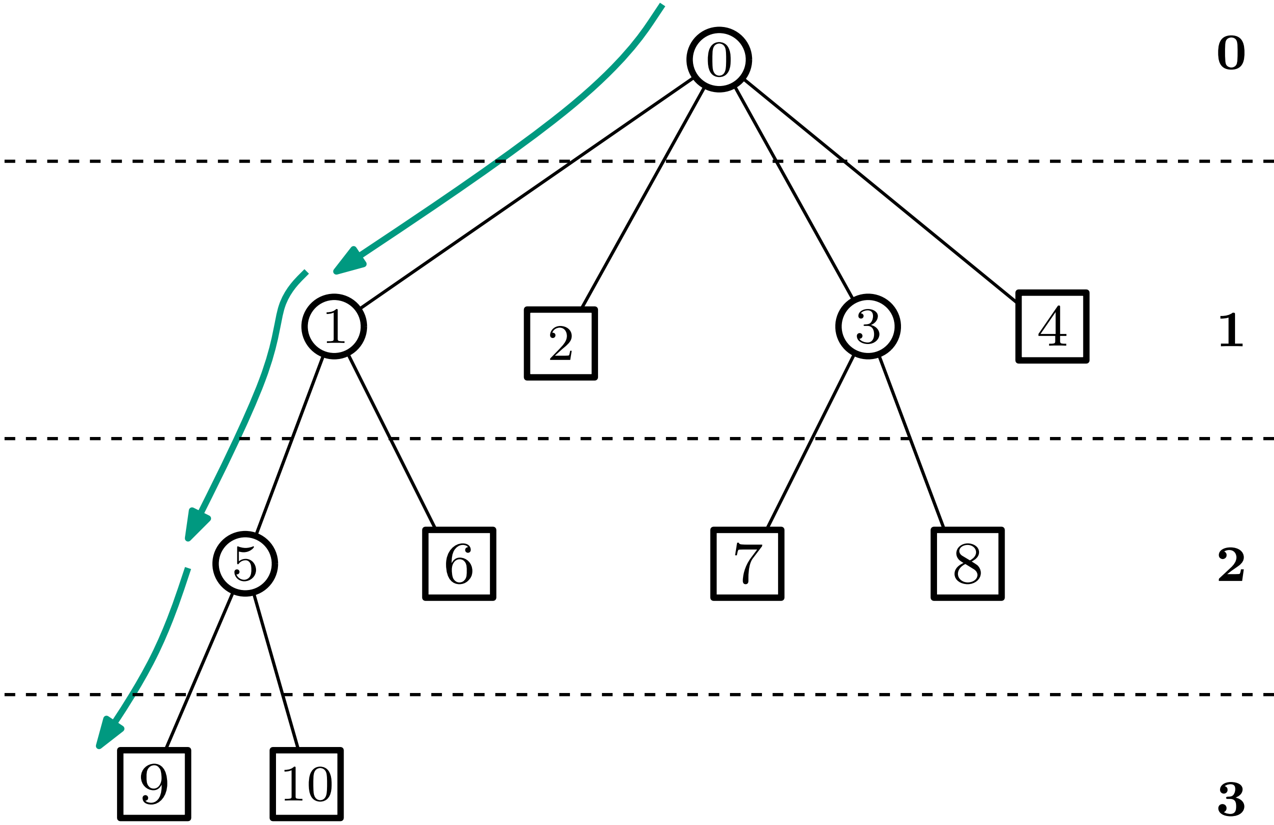

(node) N

0	1	5	9

(depth) D

0	**1**	**2**	**3**

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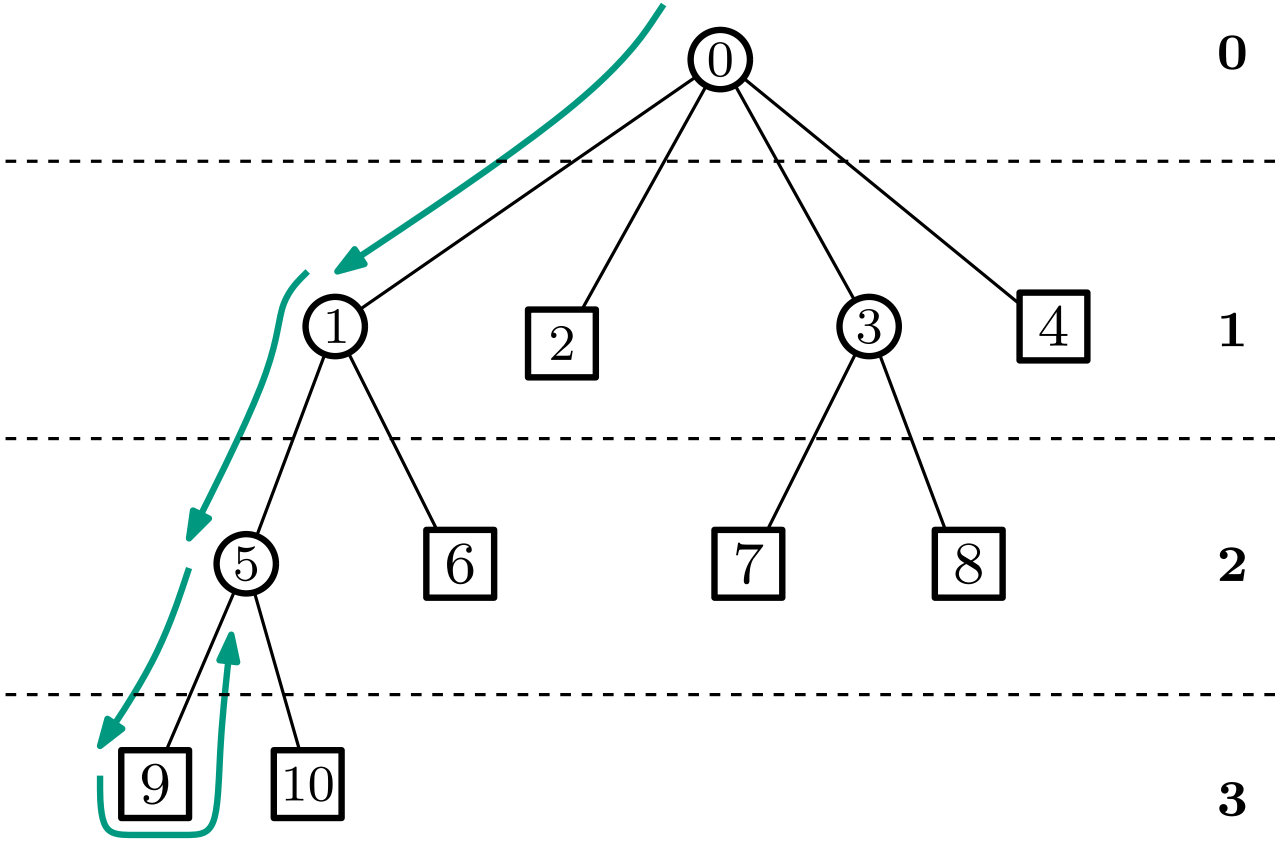

(node) N	0	1	5	9	5
(depth) D	**0**	**1**	**2**	**3**	**2**

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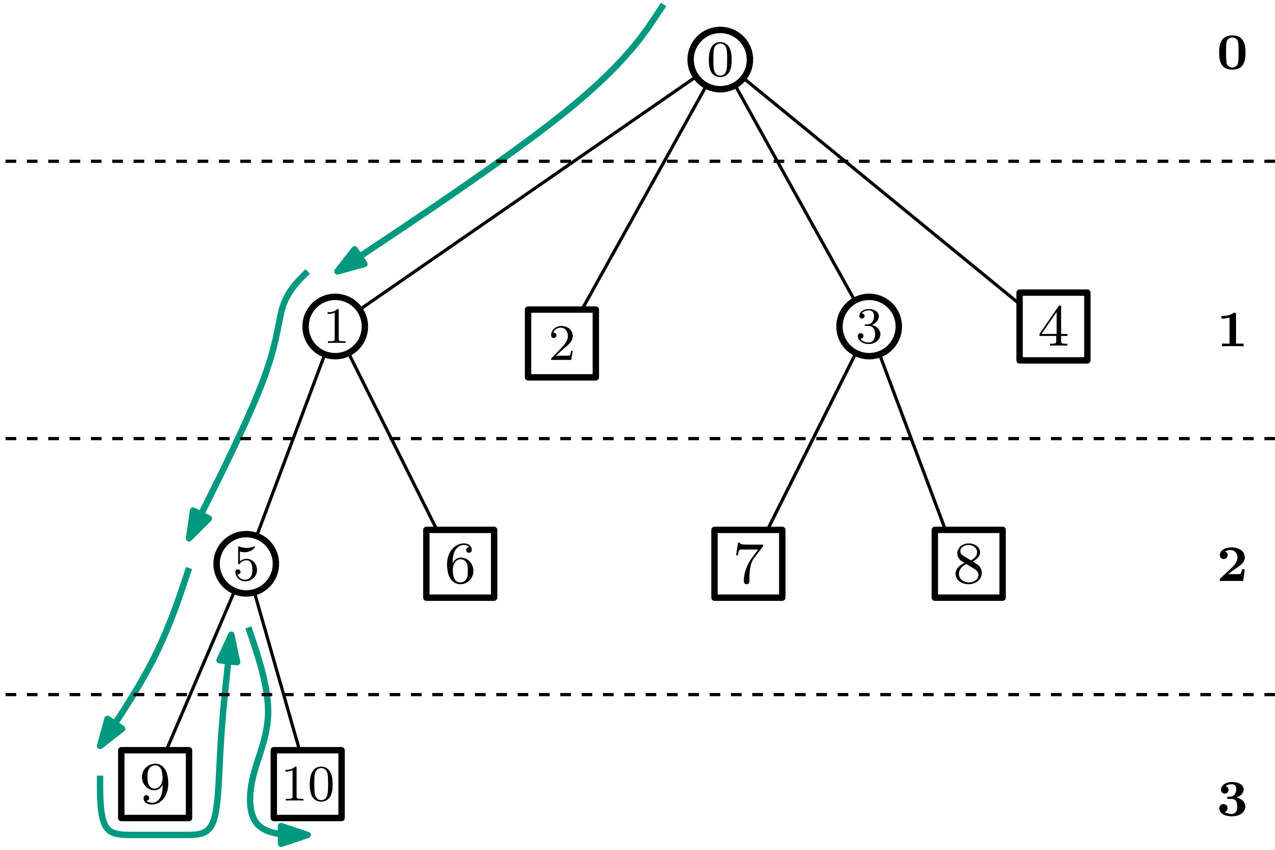

(node) N	0	1	5	9	5	10
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**

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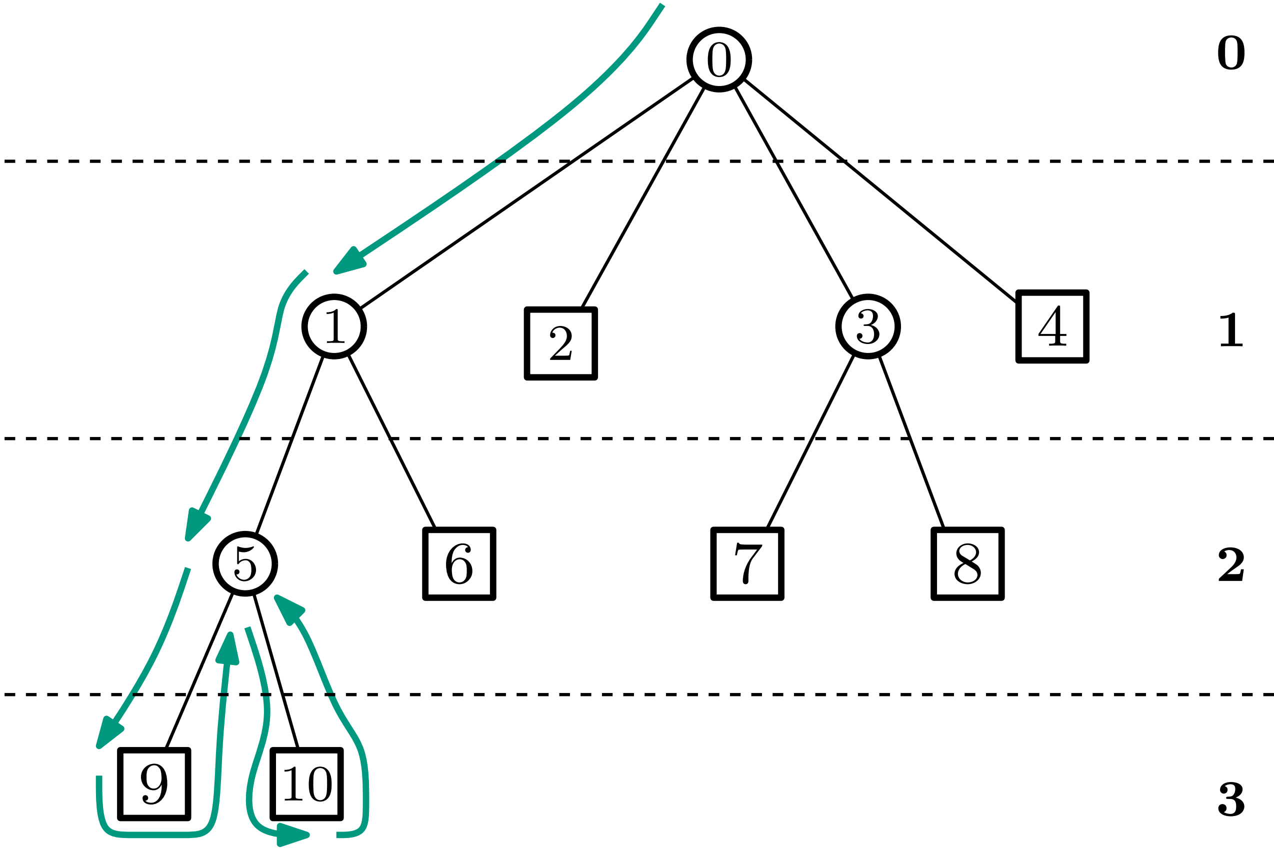

(node) N	0	1	5	9	5	10	5
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**

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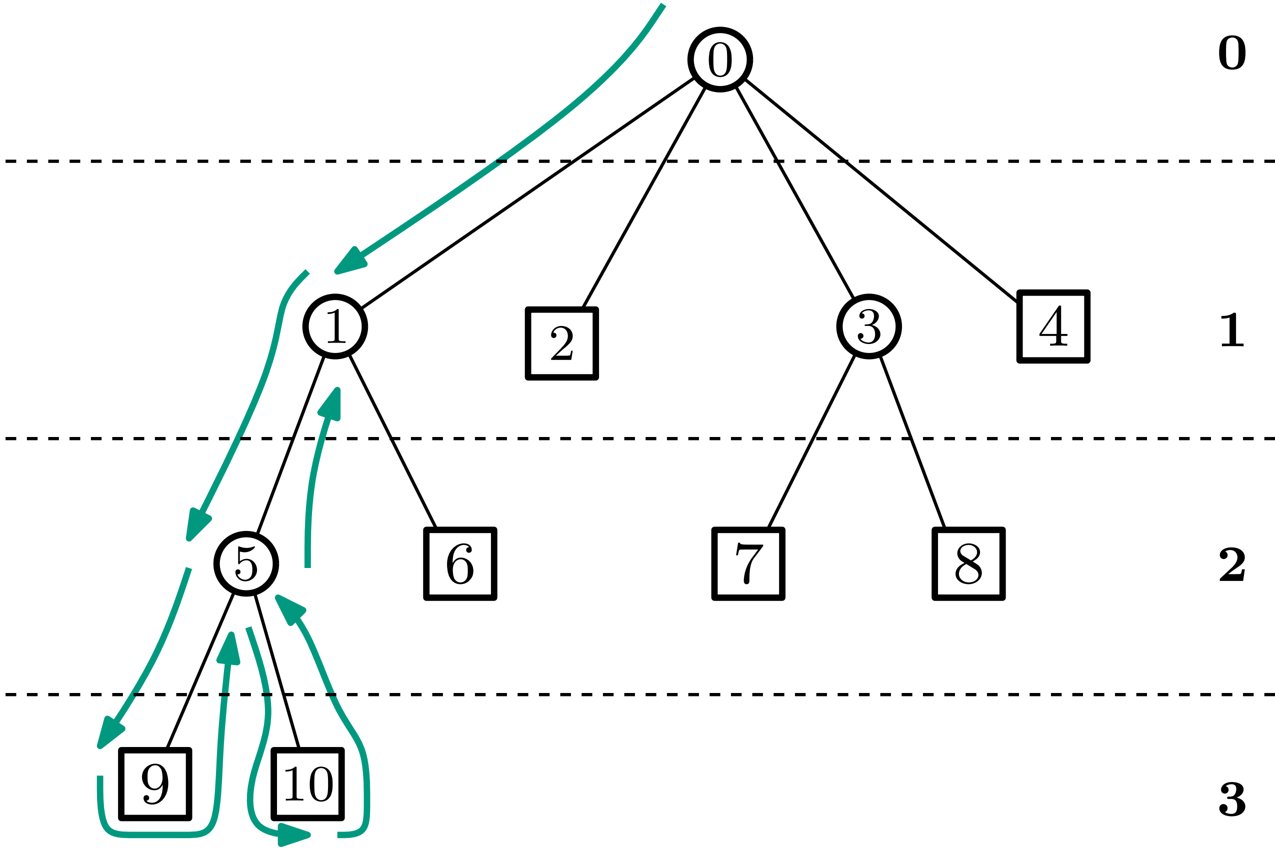

(node) N	0	1	5	9	5	10	5	1
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**

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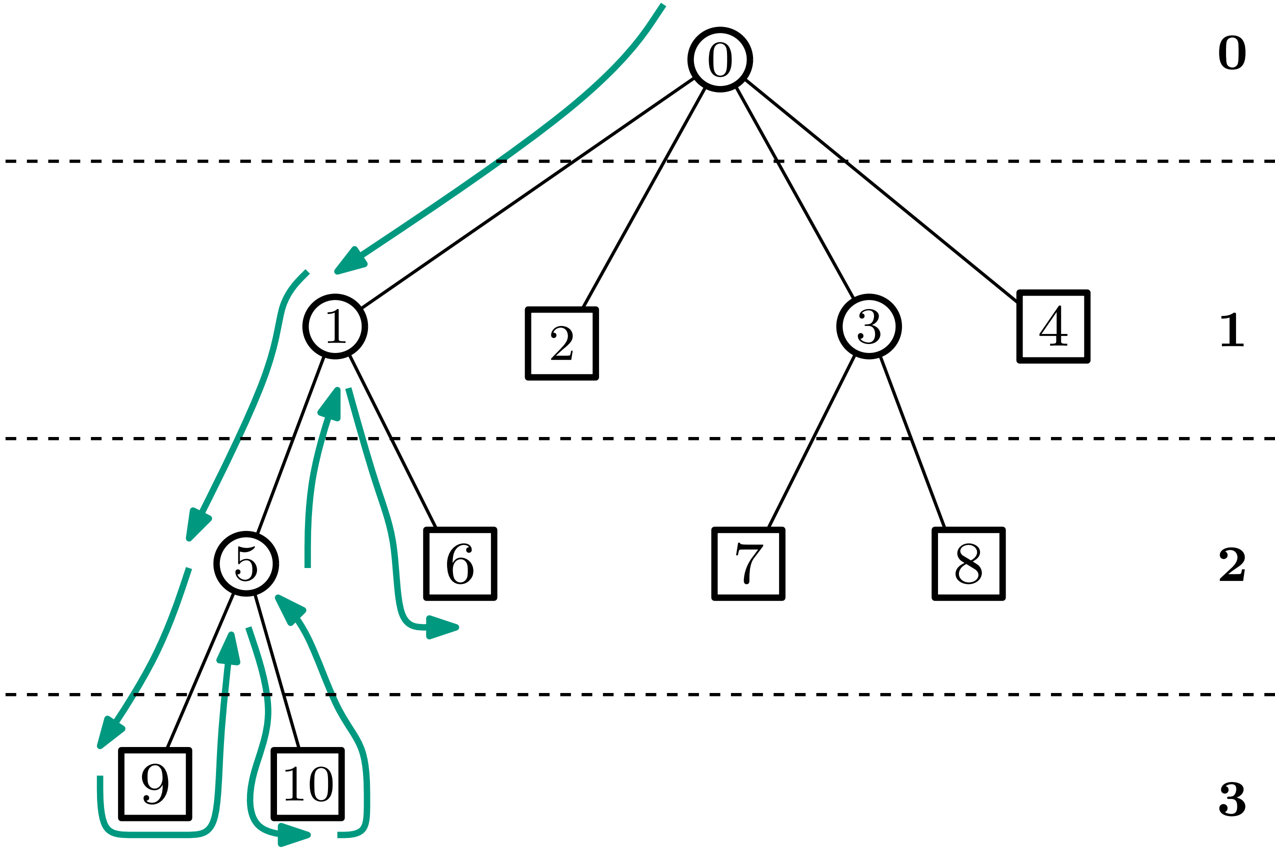

(node) N	0	1	5	9	5	10	5	1	6
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**

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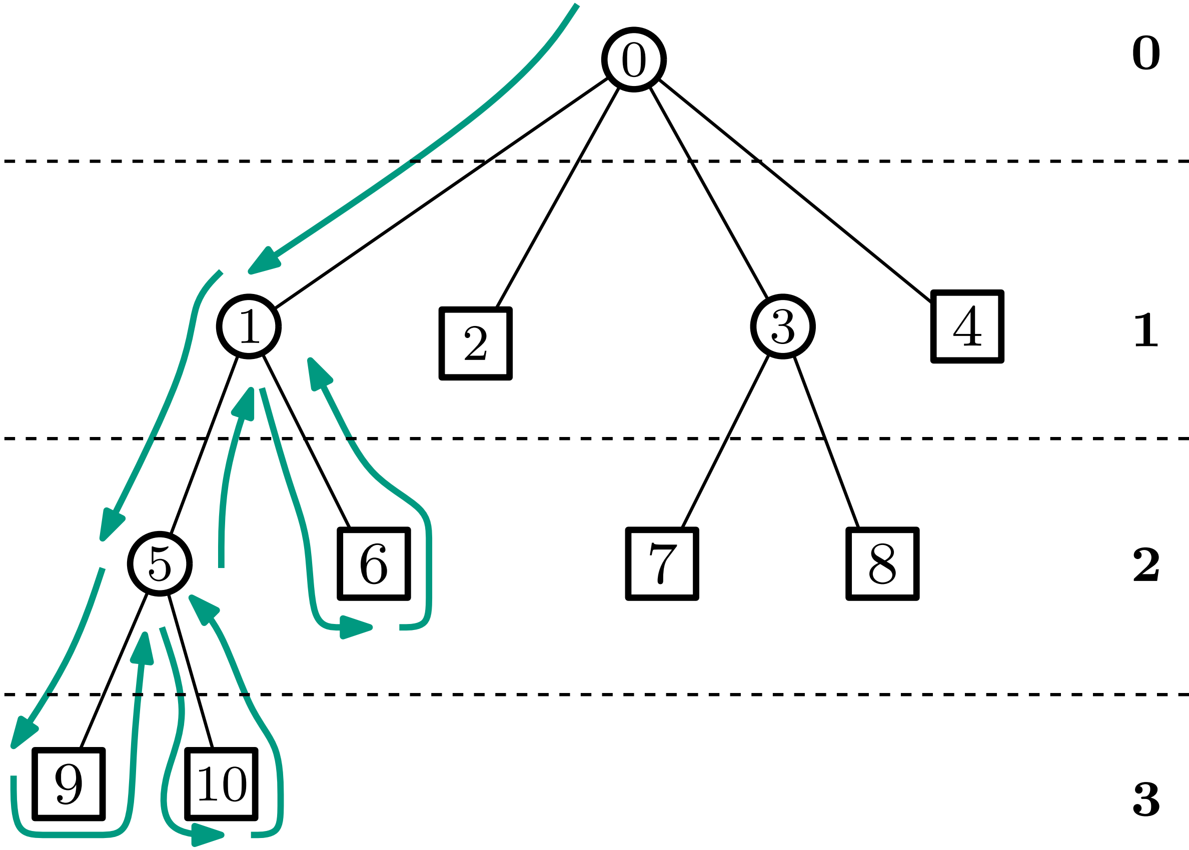

(node) N	0	1	5	9	5	10	5	1	6	1
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**

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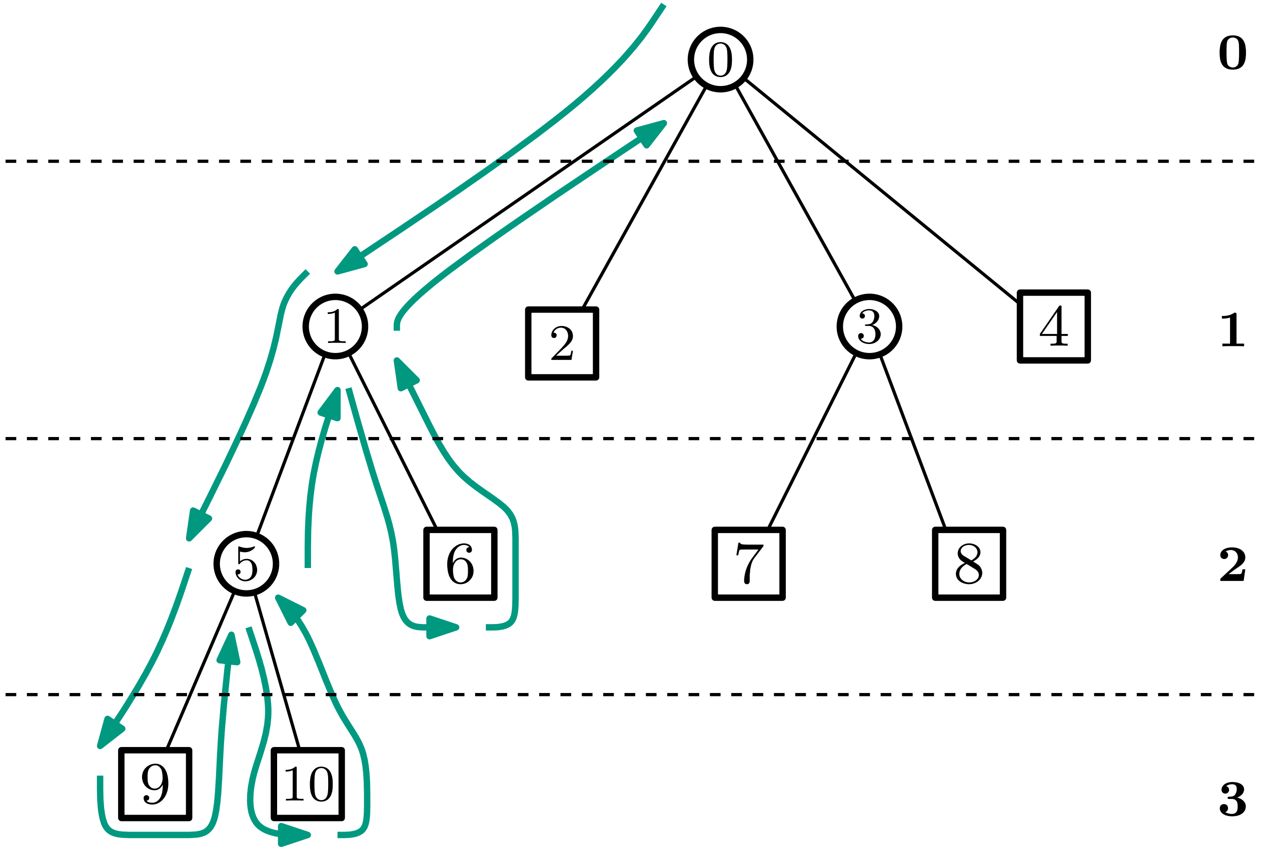

(node) N	0	1	5	9	5	10	5	1	6	1	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**

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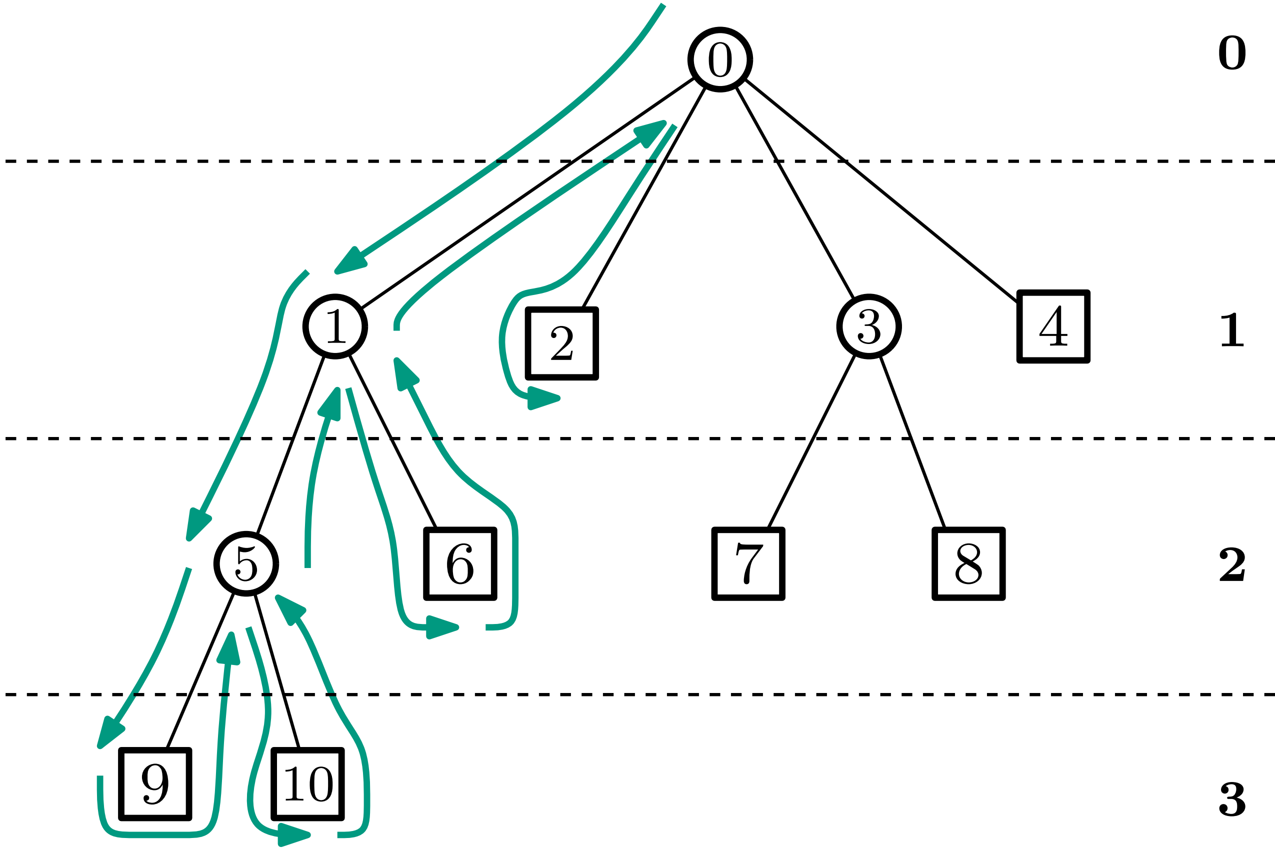

(node) N	0	1	5	9	5	10	5	1	6	1	0	2
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**

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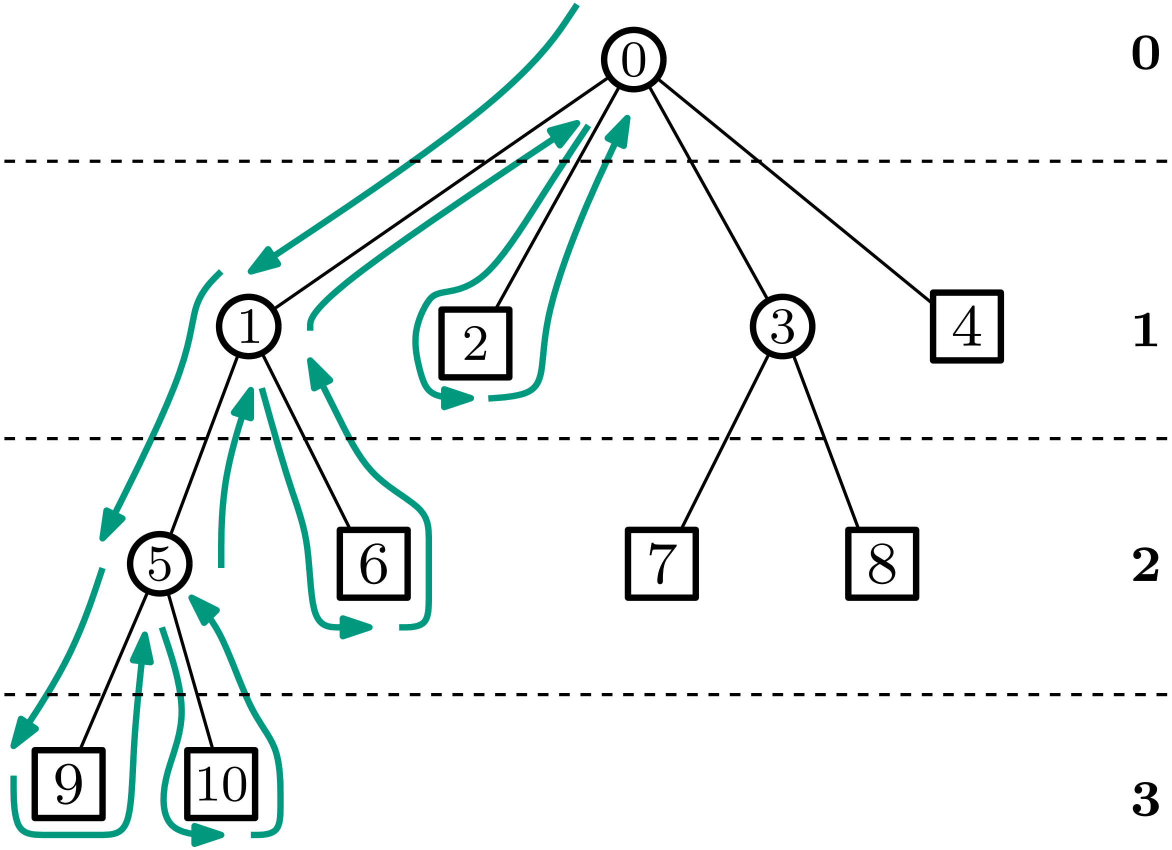

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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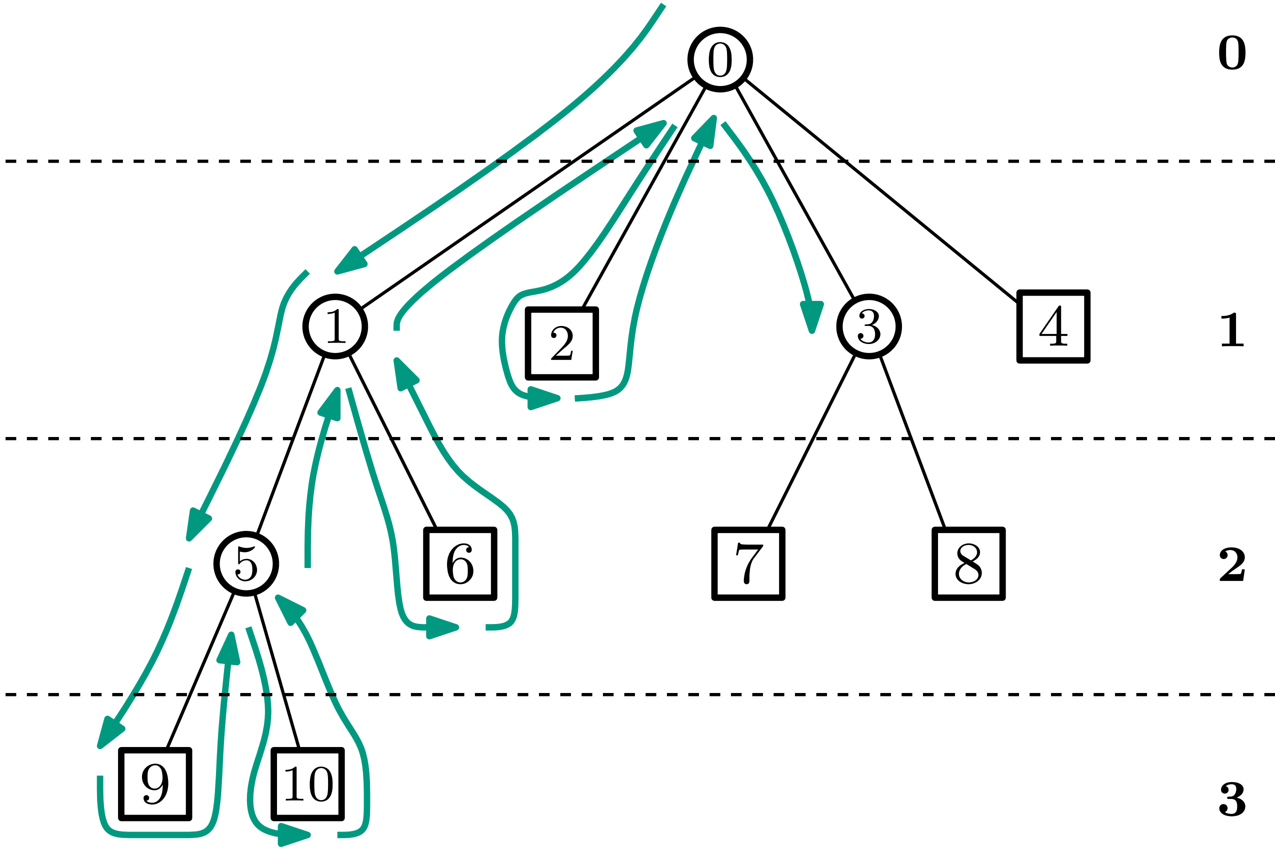

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**

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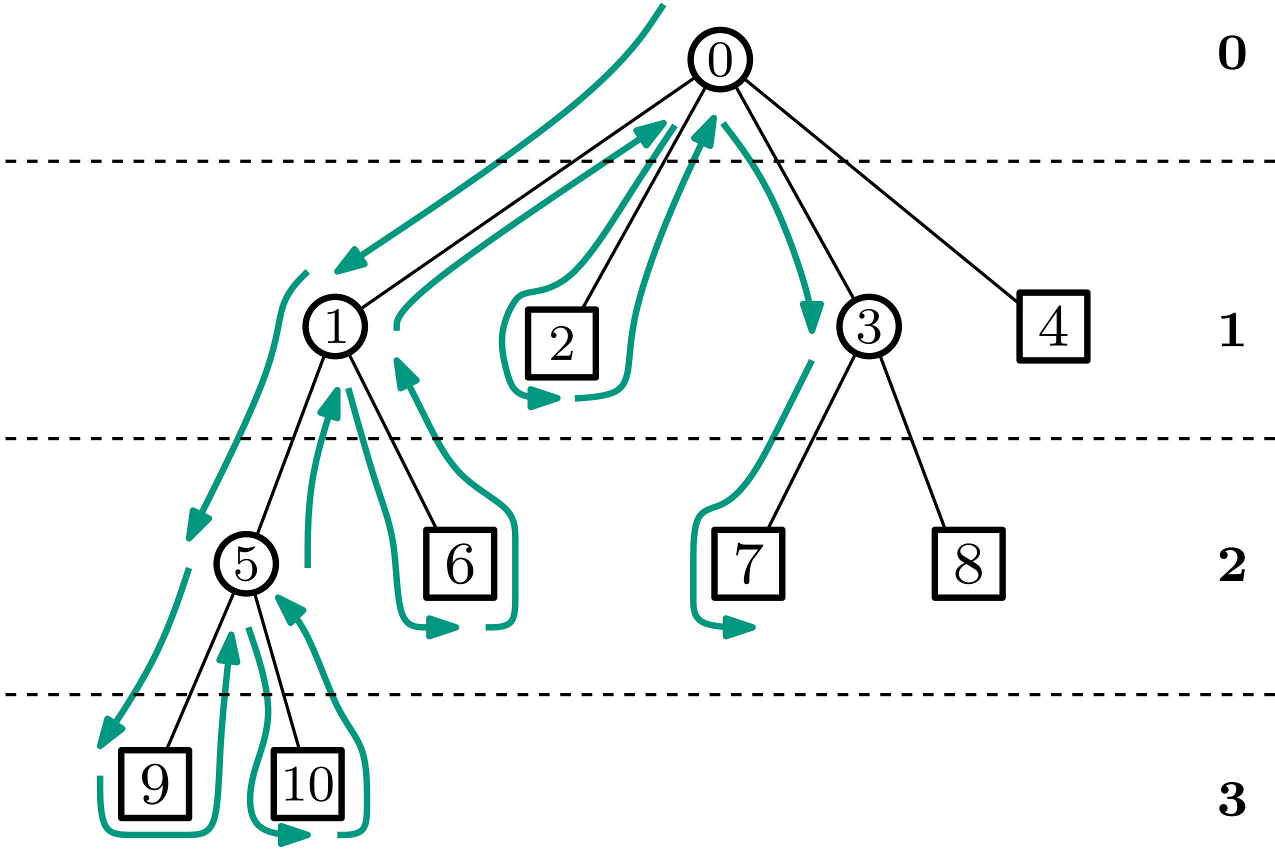


(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**

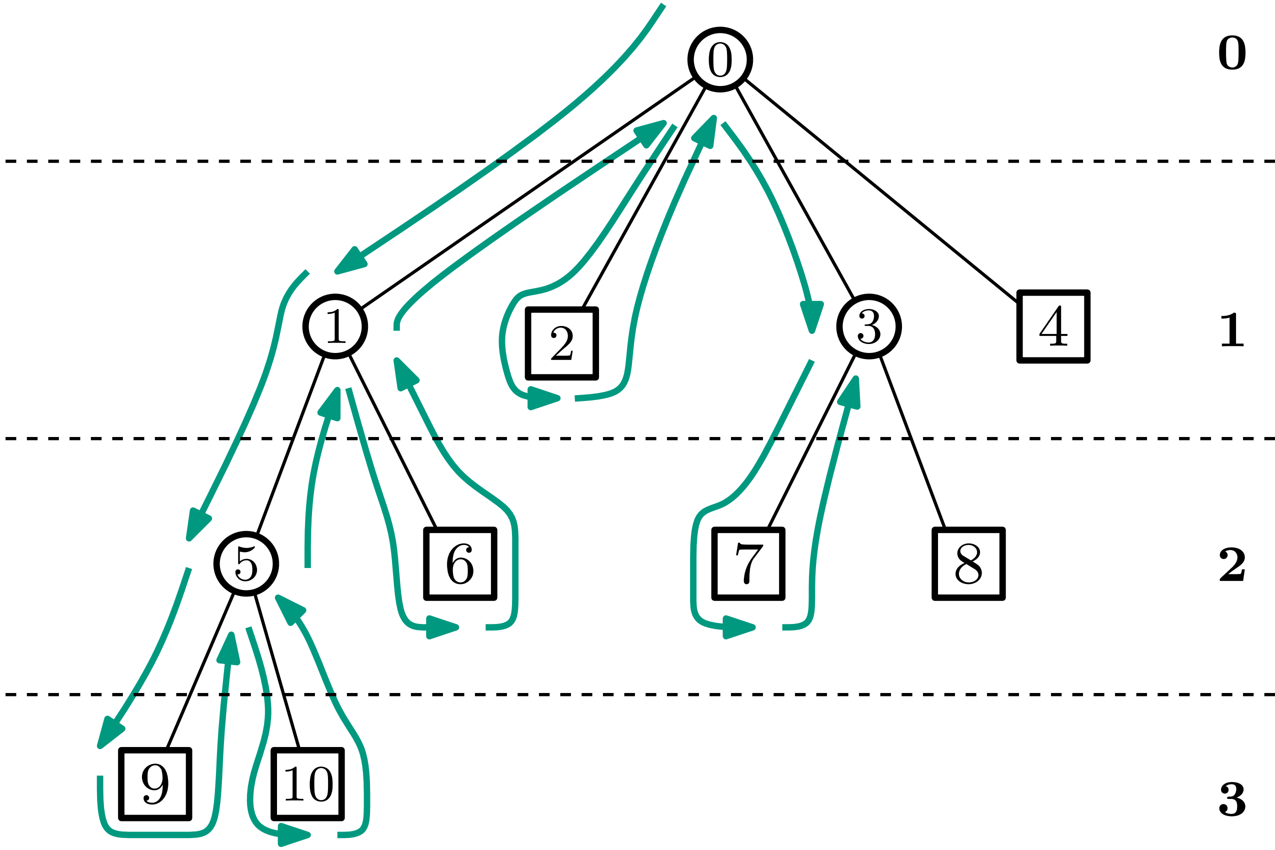
Solving LCAs using RMQs

Compute an Euler tour of T...

(a depth first search with repeats)

Write down every node you visit

...and its depth



(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**

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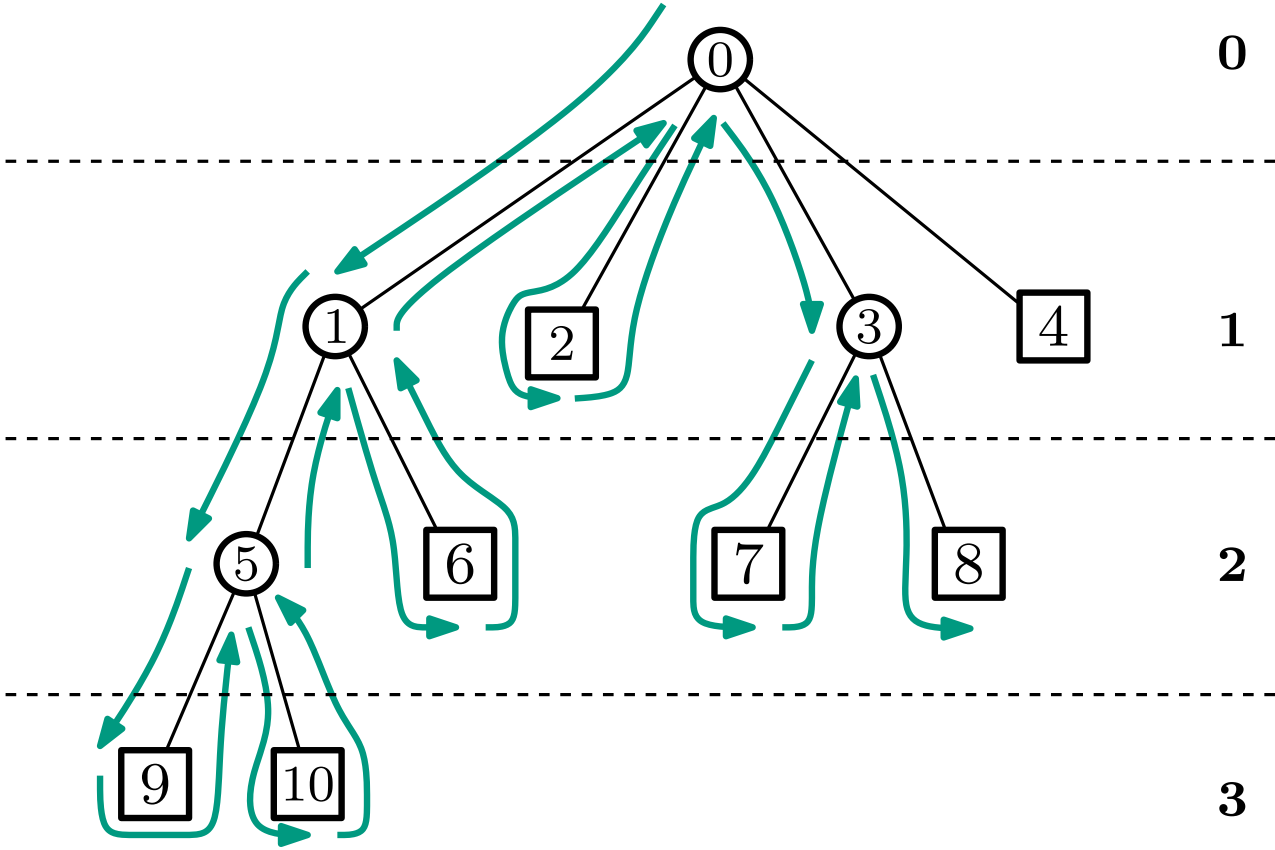

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**

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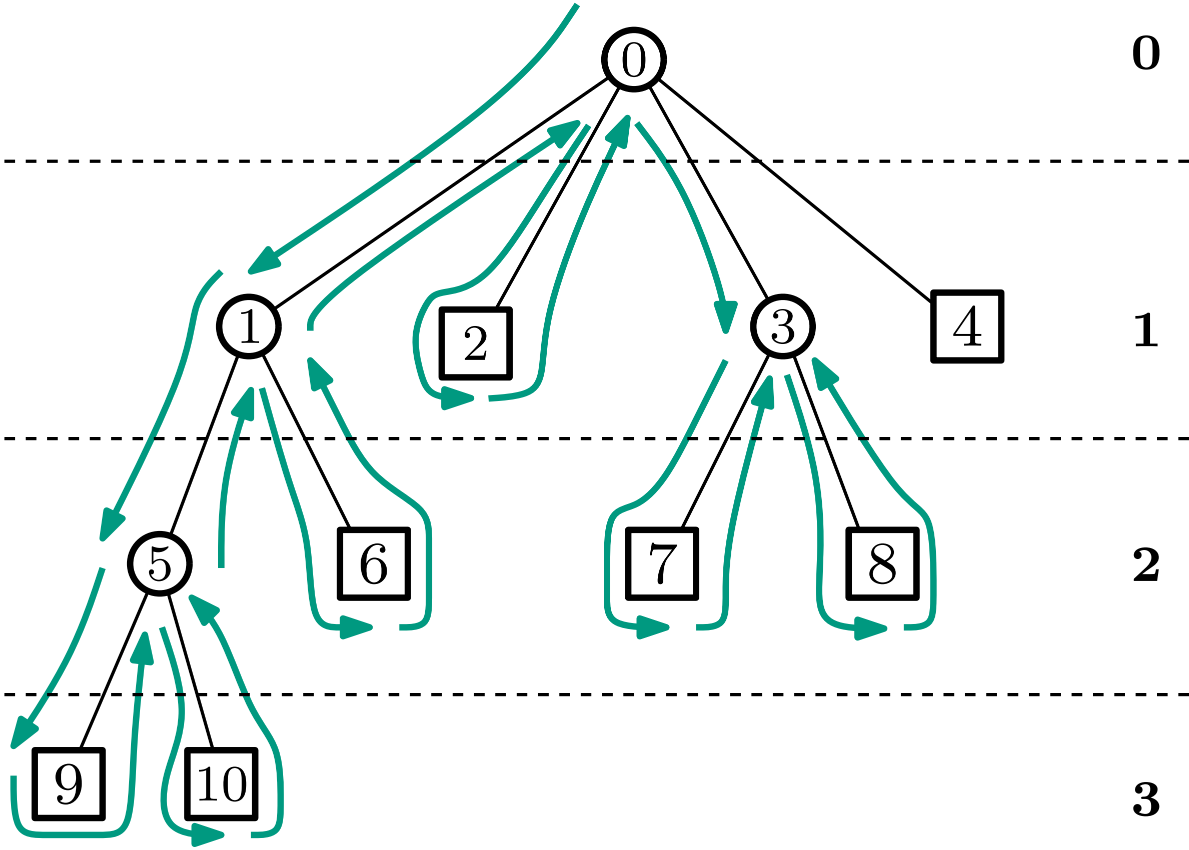

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**

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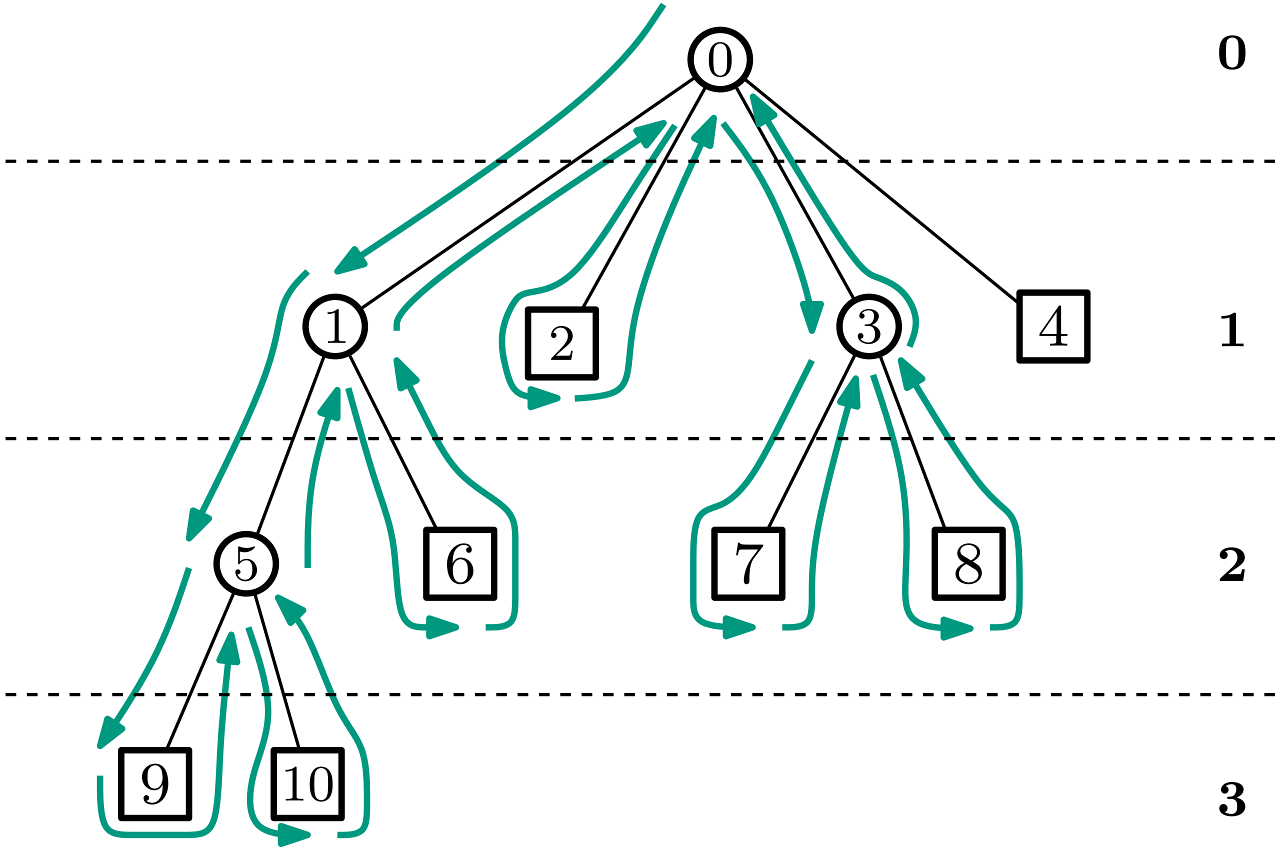

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**

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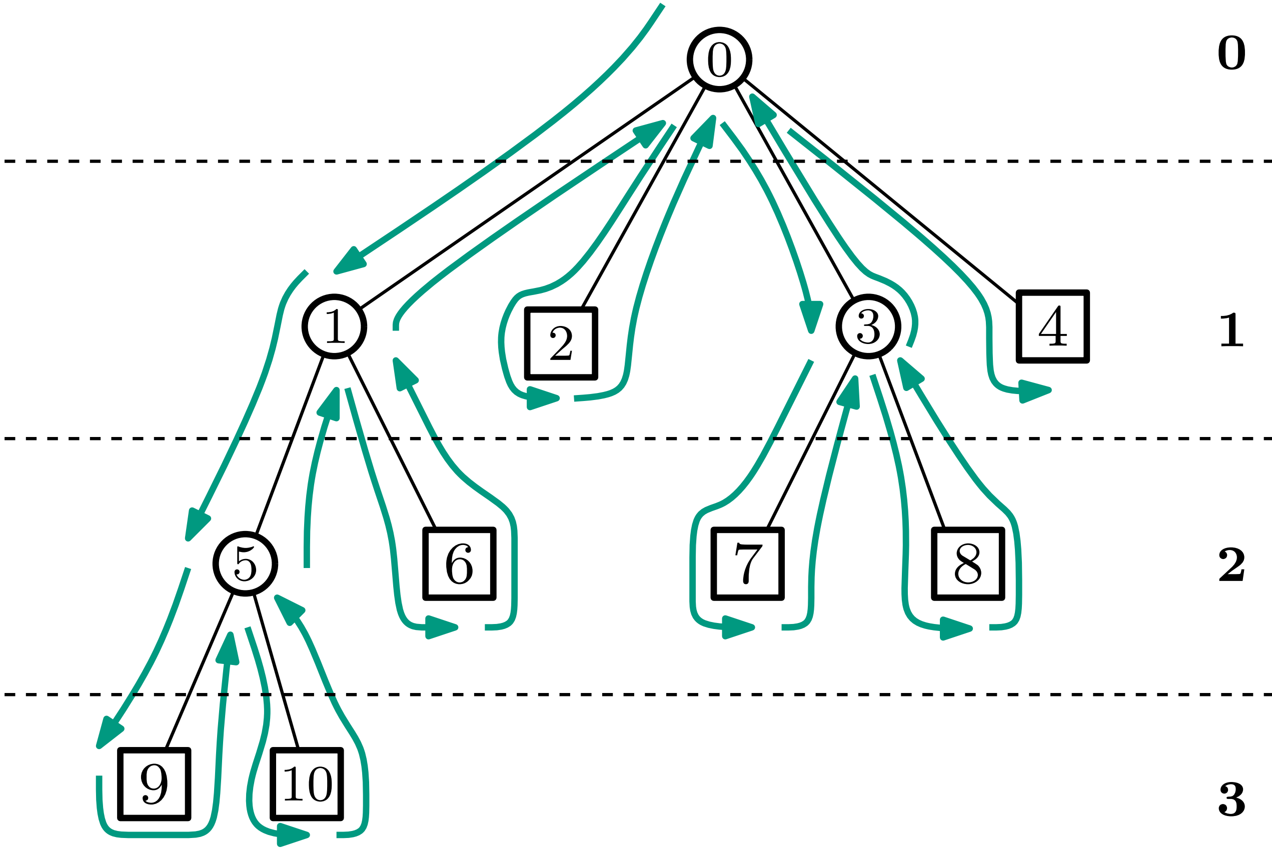

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**

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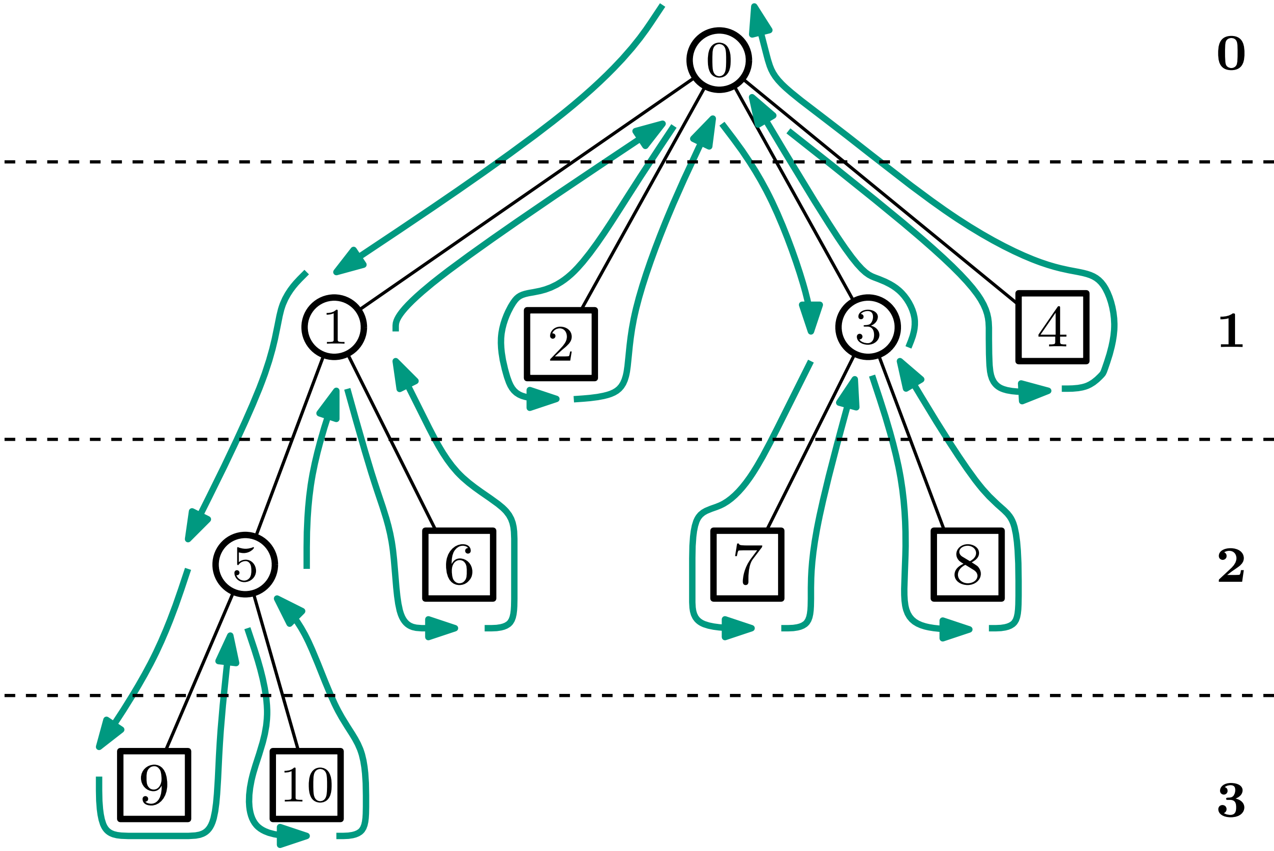

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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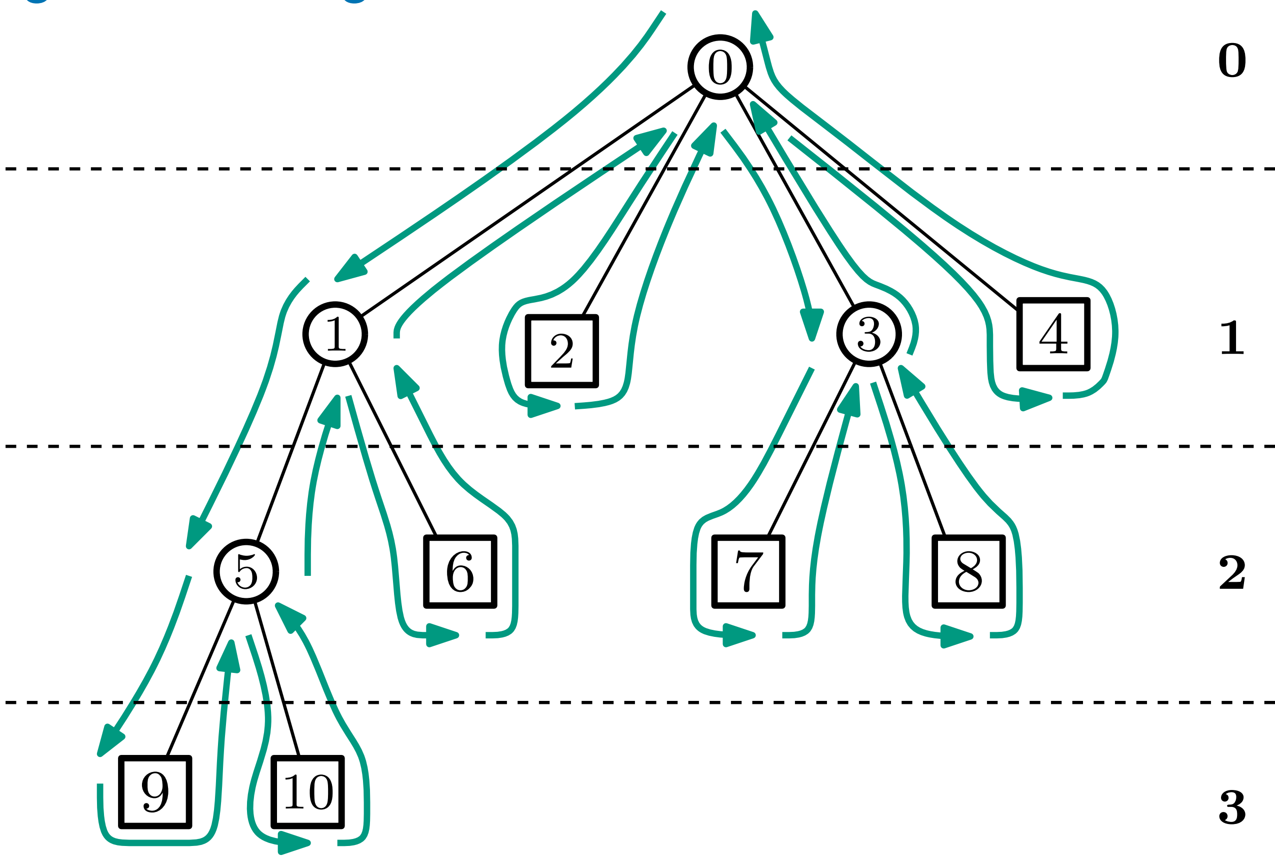

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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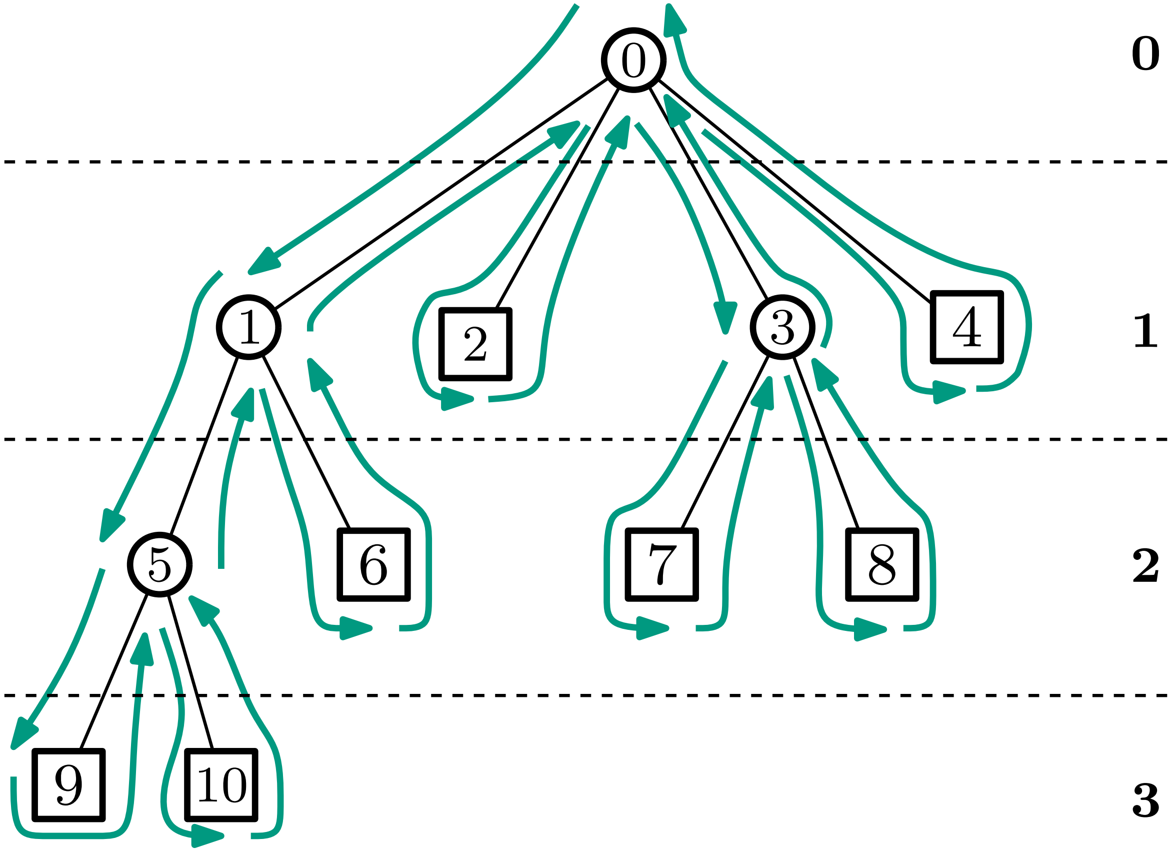
We follow each edge twice...

and there are $(n-1)$ edges



(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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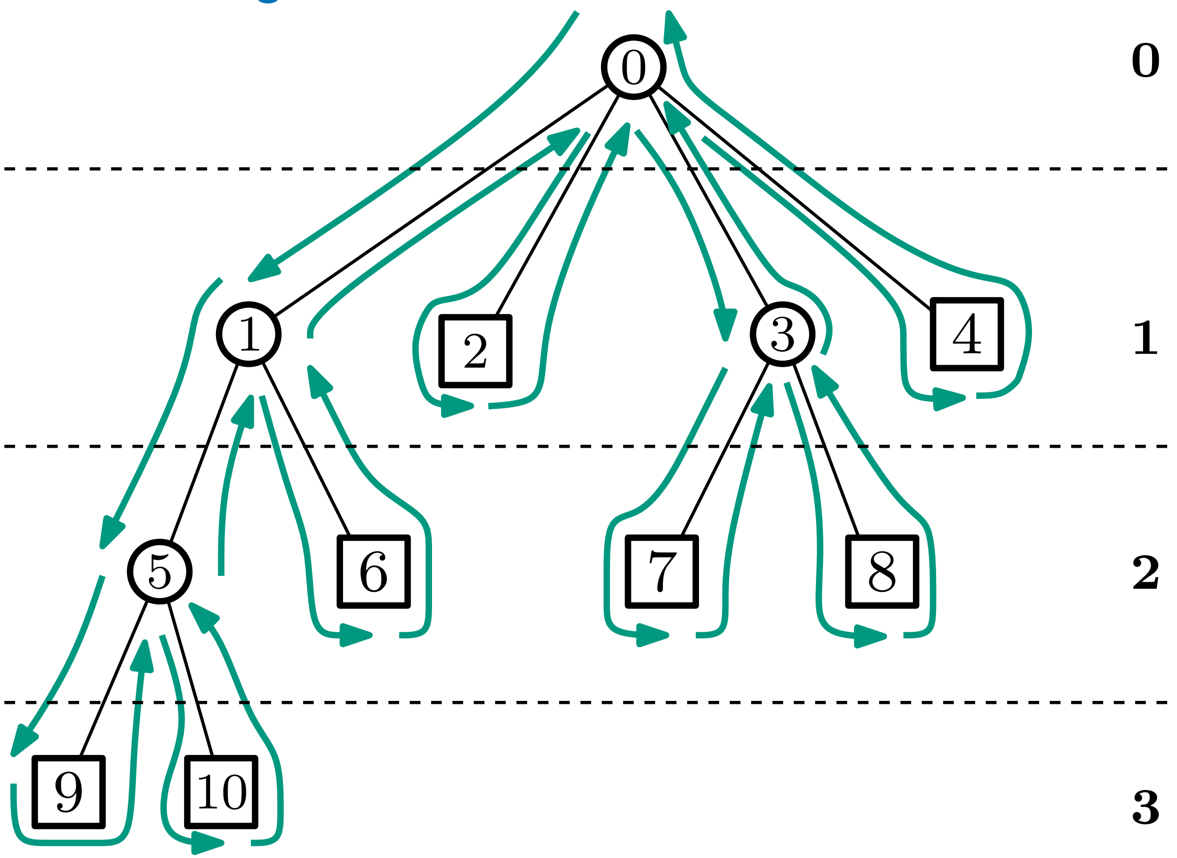

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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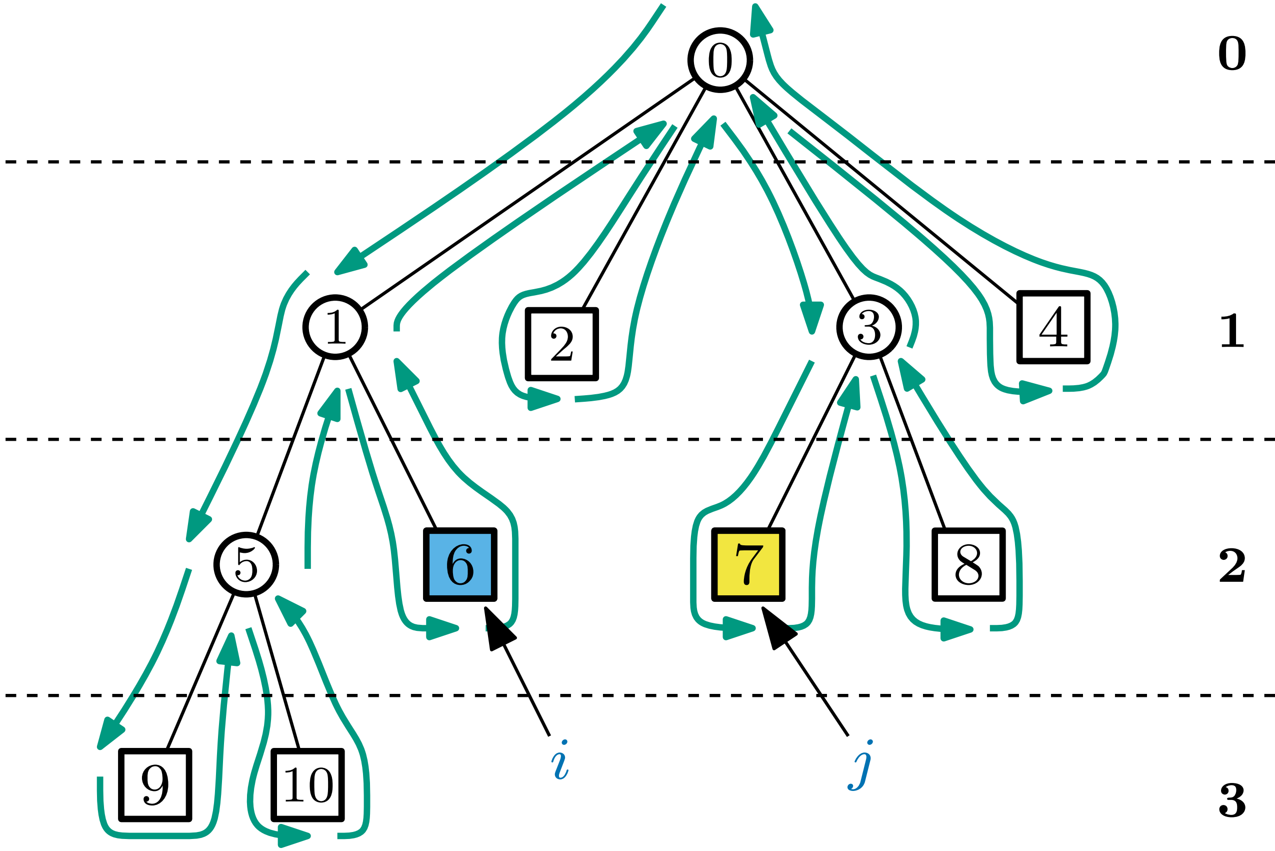

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

how do we find LCA(i,j)?

Solving LCAs using RMQs

Compute an Euler tour of T...

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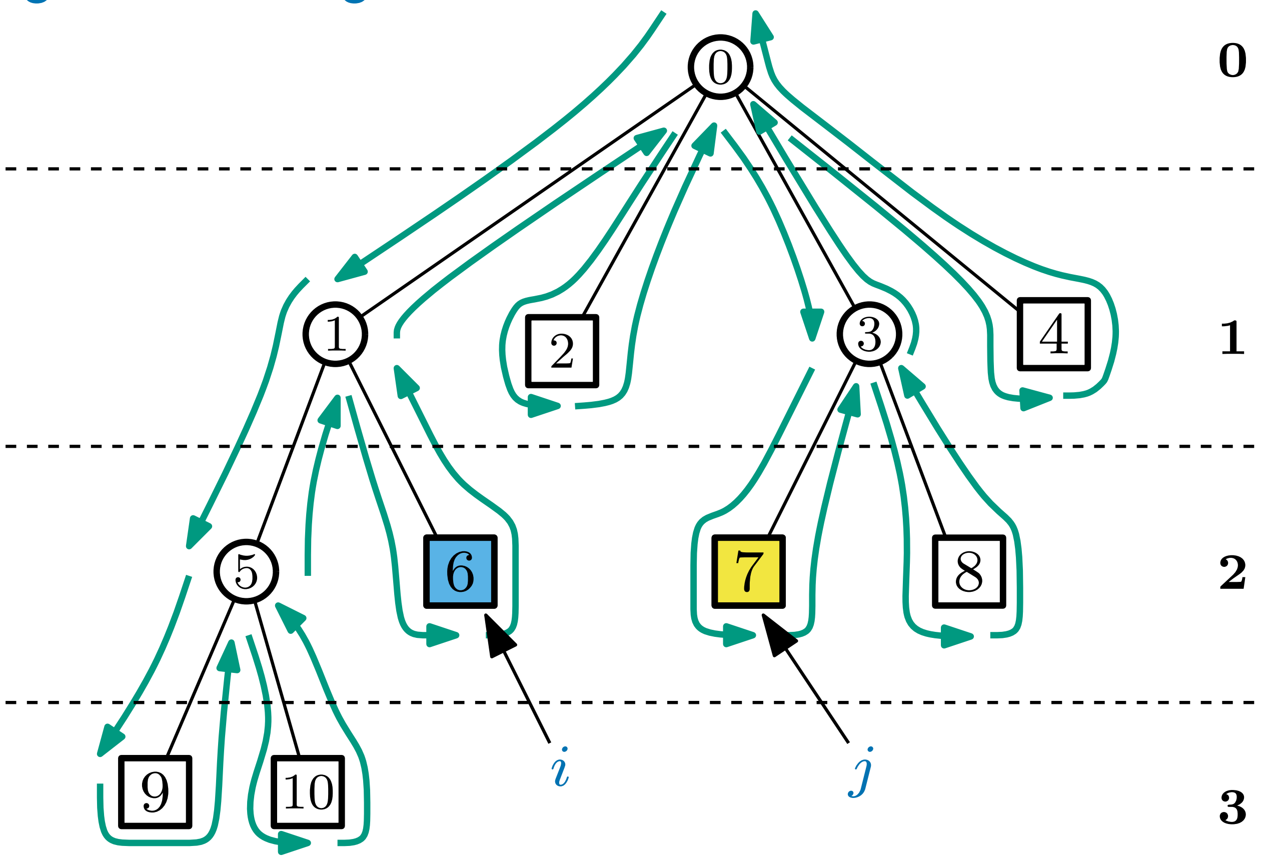

Find i and j in N

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

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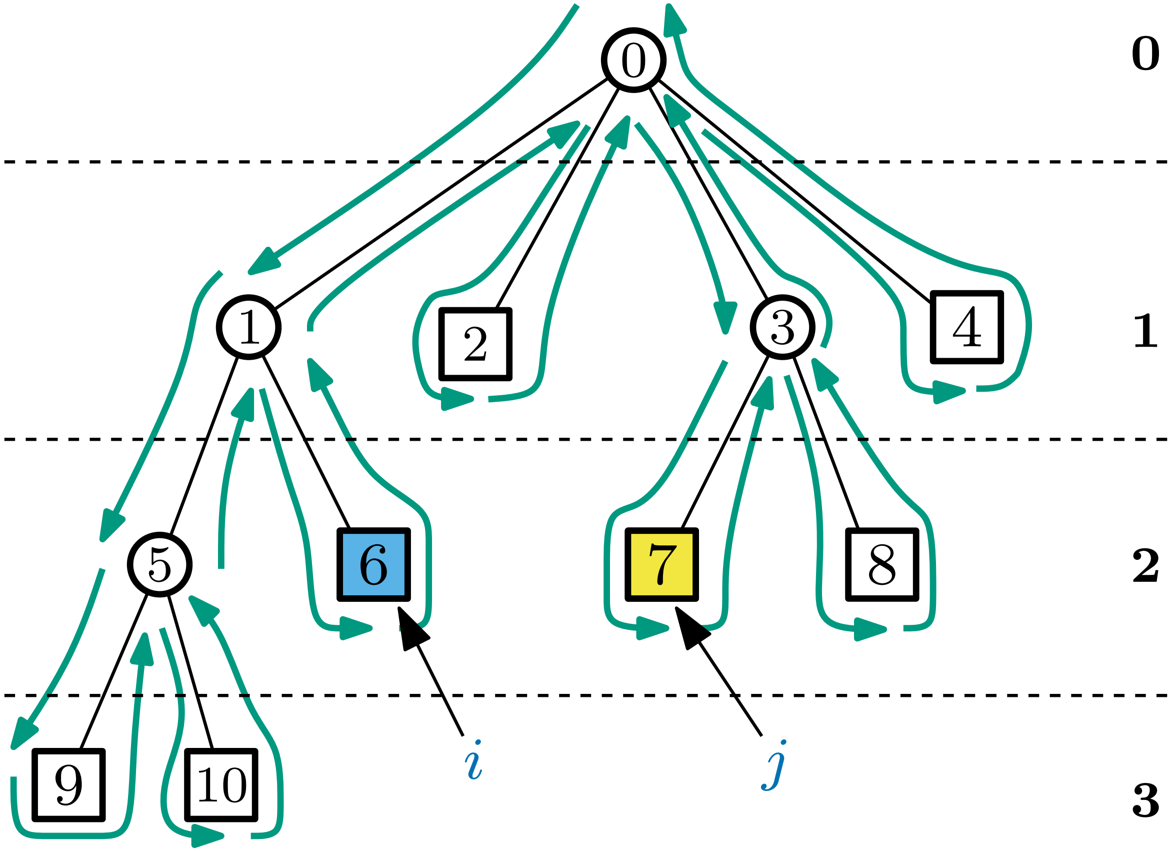

Find i and j in N

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

i' j'

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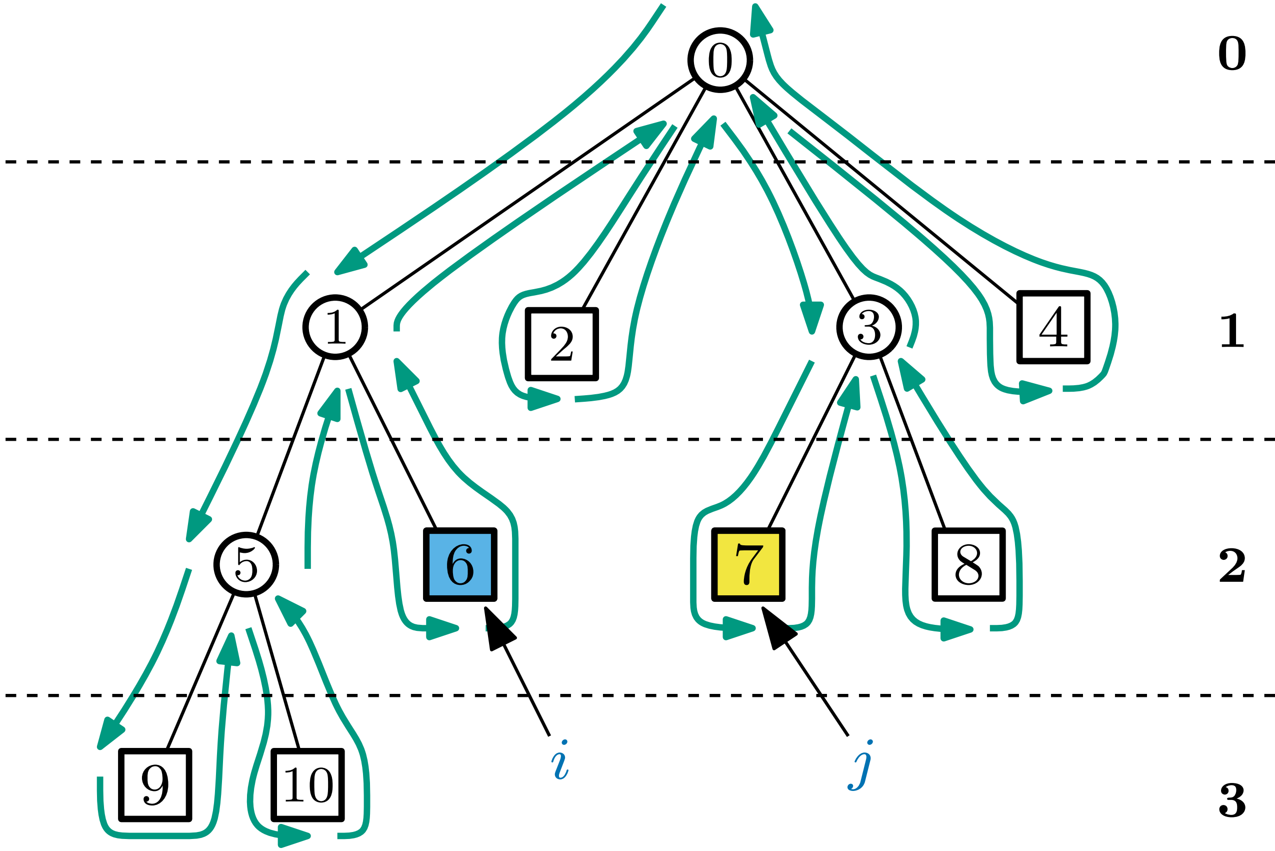

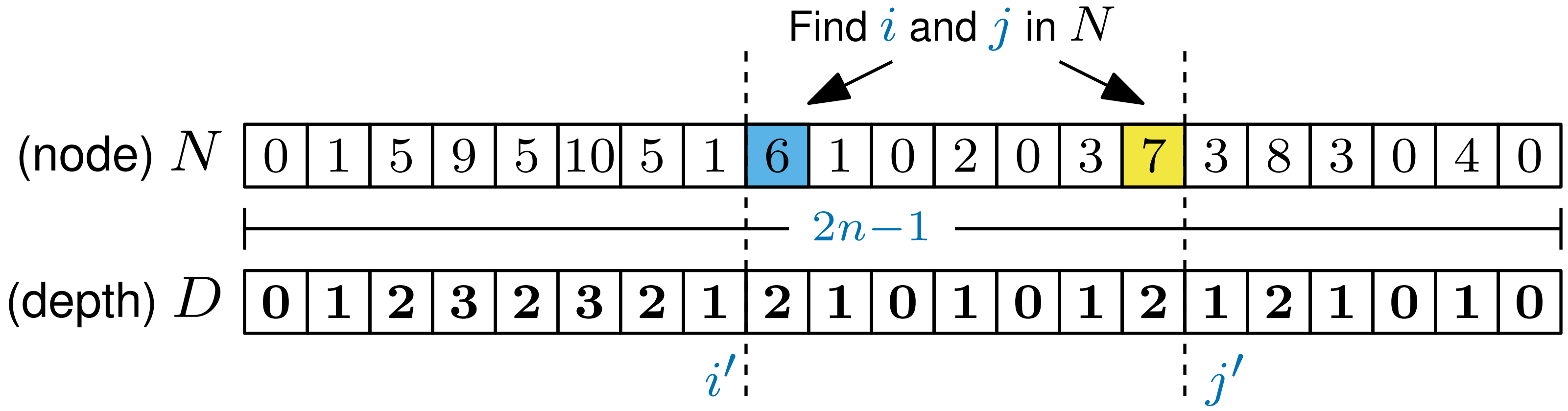

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

how do we find LCA(i,j)?

Compute RMQ(i', j') in D

Solving LCAs using RMQs

Compute an Euler tour of T...

(a depth first search with repeats)

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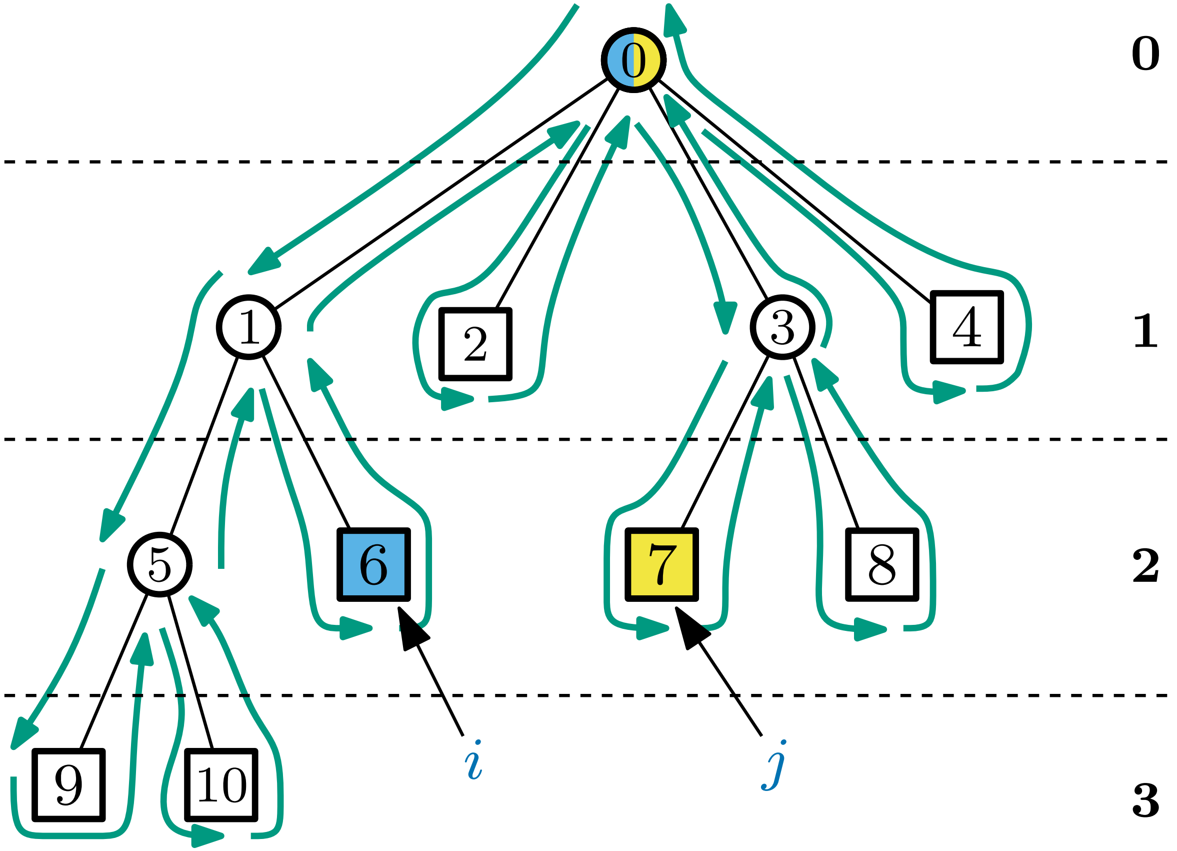
How long is the tour?

We follow each edge twice...

and there are $(n-1)$ edges



Find i and j in N

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

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Solving LCAs using RMQs

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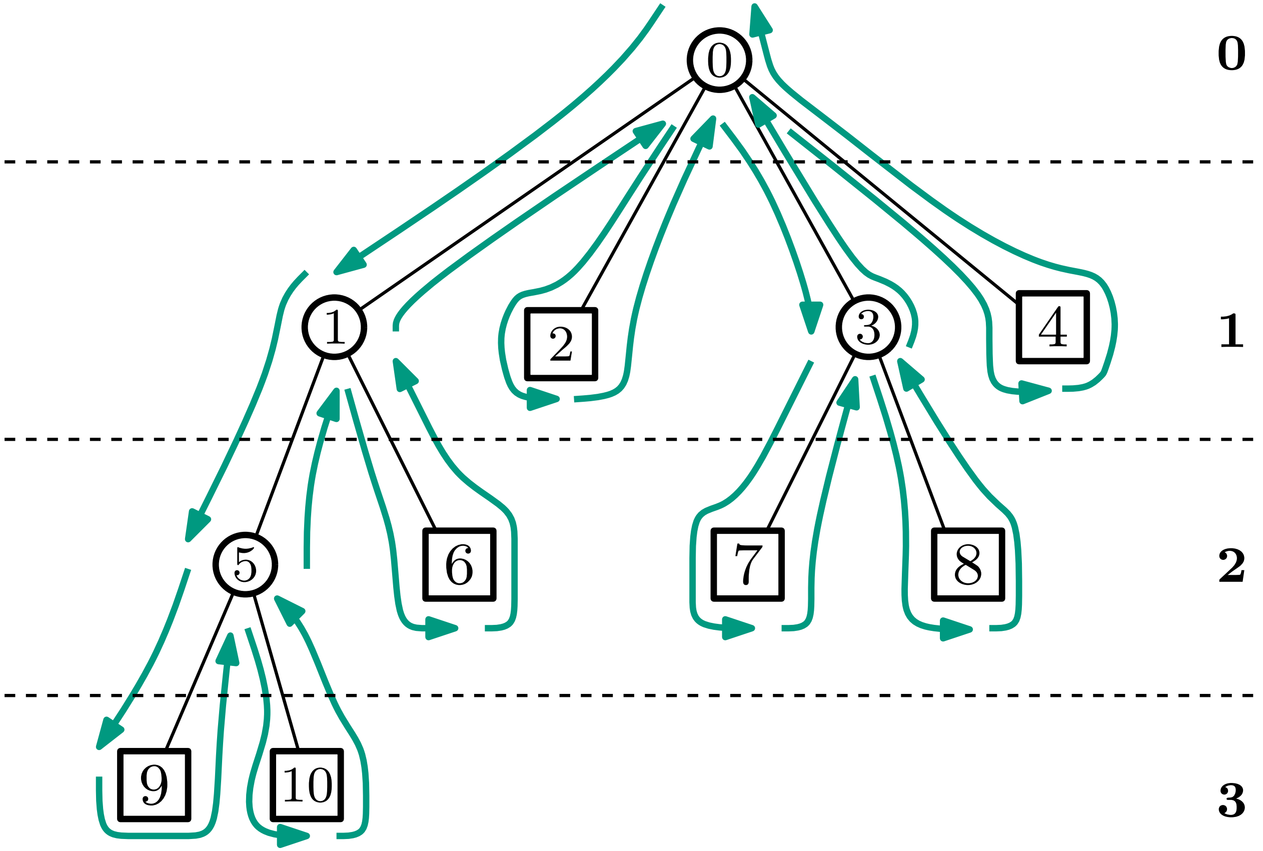


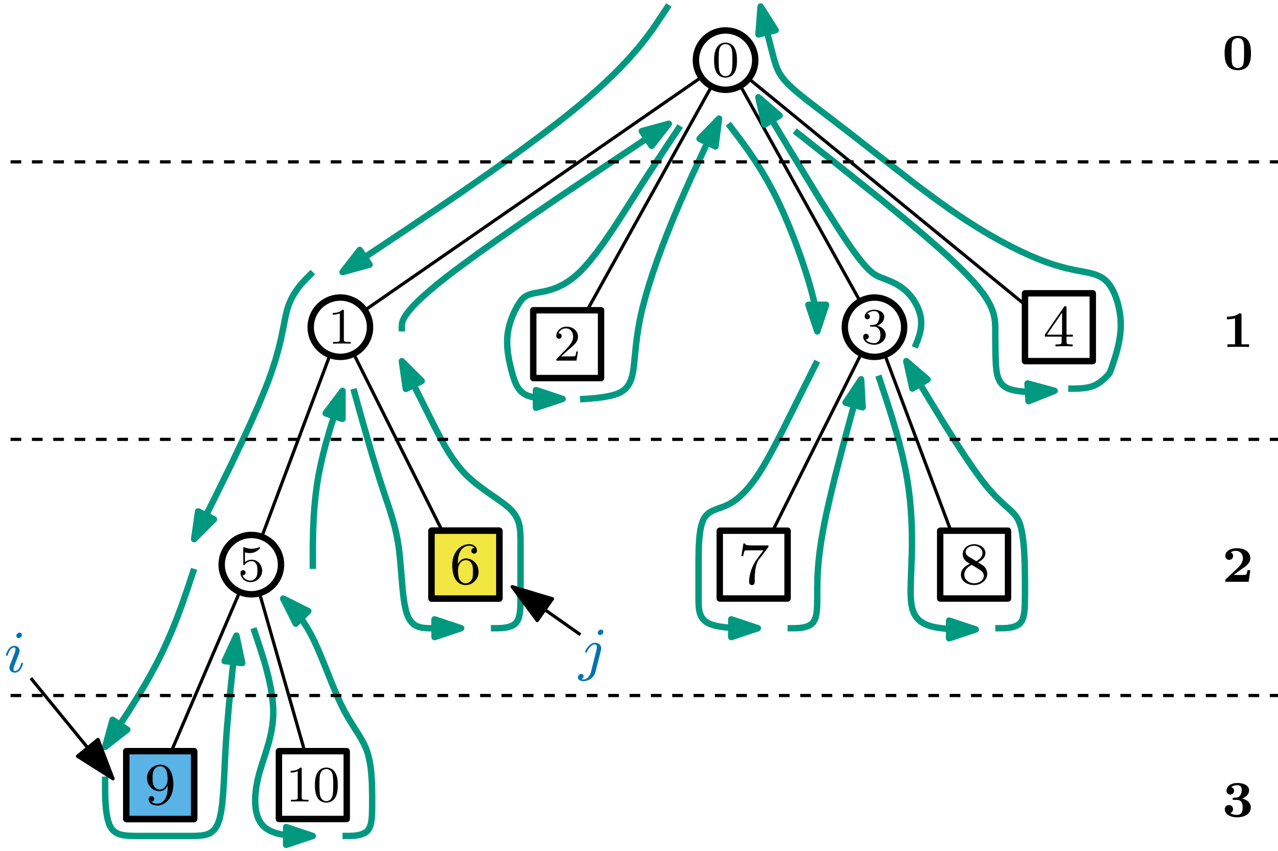
(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

how do we find LCA(i,j)?

Solving LCAs using RMQs



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(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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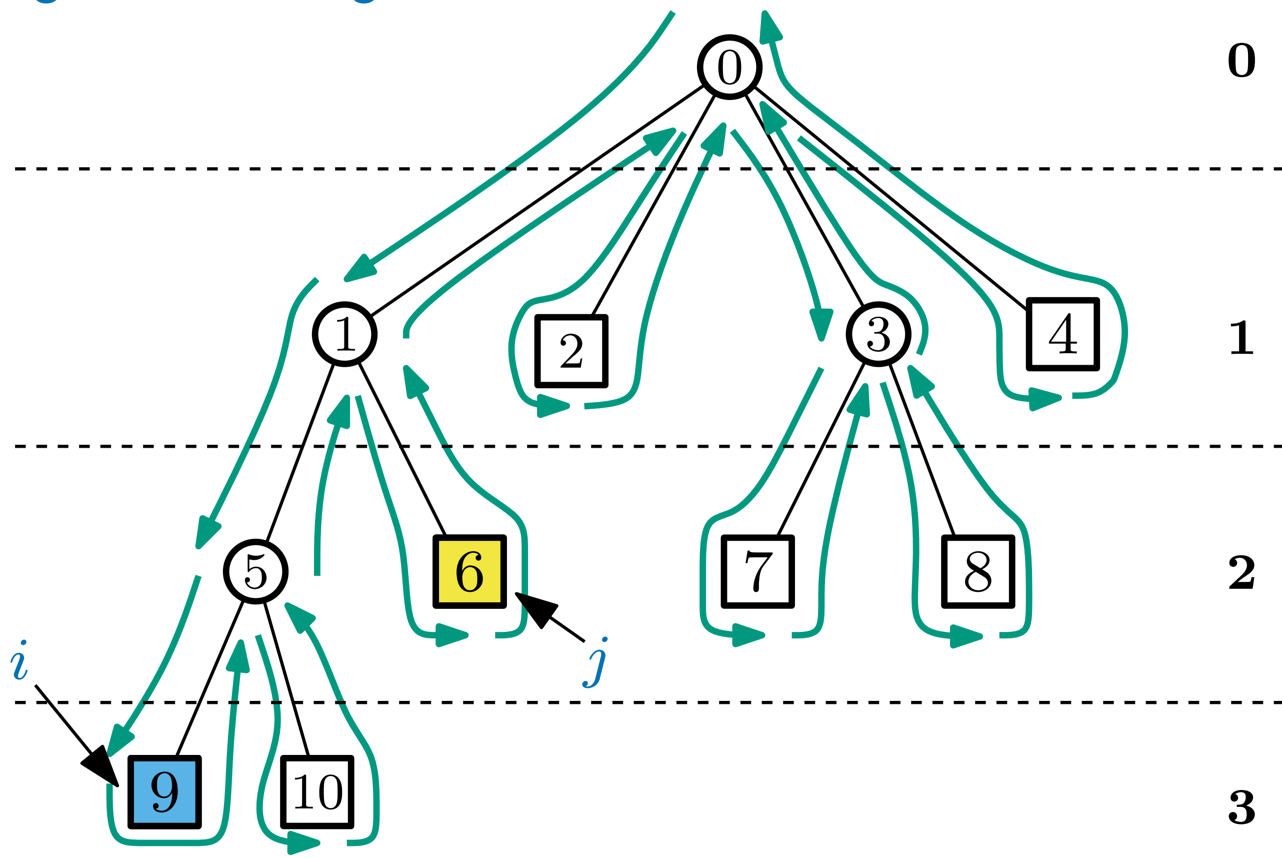

Find i and j in N

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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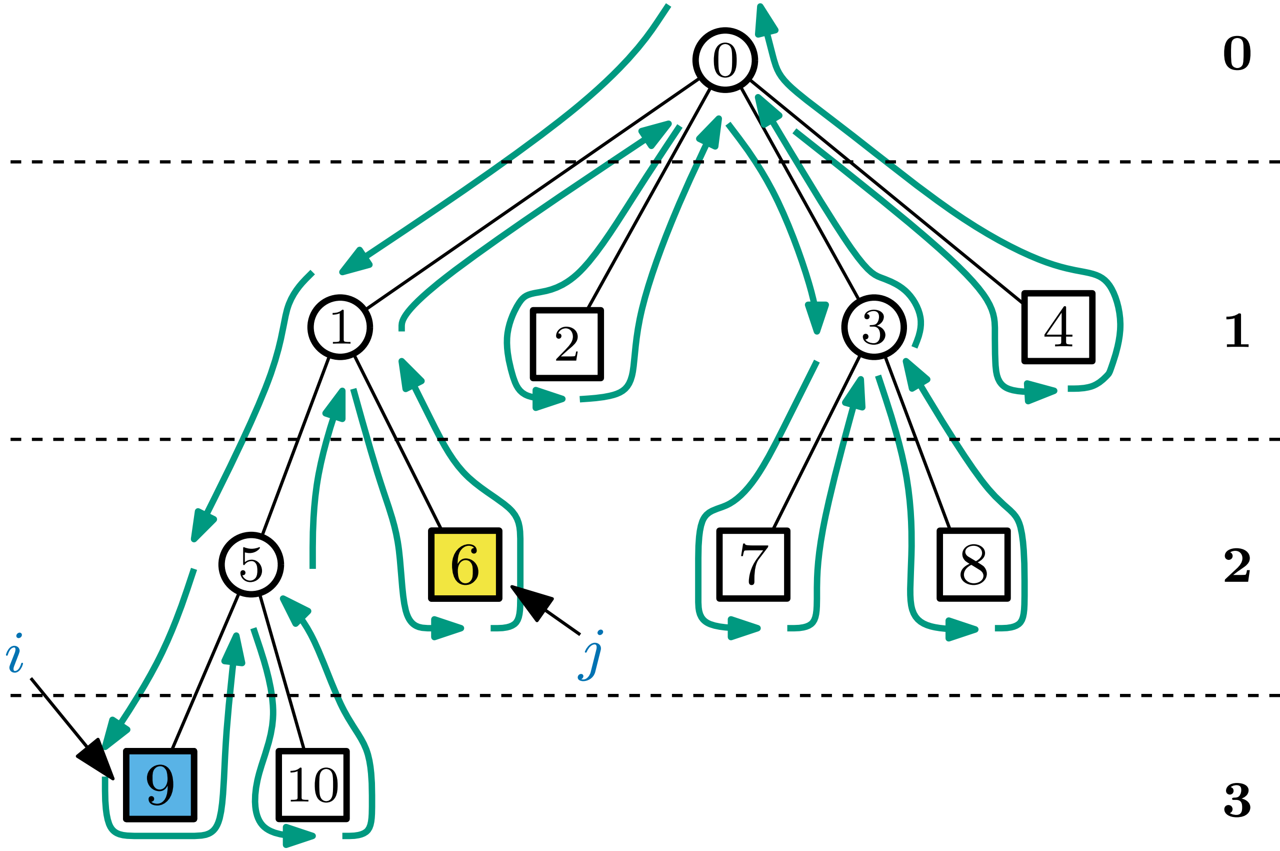

Find i and j in N

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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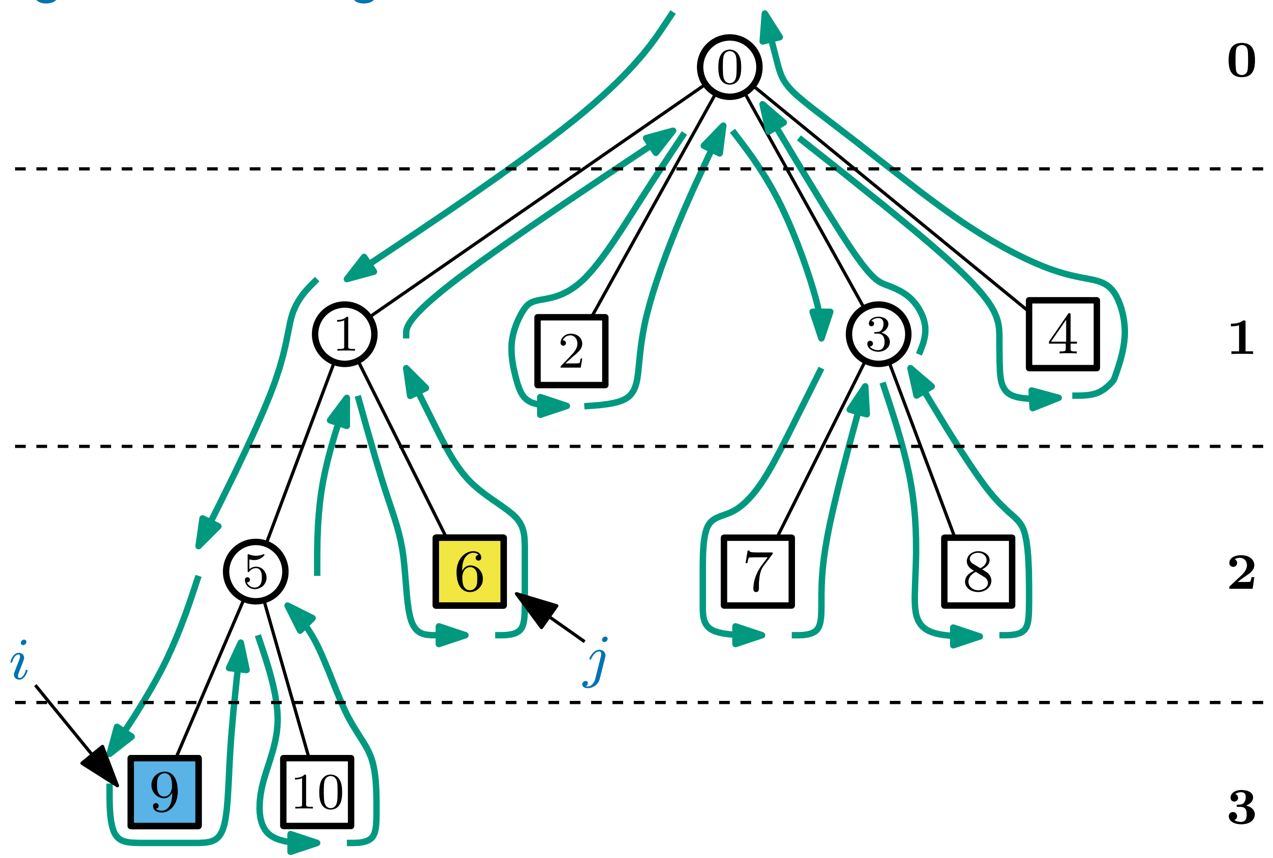

Find i and j in N

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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Compute RMQ(i', j') in D

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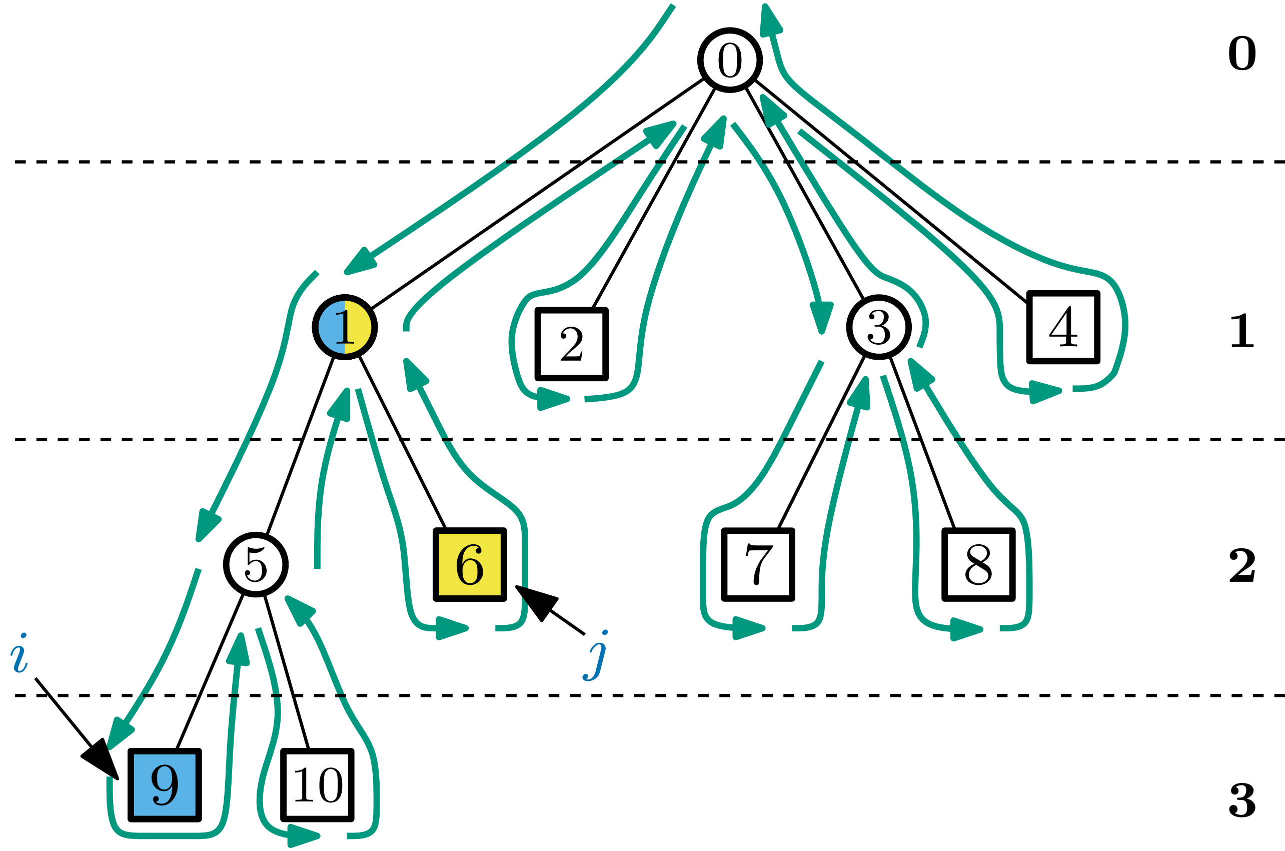

Find i and j in N

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
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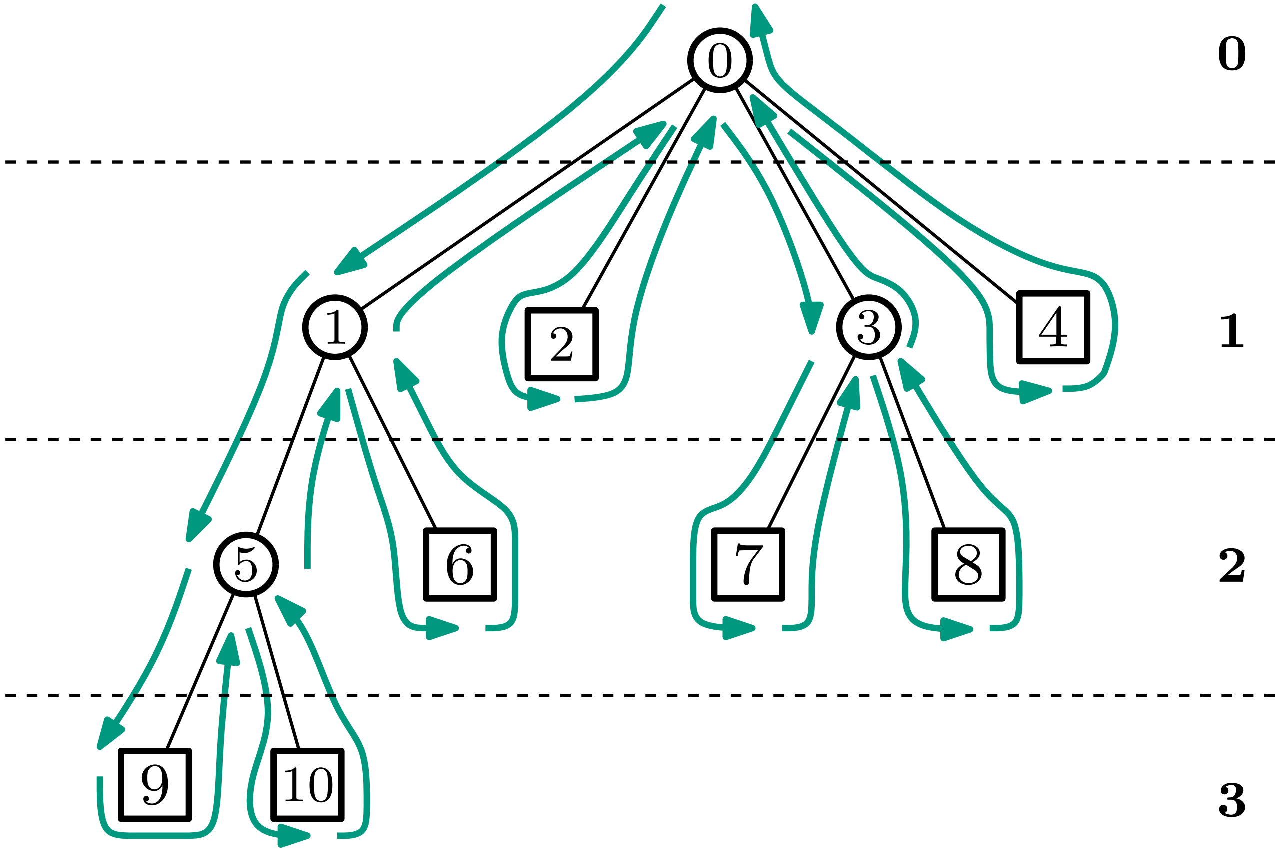

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$2n-1$

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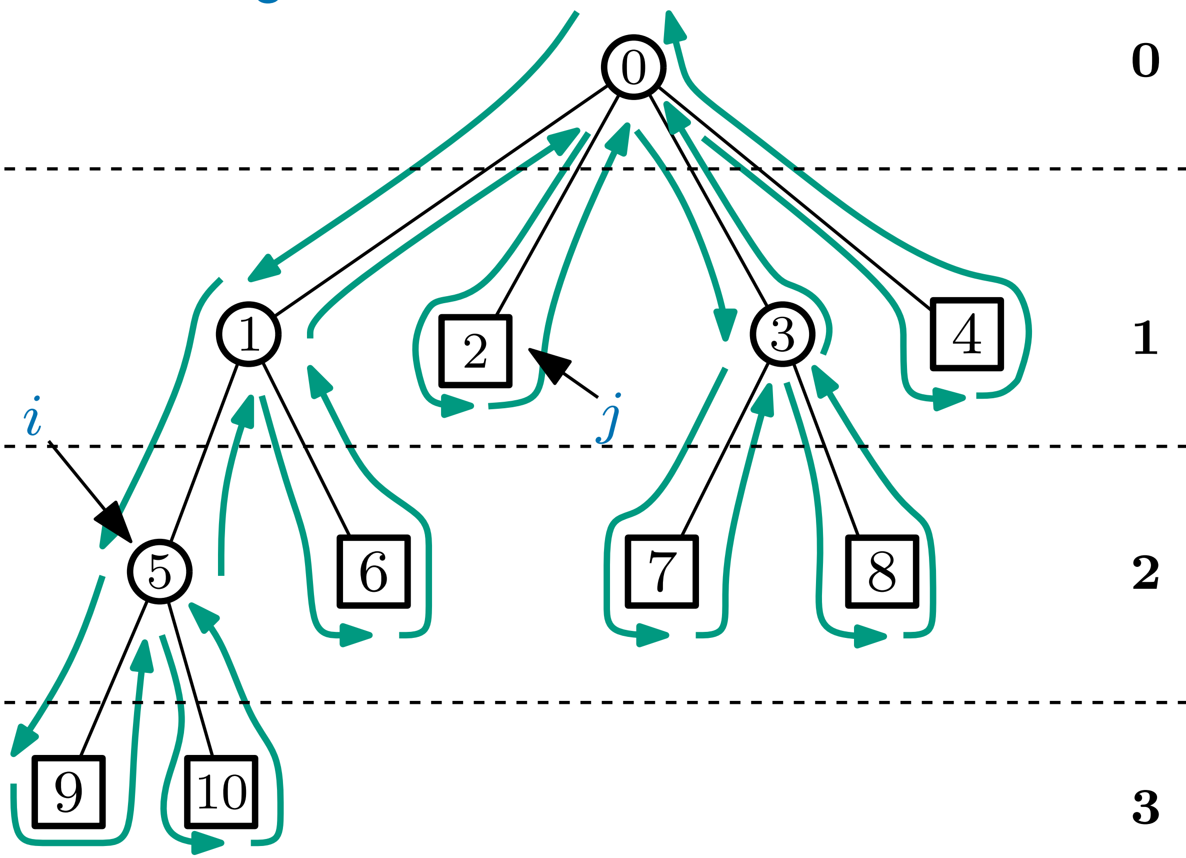

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$2n-1$

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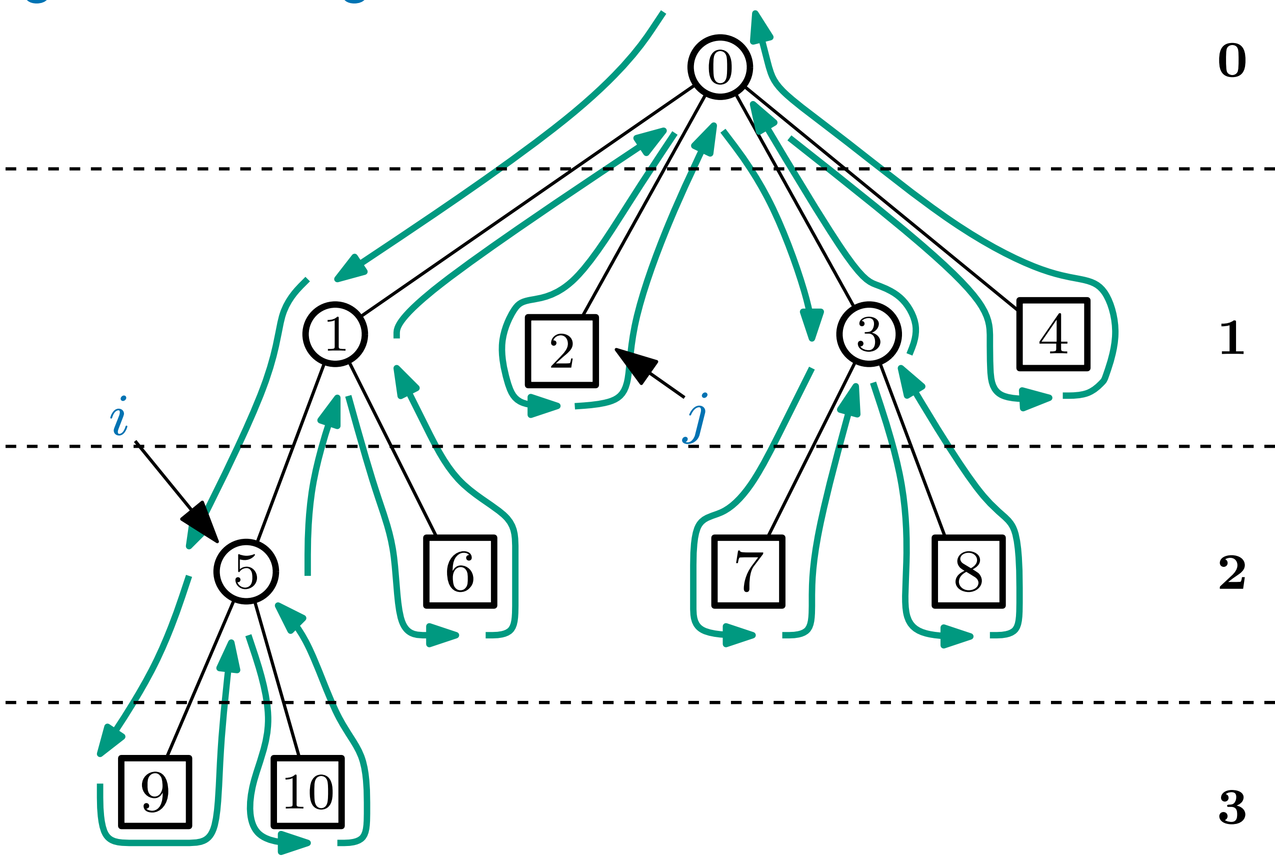

Find i and j in N...

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

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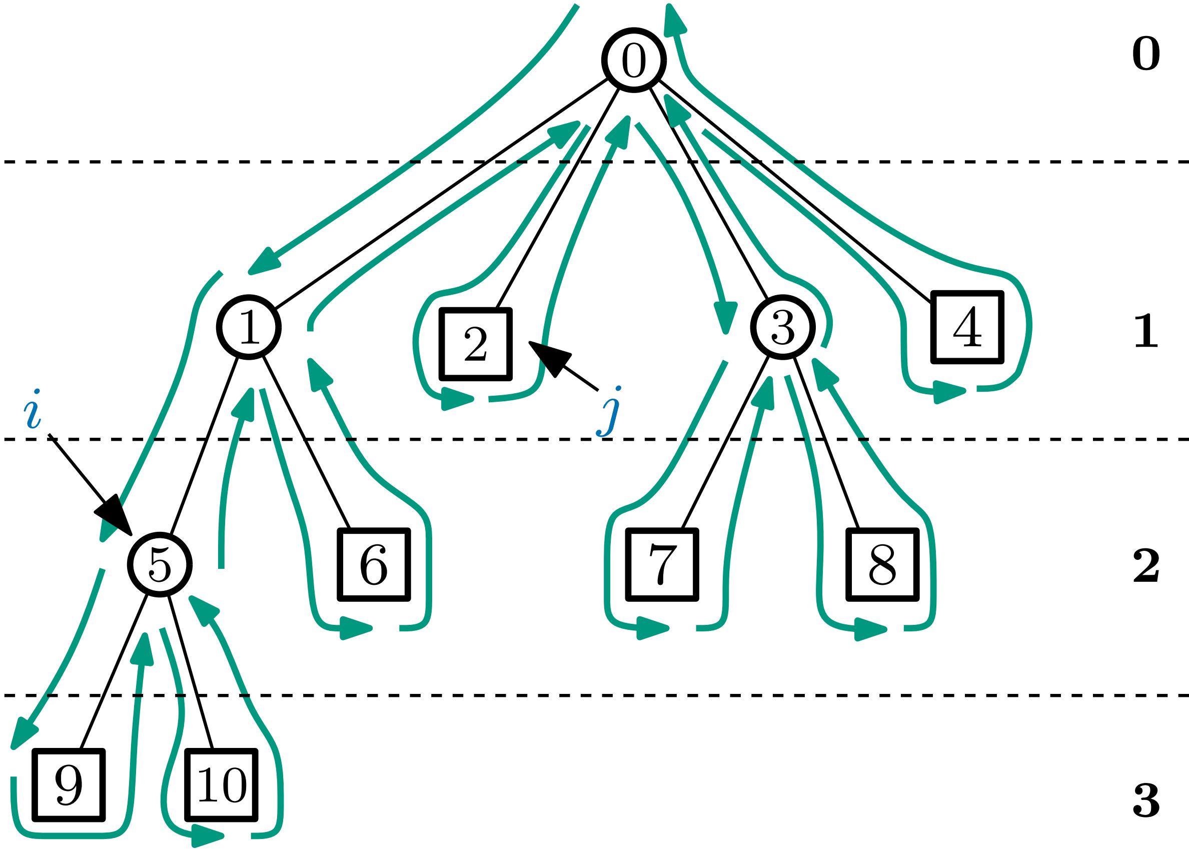

Find i and j in N... which copy of i?

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

how do we find LCA(i,j)?

Solving LCAs using RMQs

Compute an Euler tour of T...

(a depth first search with repeats)

Write down every node you visit

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How long is the tour?

We follow each edge twice...

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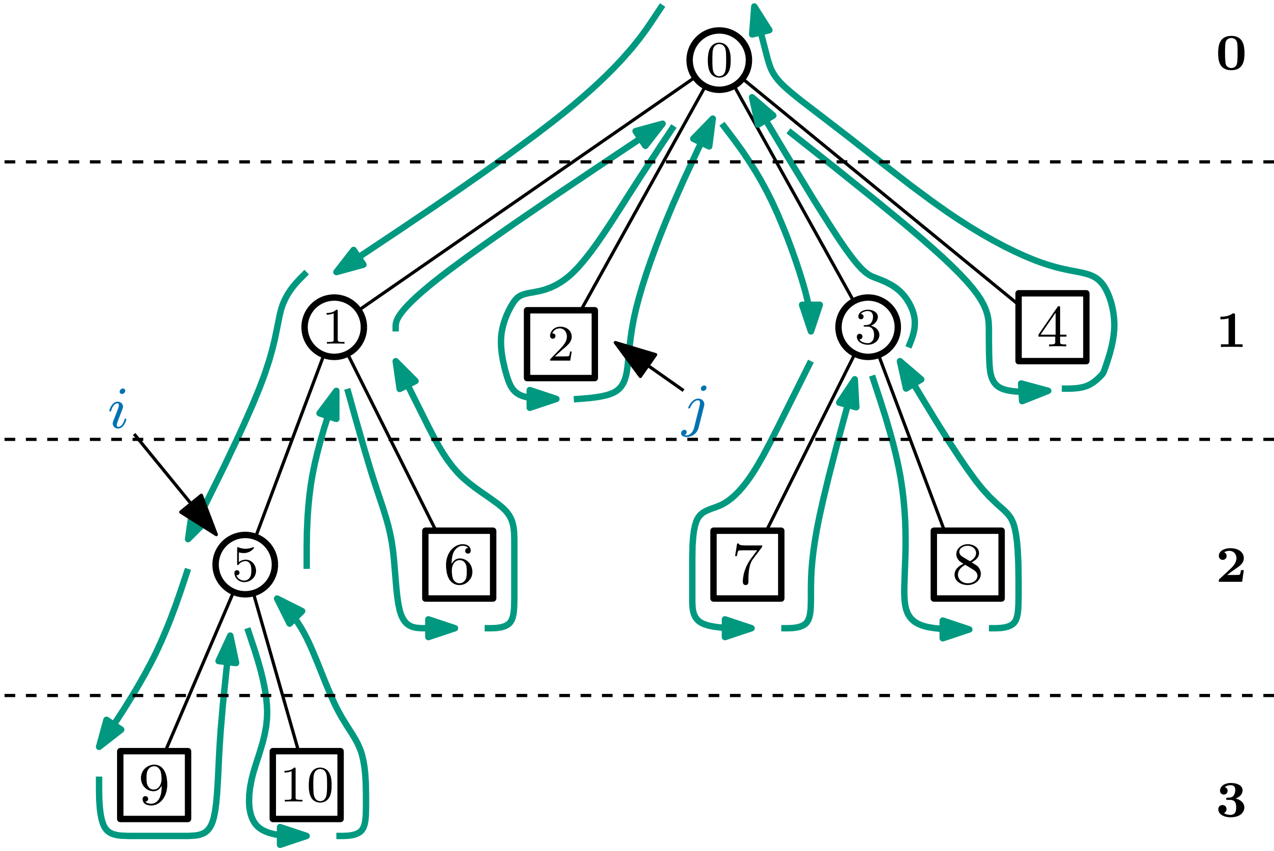


Find i and j in N... which copy of i?

any copy is fine

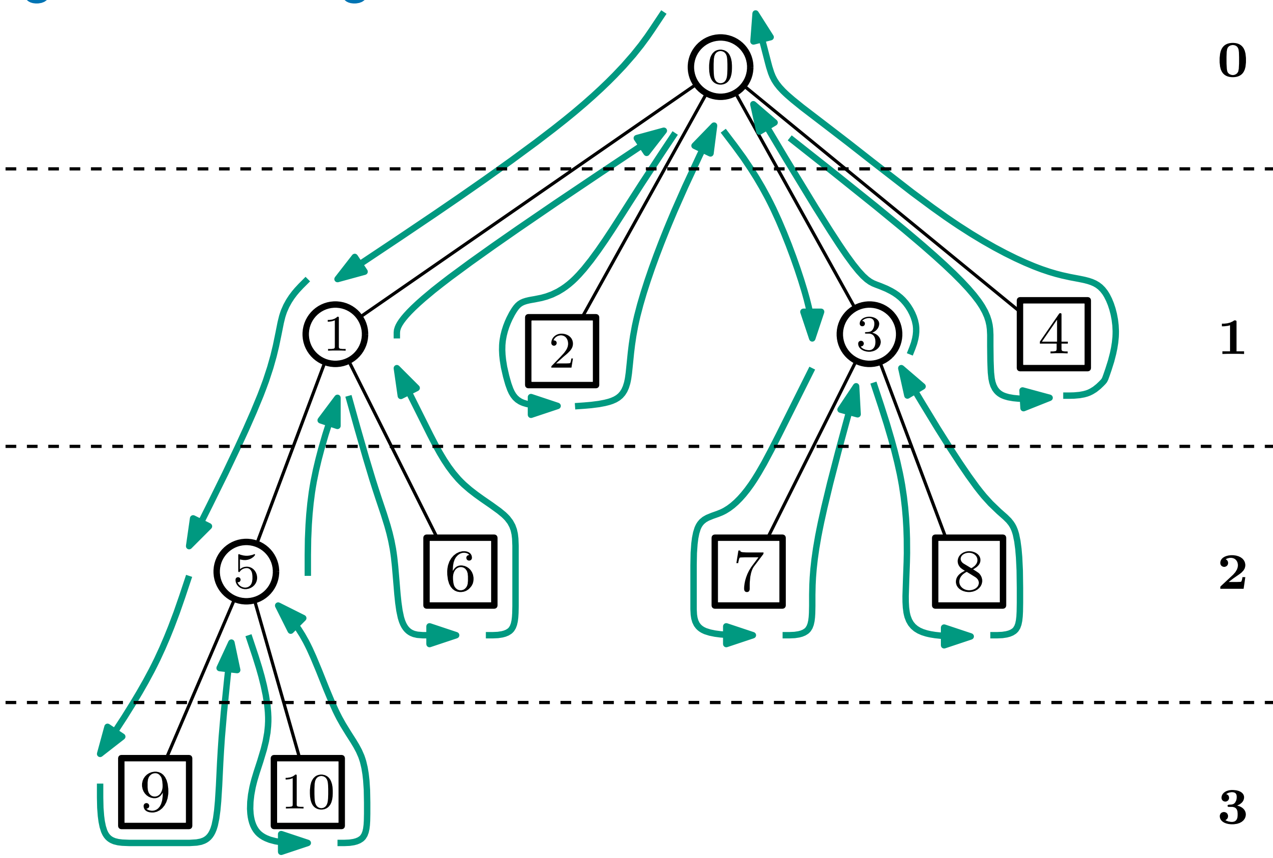
(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

how do we find LCA(i,j)?

Solving LCAs using RMQs



(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

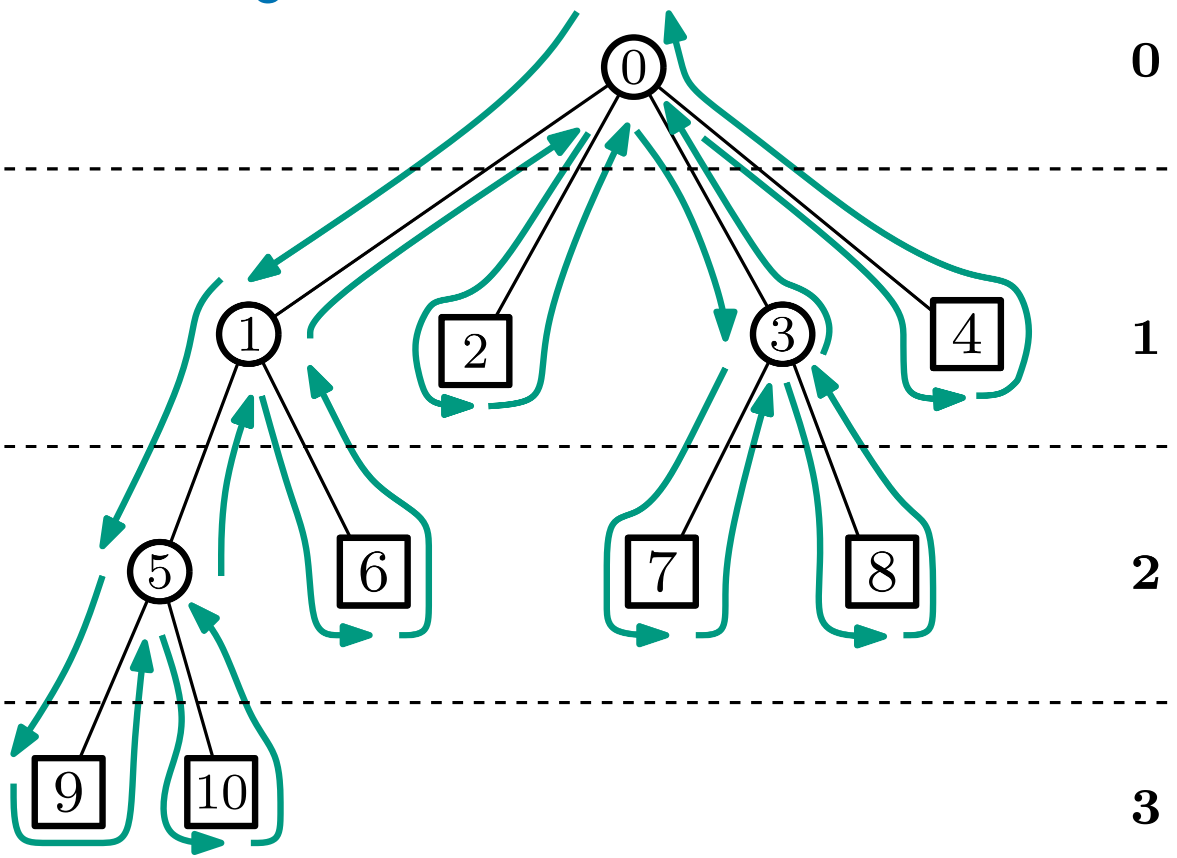
$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

Solving LCAs using RMQs

Preprocessing Summary

1. Construct N and D from T
2. Add a pointer from each node i to some $N[i'] = i$
3. Preprocess D for RMQs



(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

Solving LCAs using RMQs

Preprocessing Summary

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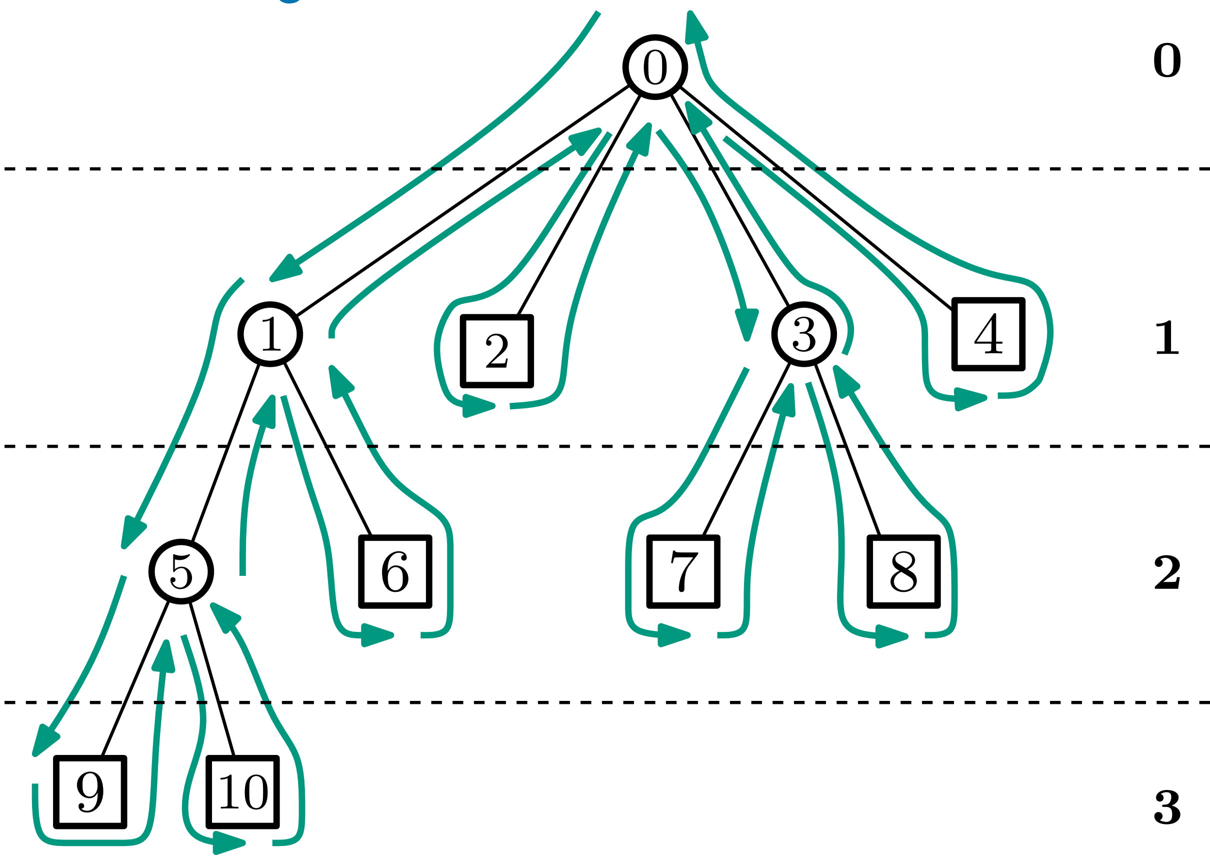
Query Summary - LCA(i,j)

1. Find (any) i' st. $N[i'] = i$
2. Find (any) j' st. $N[j'] = j$
3. Compute RMQ(i', j') in D
4. LCA$(i, j) = N[$RMQ$(i', j')]$



(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

2. Add a pointer from each node i to some $N[i'] = i$

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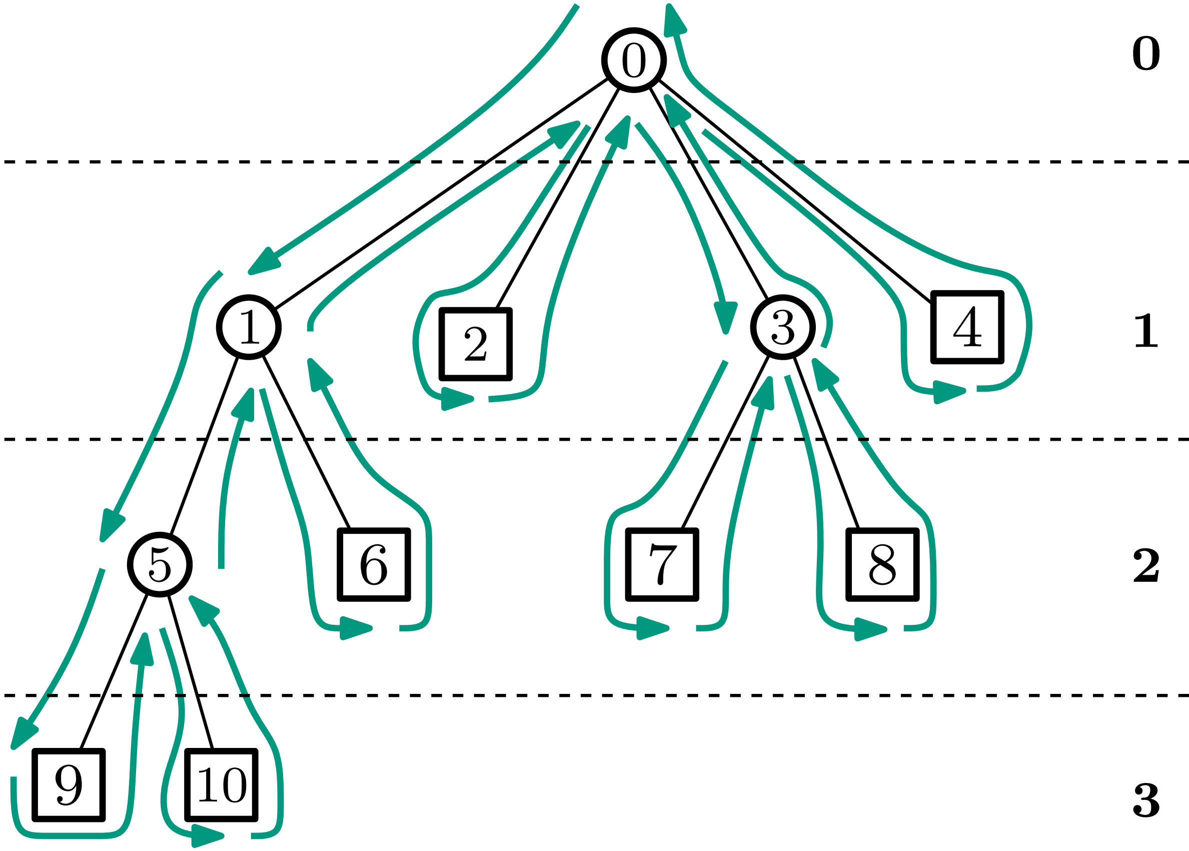

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

$O(n)$ 2. Add a pointer from each node i to some $N[i'] = i$

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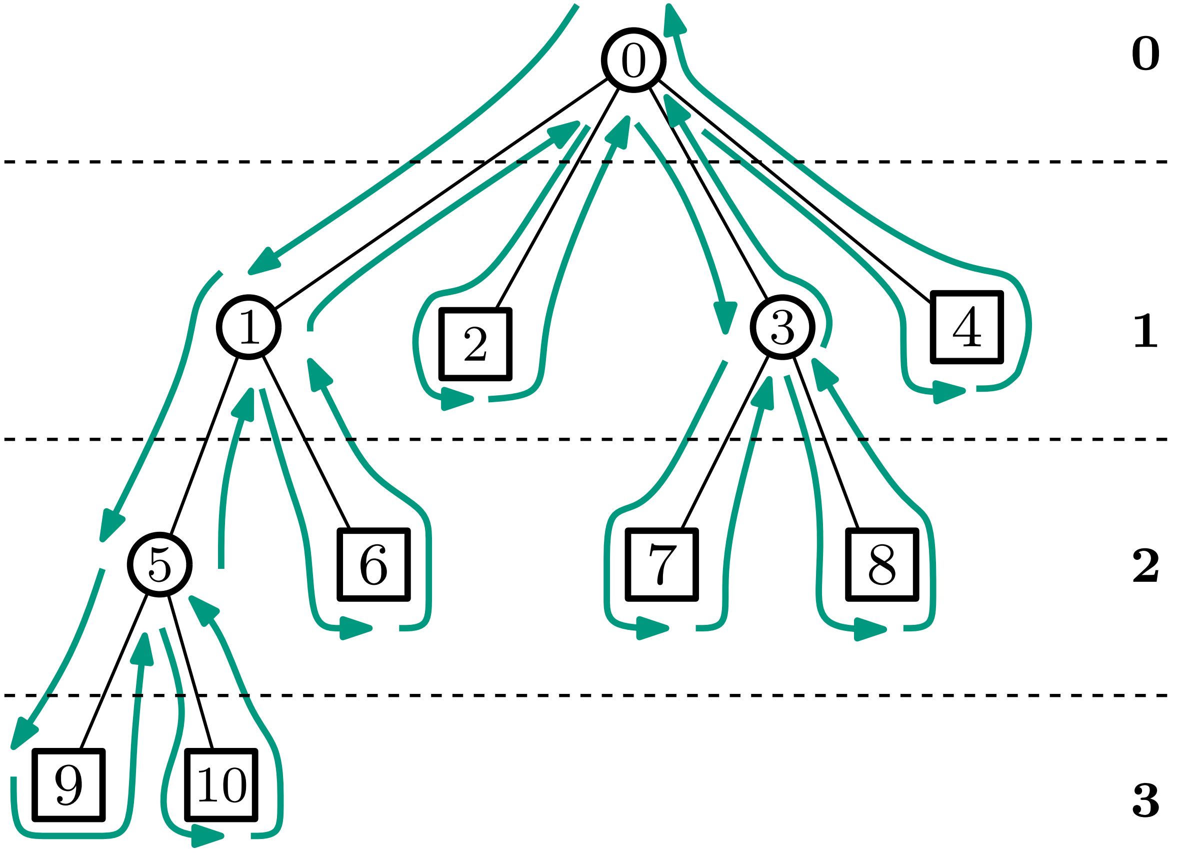

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

$O(n)$ 2. Add a pointer from each node i to some $N[i'] = i$

$O(?)$ 3. Preprocess D for RMQs

Query Summary - LCA(i,j)

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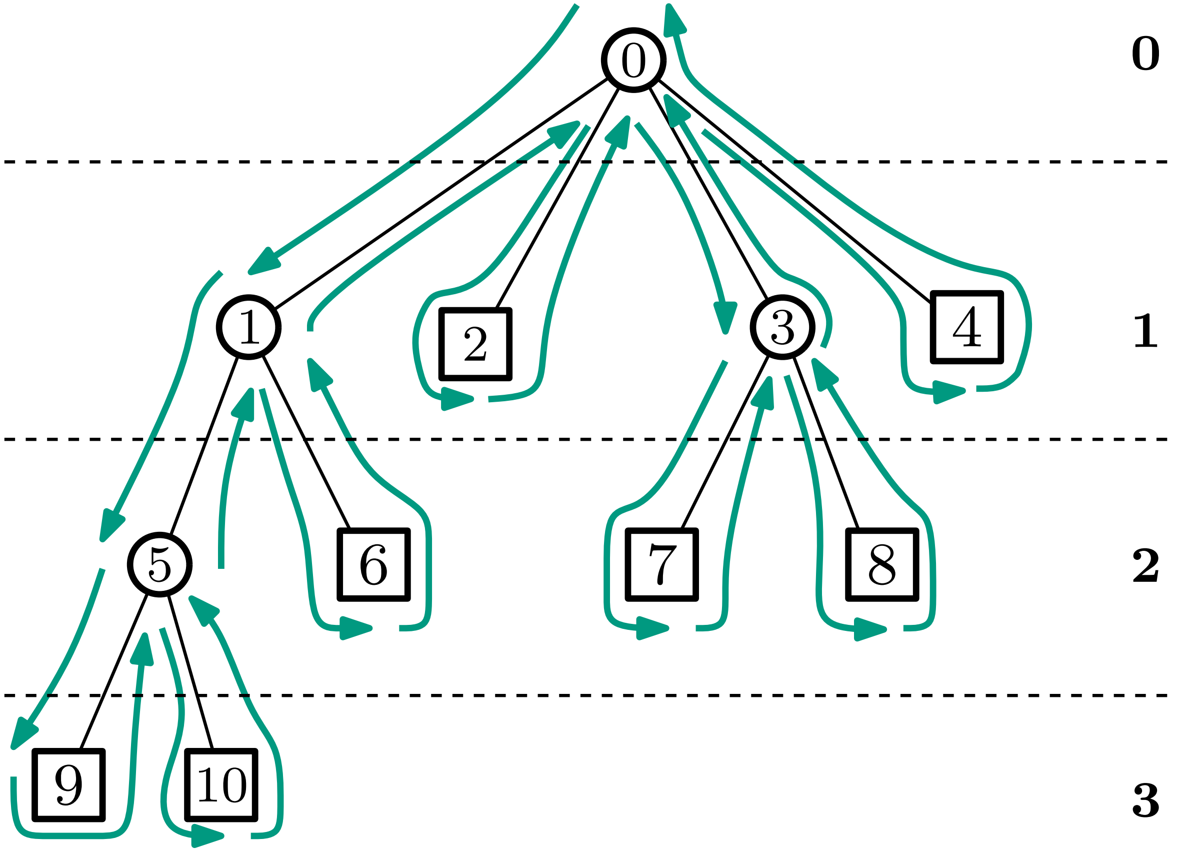

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

Solving LCAs using RMQs

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$O(n)$ 1. Construct N and D from T

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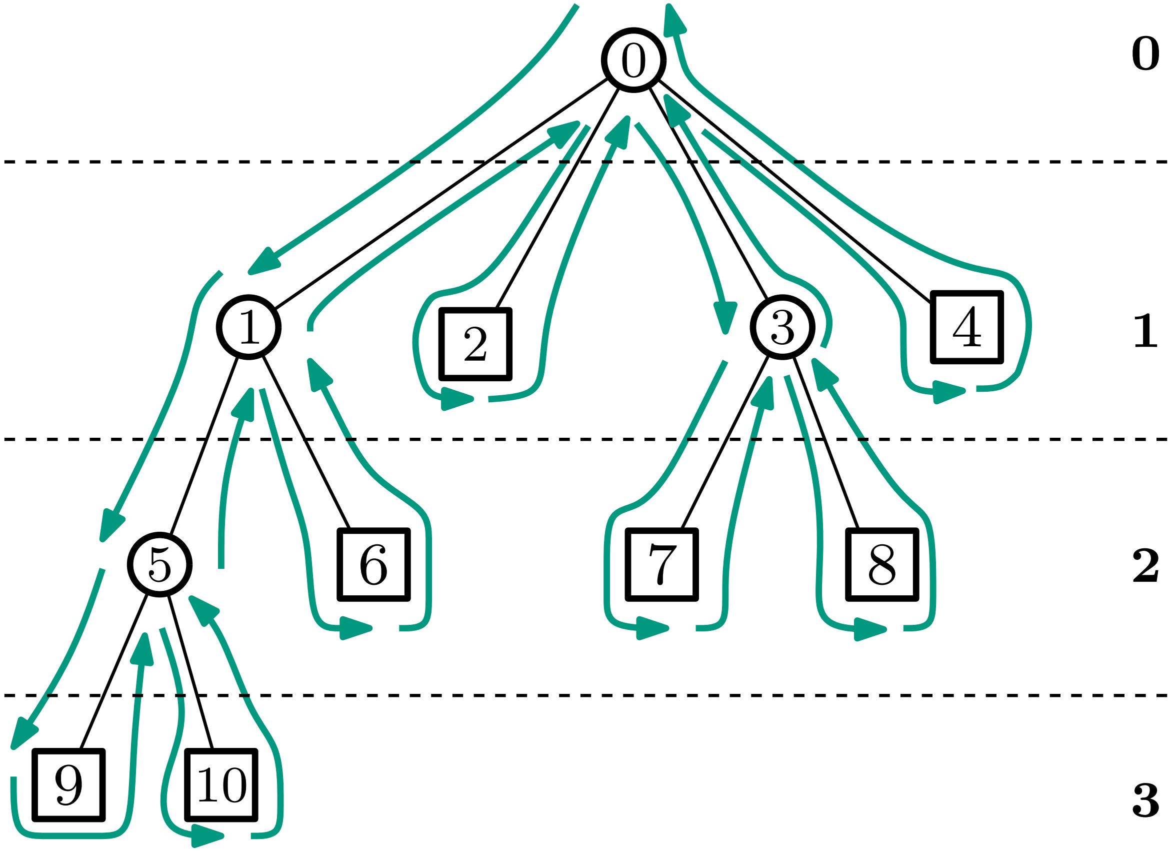

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Solving LCAs using RMQs

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$O(n)$ 1. Construct N and D from T

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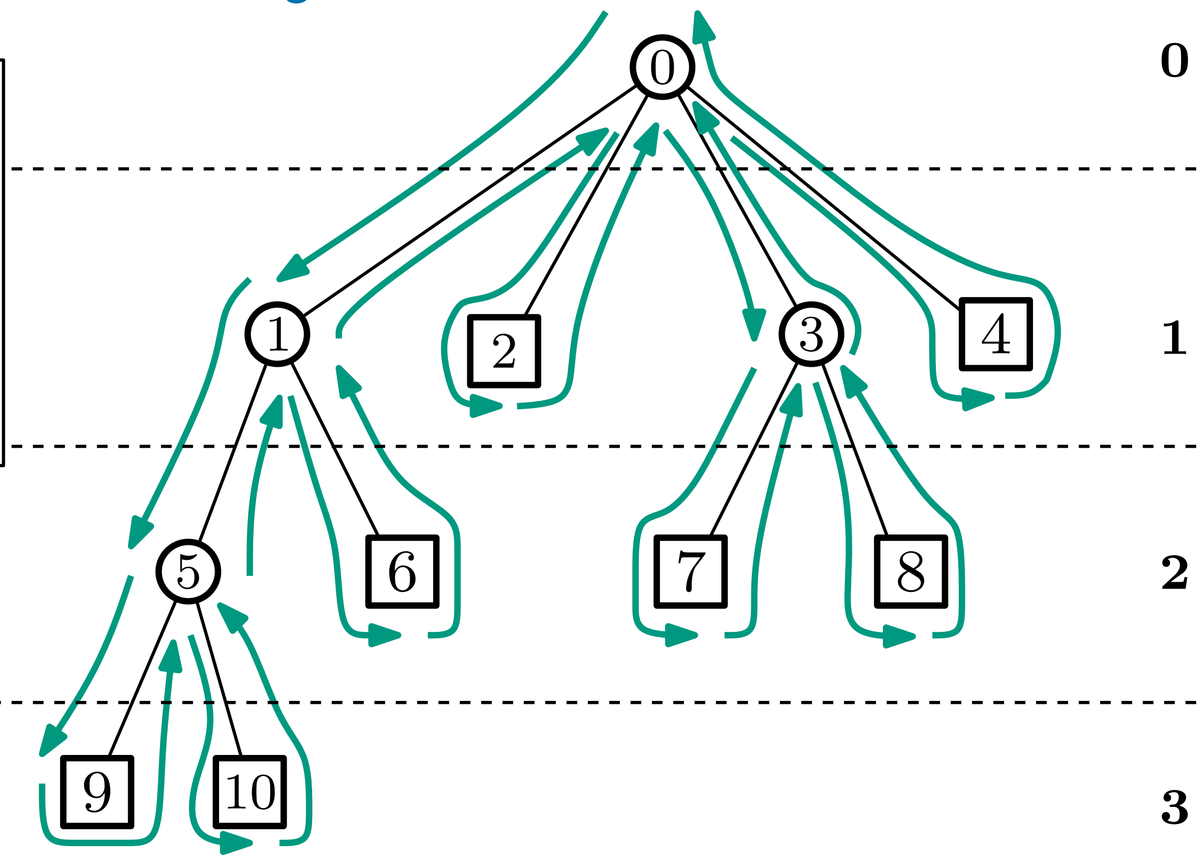

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

$O(n)$ 2. Add a pointer from each node i to some $N[i'] = i$

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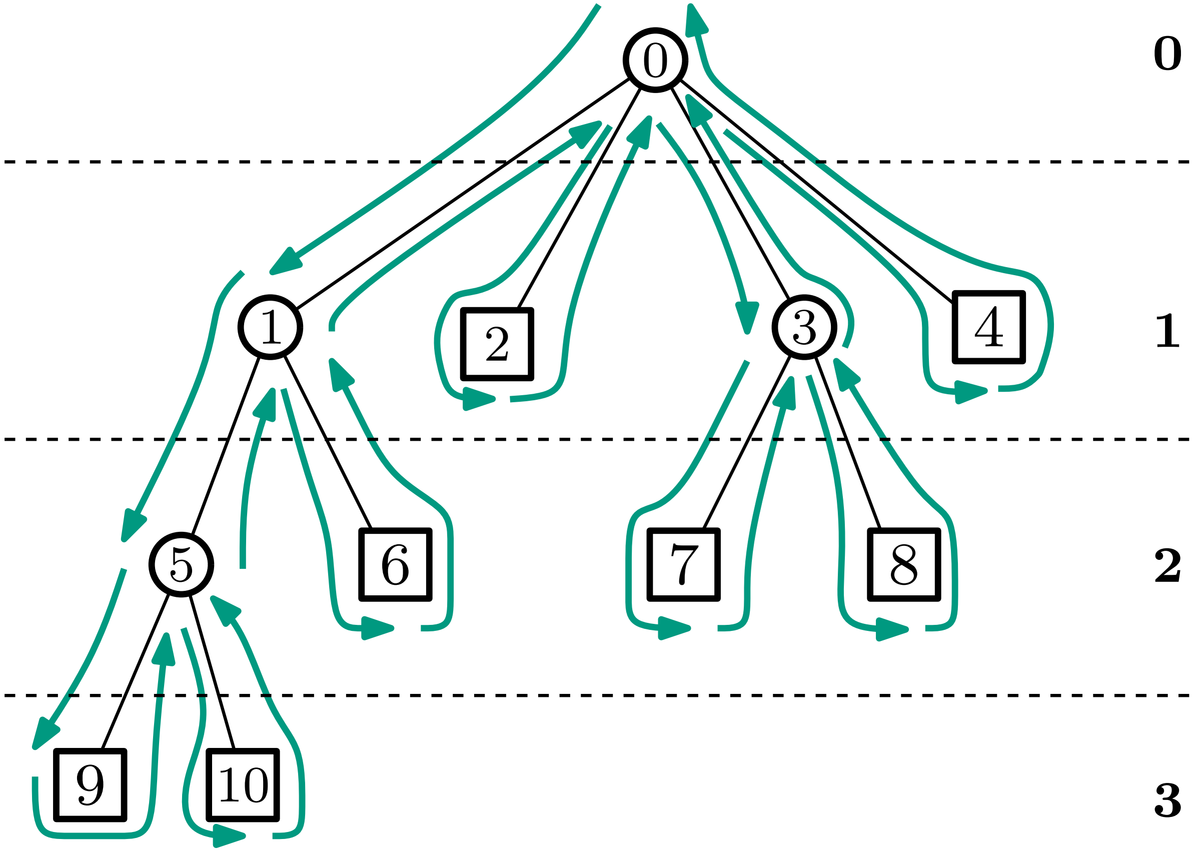

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

$O(n)$ 2. Add a pointer from each node i to some $N[i'] = i$

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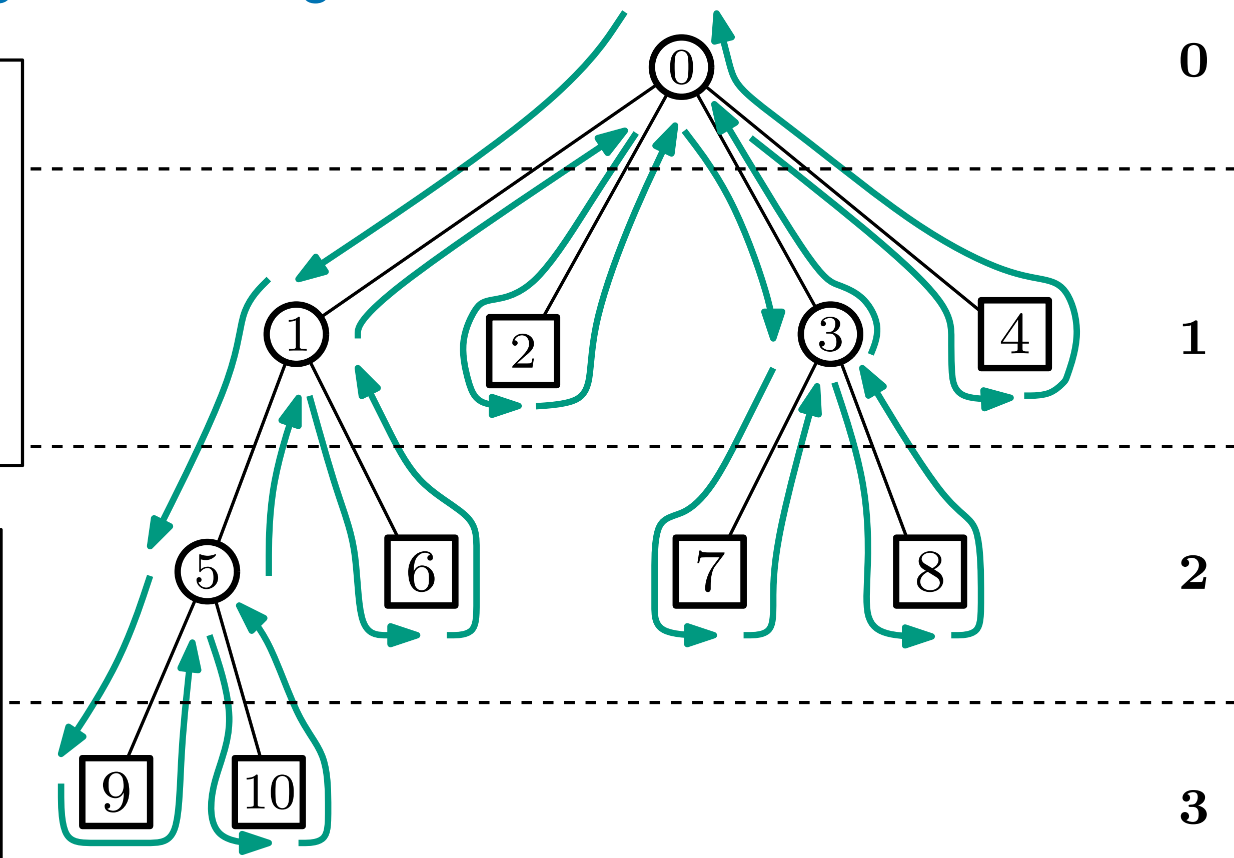

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Solving LCAs using RMQs

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$O(n)$ 1. Construct N and D from T

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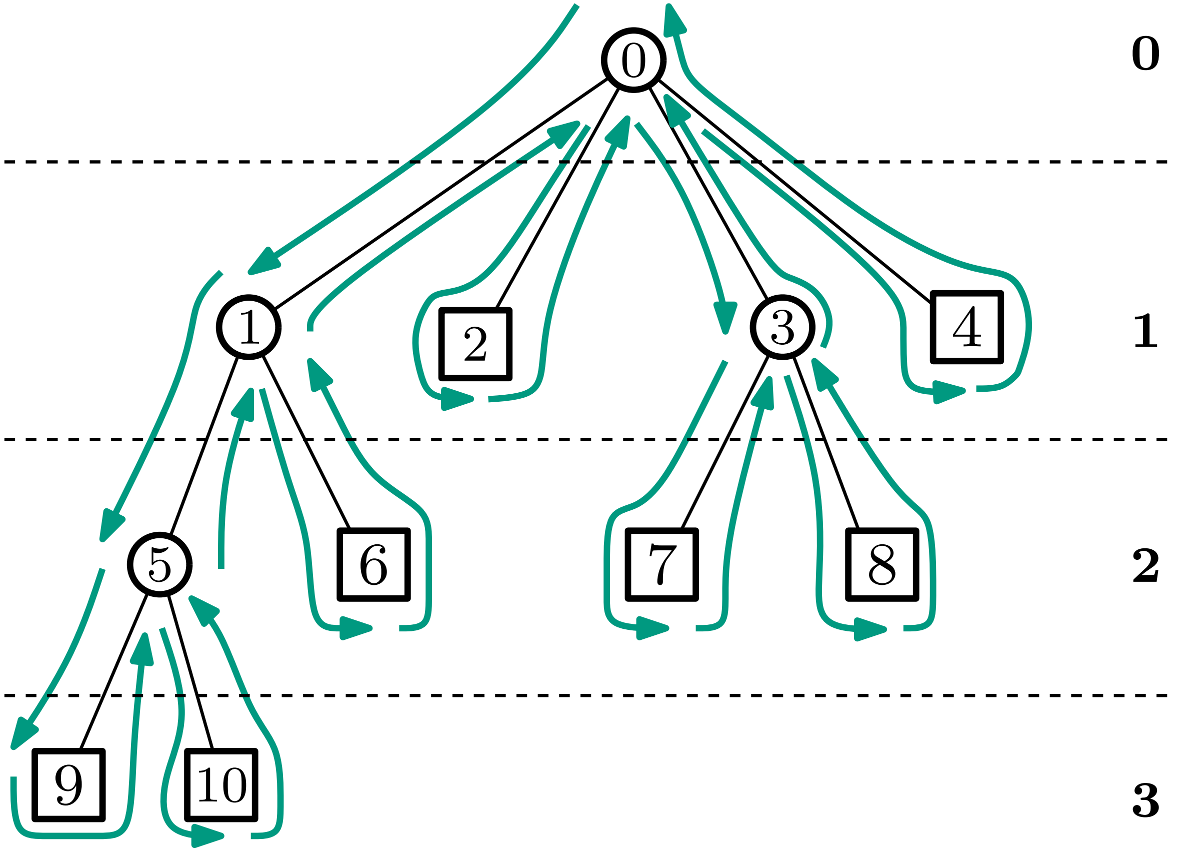

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Prep. time $O(n + \text{prepRMQ}(n))$

Space $O(n + \text{spaceRMQ}(n))$

Query time $O(1 + \text{queryRMQ}(n))$

depends on the RMQ structure used

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

$O(n)$ 2. Add a pointer from each node i to some $N[i'] = i$

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$O(?)$ 3. Compute RMQ(i', j') in D

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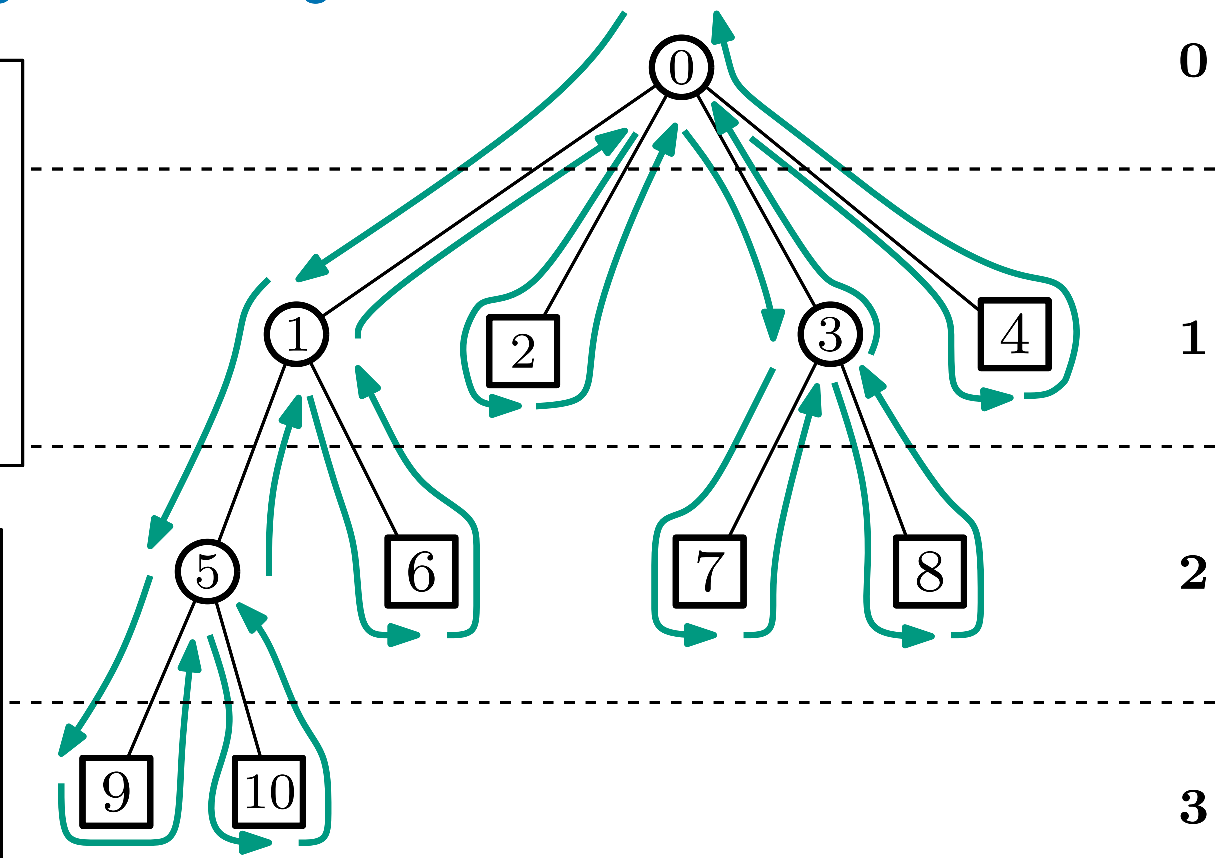

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$\underbrace{\qquad\qquad}_{2n-1}$

Prep. time $O(n \log\log n)$ Space $O(n \log\log n)$

Query time $O(1)$

using the best result from last lecture

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

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$O(?)$ 3. Compute RMQ(i', j') in D

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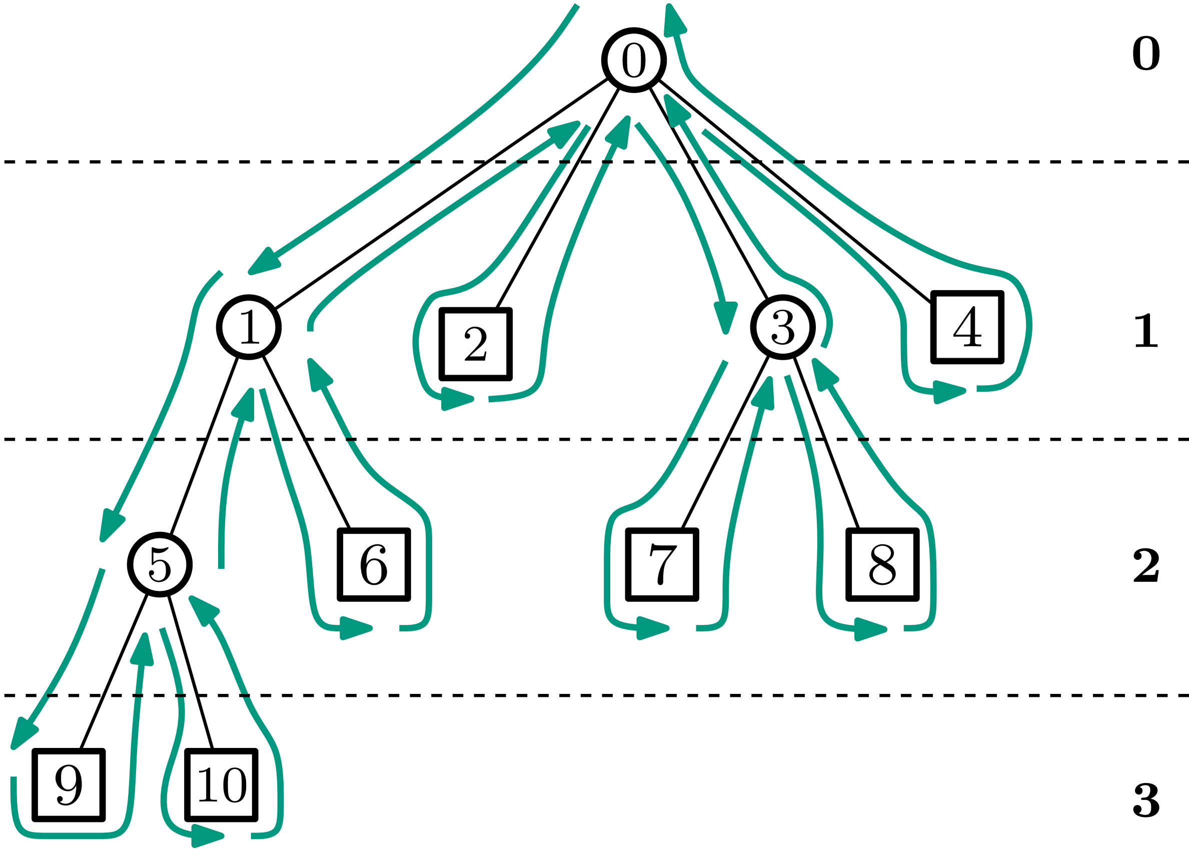


why does this work?

(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Prep. time $O(n \log\log n)$

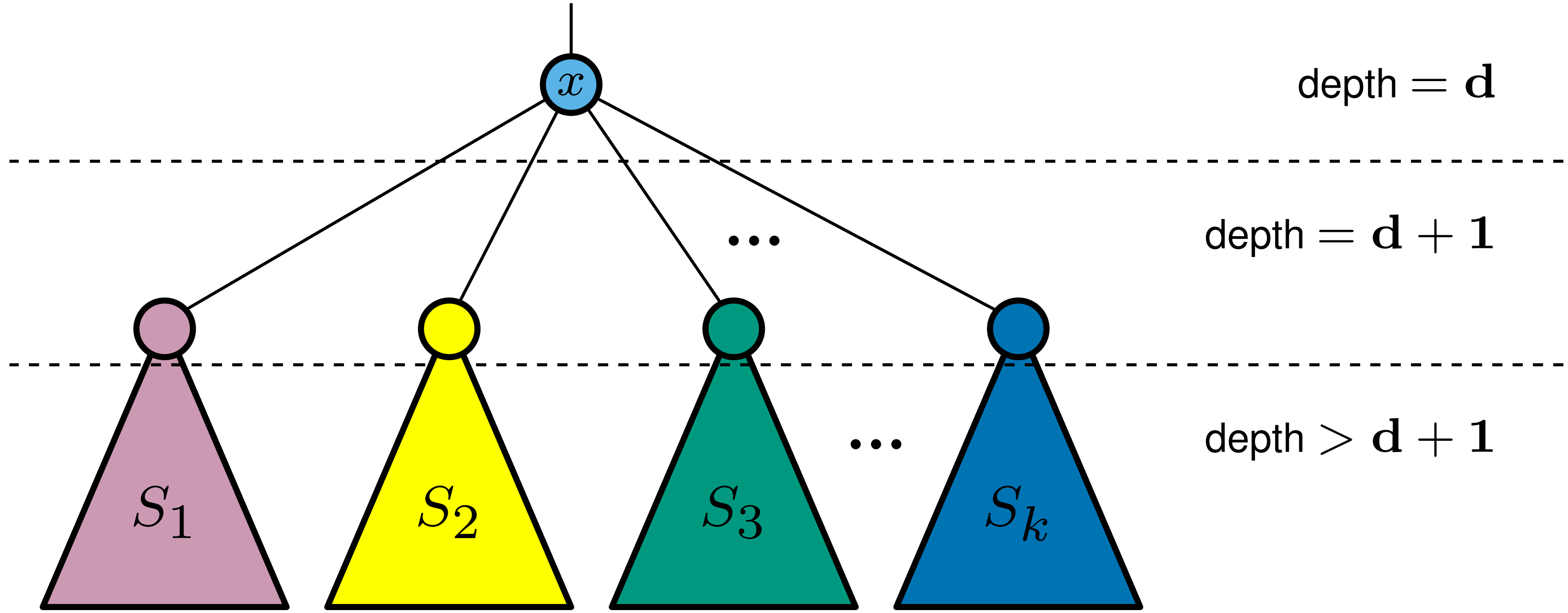
Space $O(n \log\log n)$

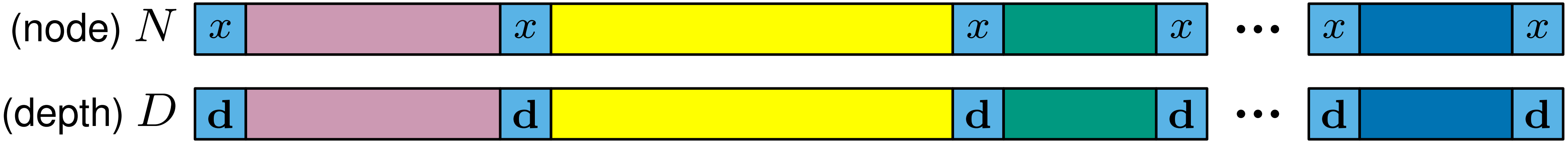
Query time $O(1)$

using the best result from last lecture

Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively. . .





Solving LCA using RMQ - correctness

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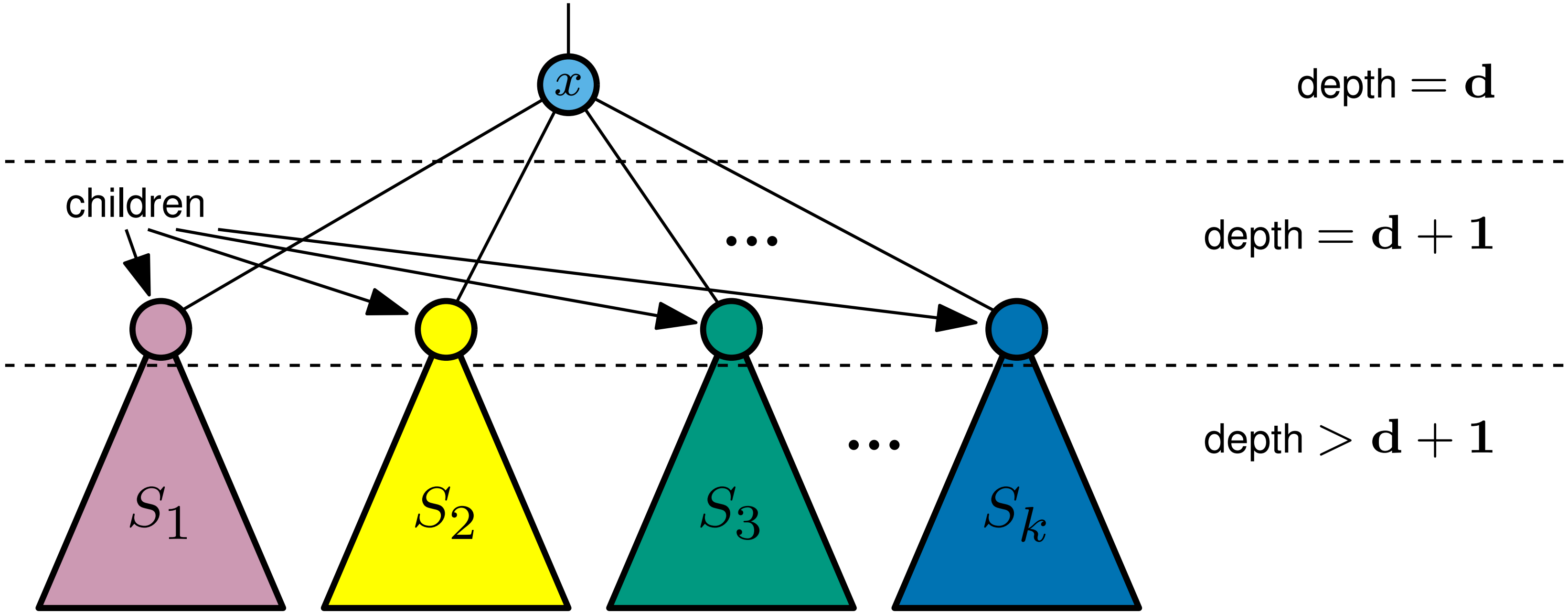

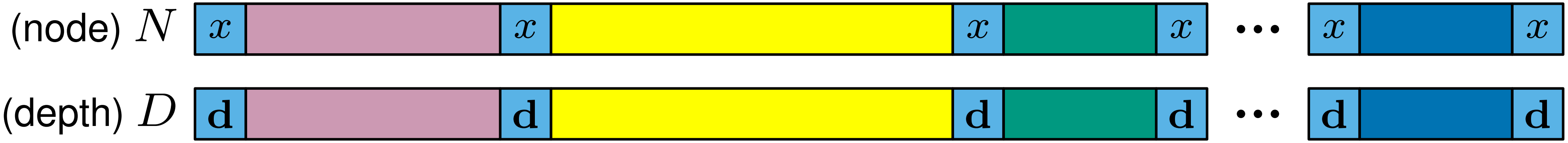
(node) N
x x x x ... x x
(depth) D
$\mathbf{d}$ $\mathbf{d}$ $\mathbf{d}$ $\mathbf{d}$ ... $\mathbf{d}$ $\mathbf{d}$

Solving LCA using RMQ - correctness

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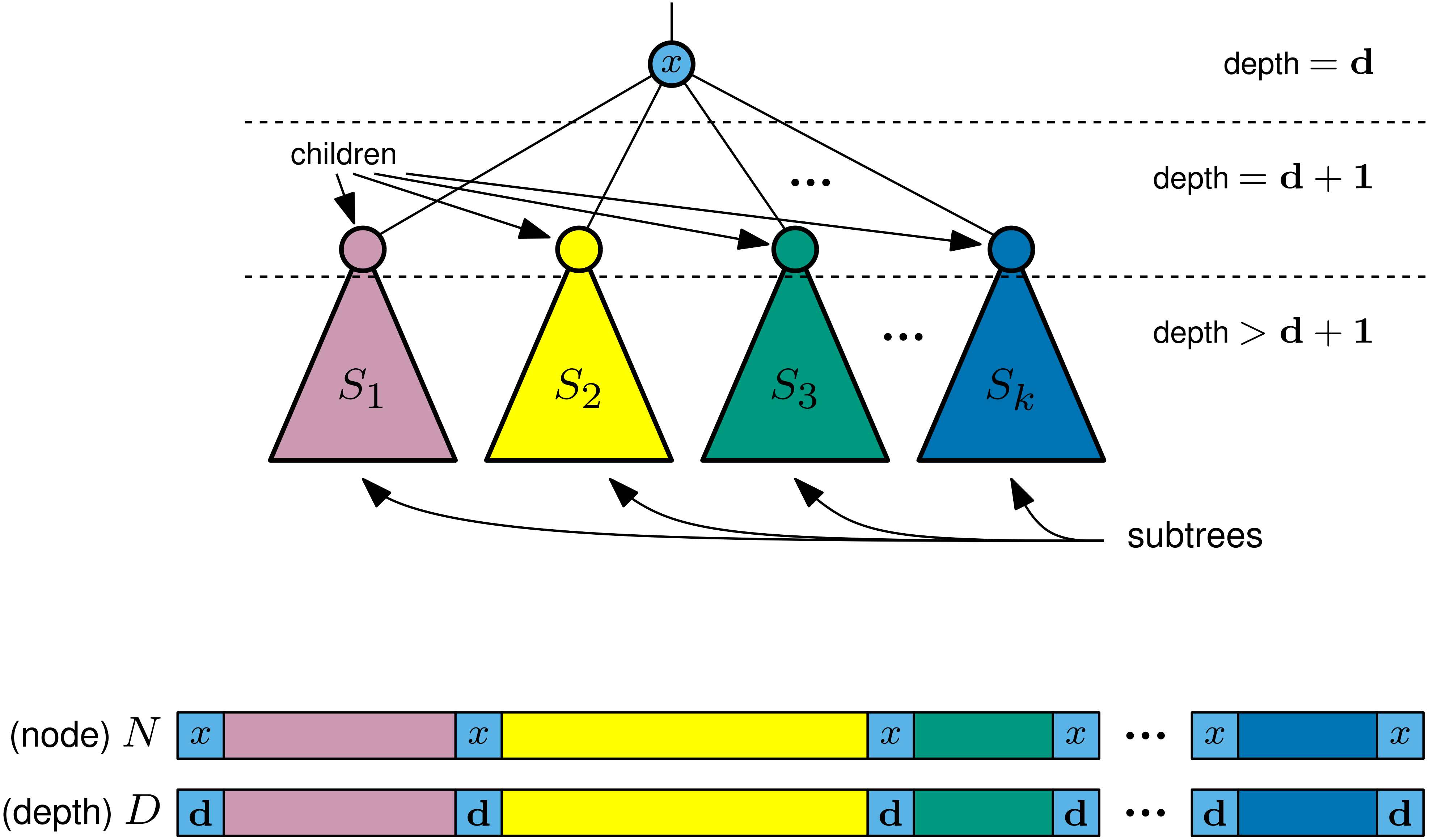
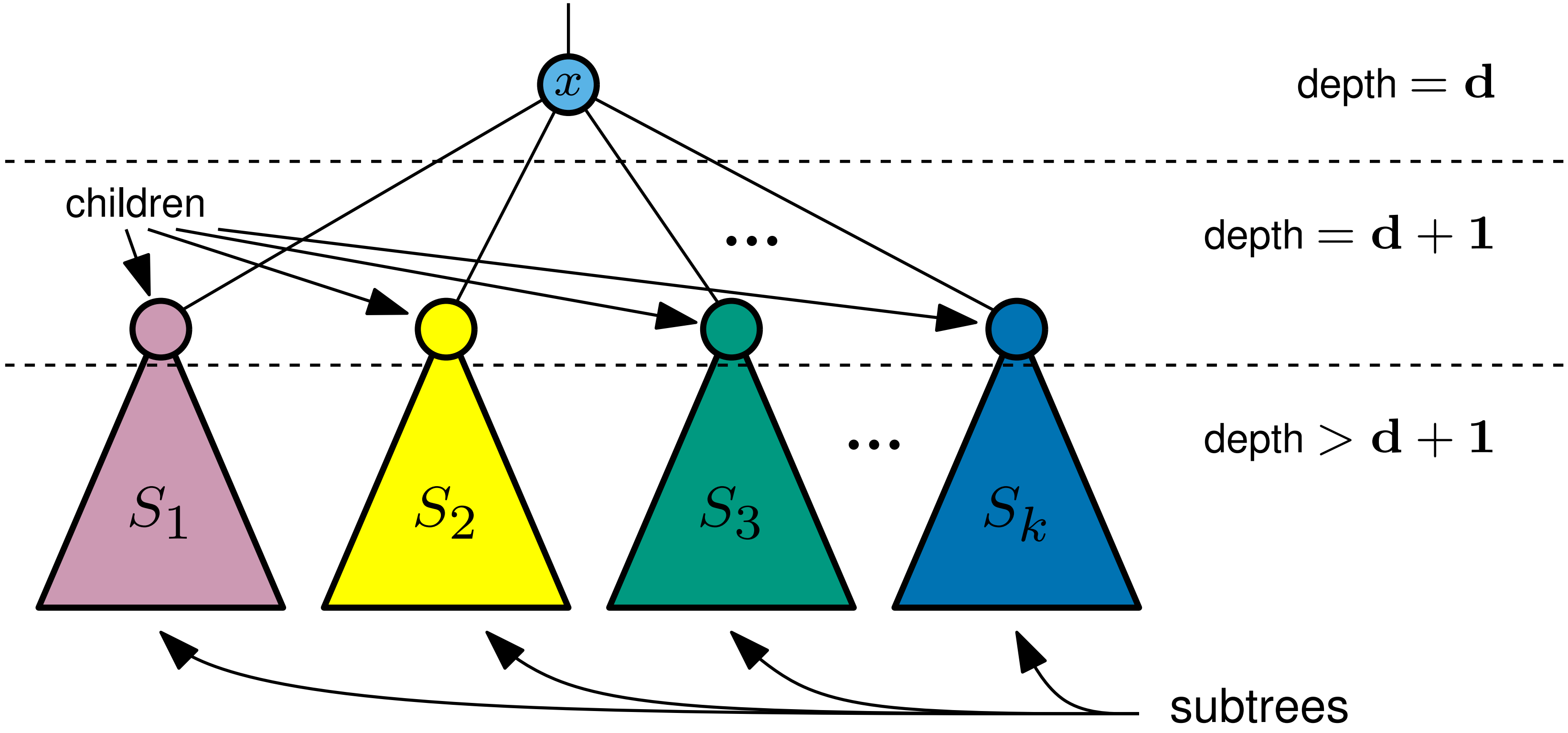
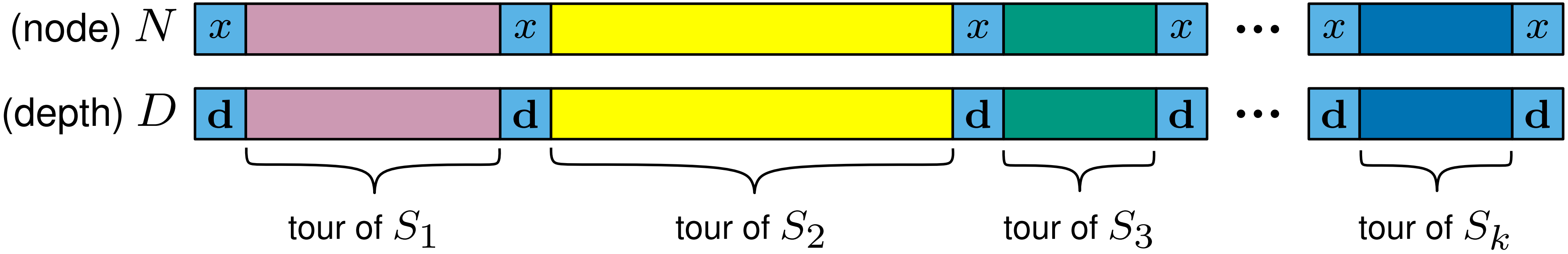

x
depth $= \mathbf{d}$
children
$\cdots$
depth $= \mathbf{d}+\mathbf{1}$
S_1
S_2
S_3
$\cdots$
S_k
depth $> \mathbf{d}+\mathbf{1}$
subtrees
(node) N
x x x x $\cdots$ x x
(depth) D
$\mathbf{d}$ $\mathbf{d}$ $\mathbf{d}$ $\mathbf{d}$ $\cdots$ $\mathbf{d}$ $\mathbf{d}$

Solving LCA using RMQ - correctness

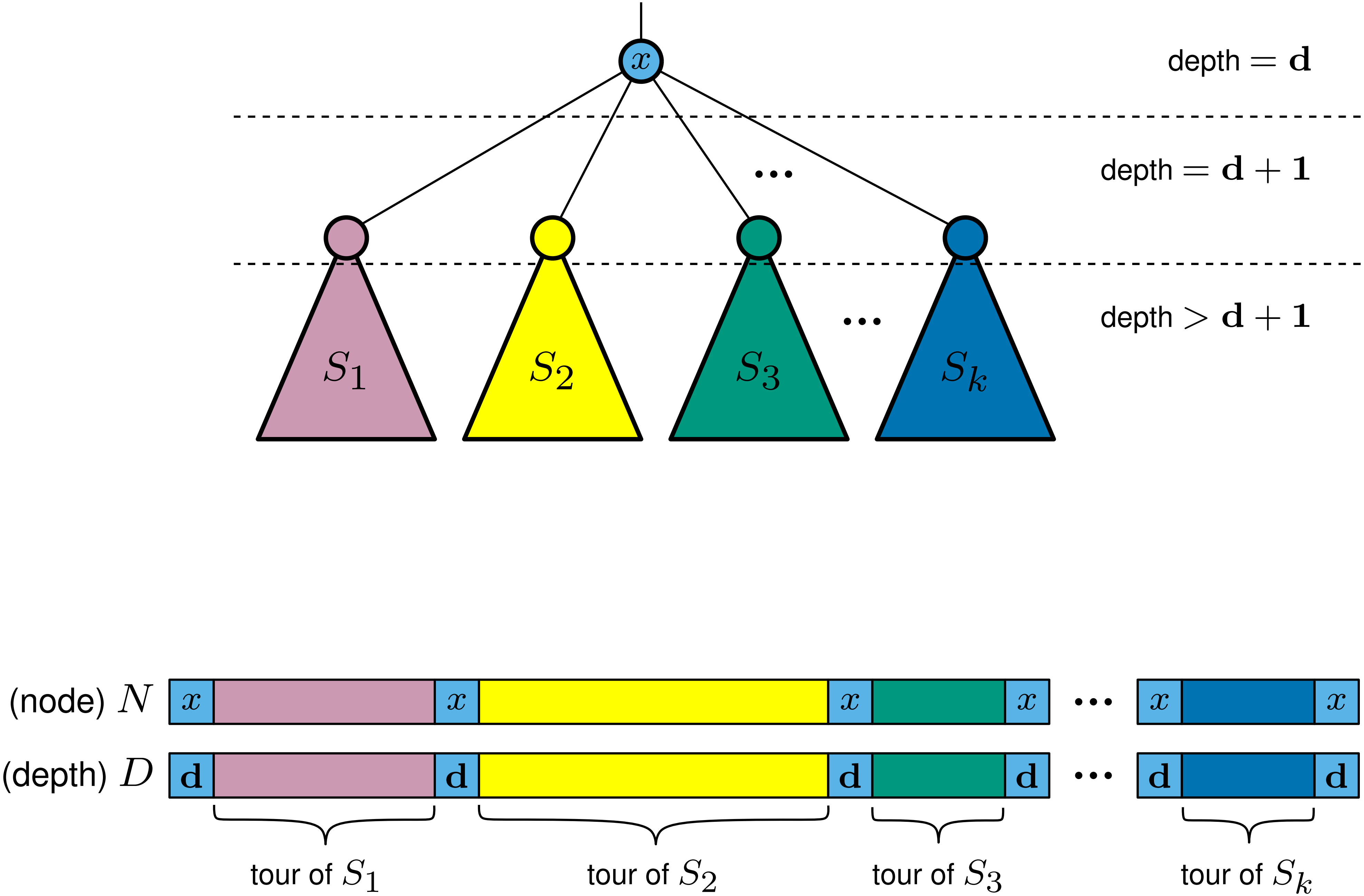
We can also define a Euler tour of T recursively...





Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively...



Solving LCA using RMQ - correctness

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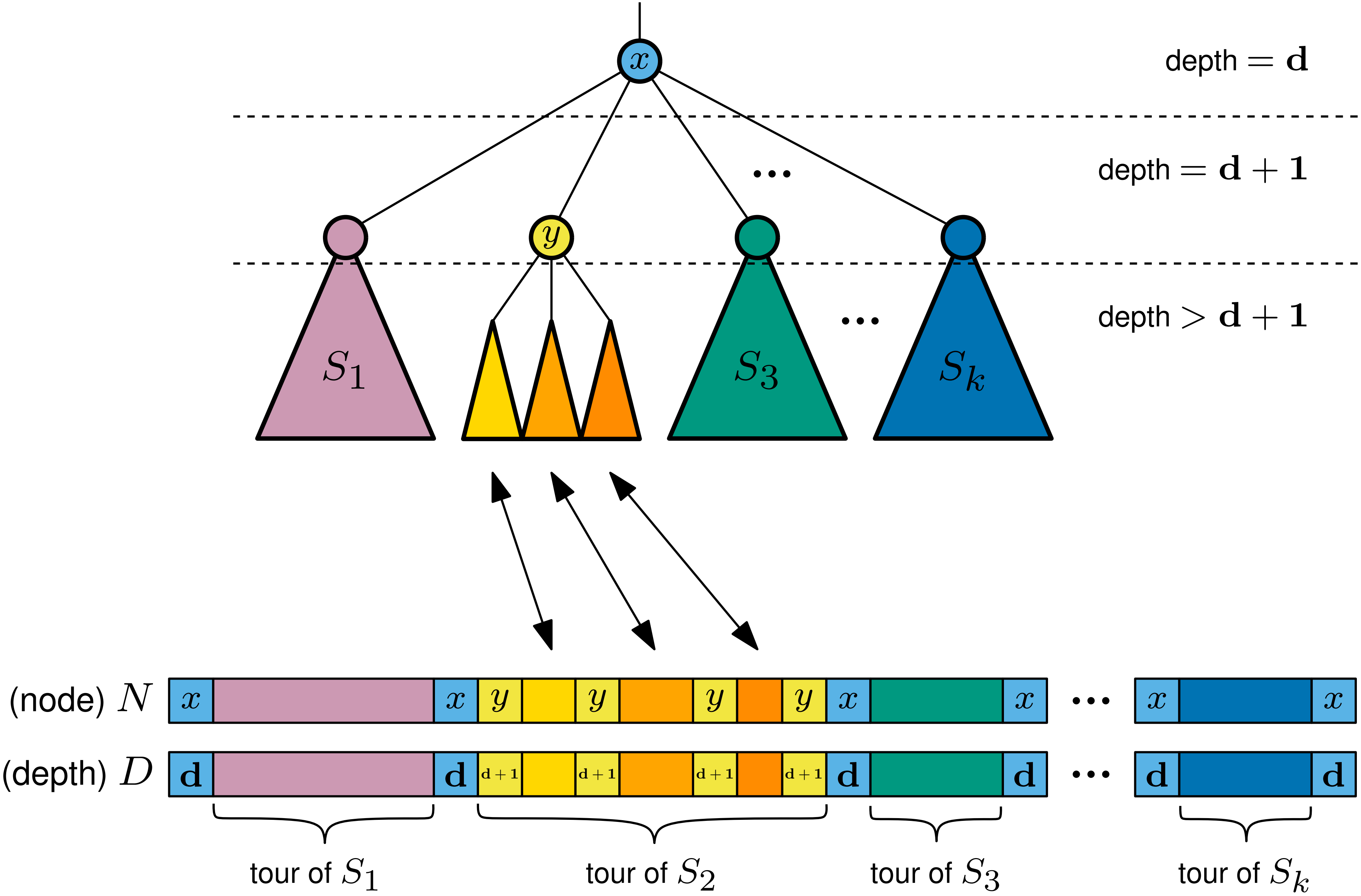
x
depth $= \mathbf{d}$
depth $= \mathbf{d+1}$
y
...
depth $> \mathbf{d+1}$
S_1
S_3
...
S_k
(node) N
x x y y y y x x ... x x
(depth) D
$\mathbf{d}$ $\mathbf{d}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathbf{d}$ $\mathbf{d}$ ... $\mathbf{d}$ $\mathbf{d}$
tour of S_1
tour of S_2
tour of S_3
tour of S_k

Solving LCA using RMQ - correctness

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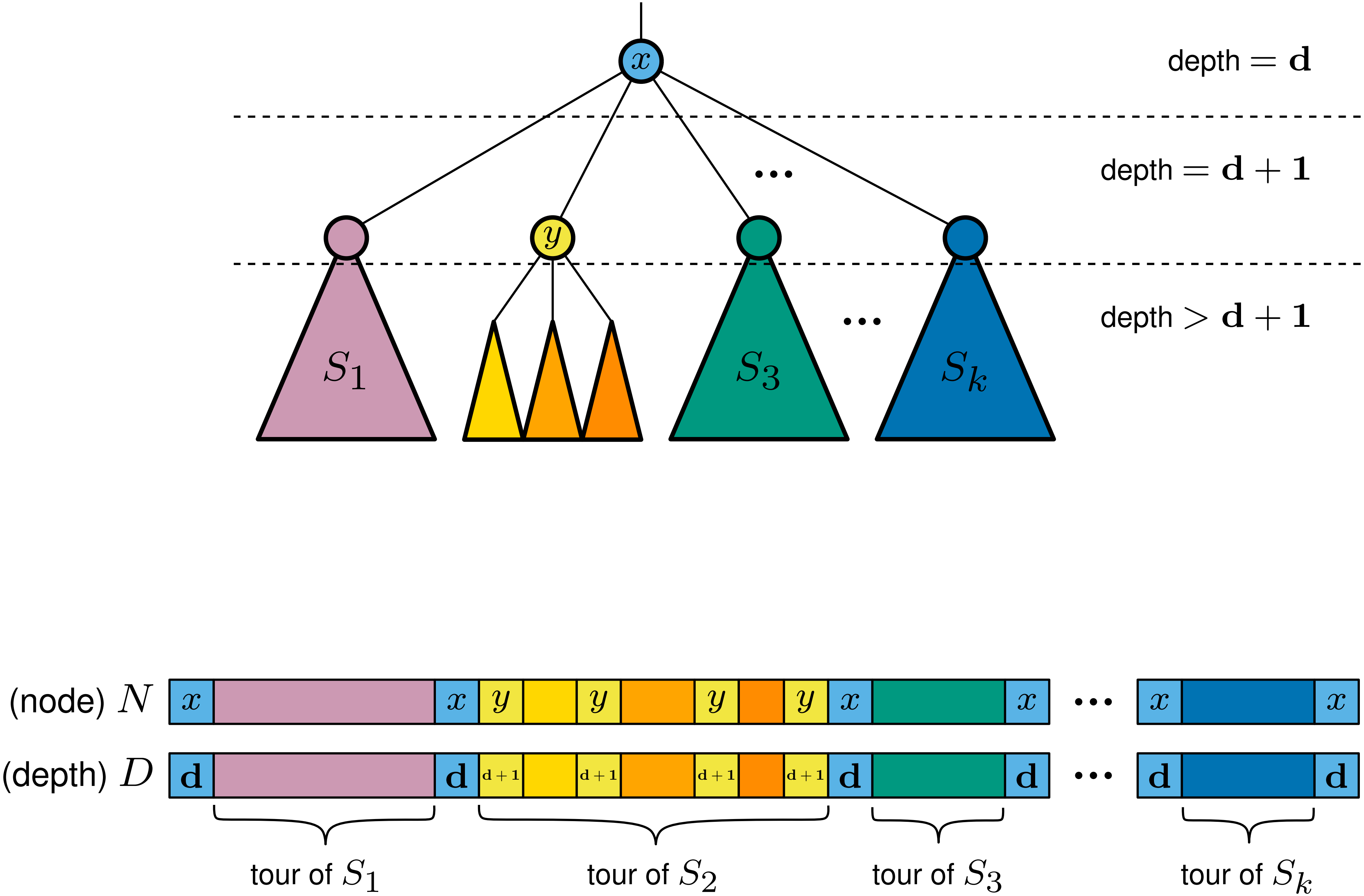
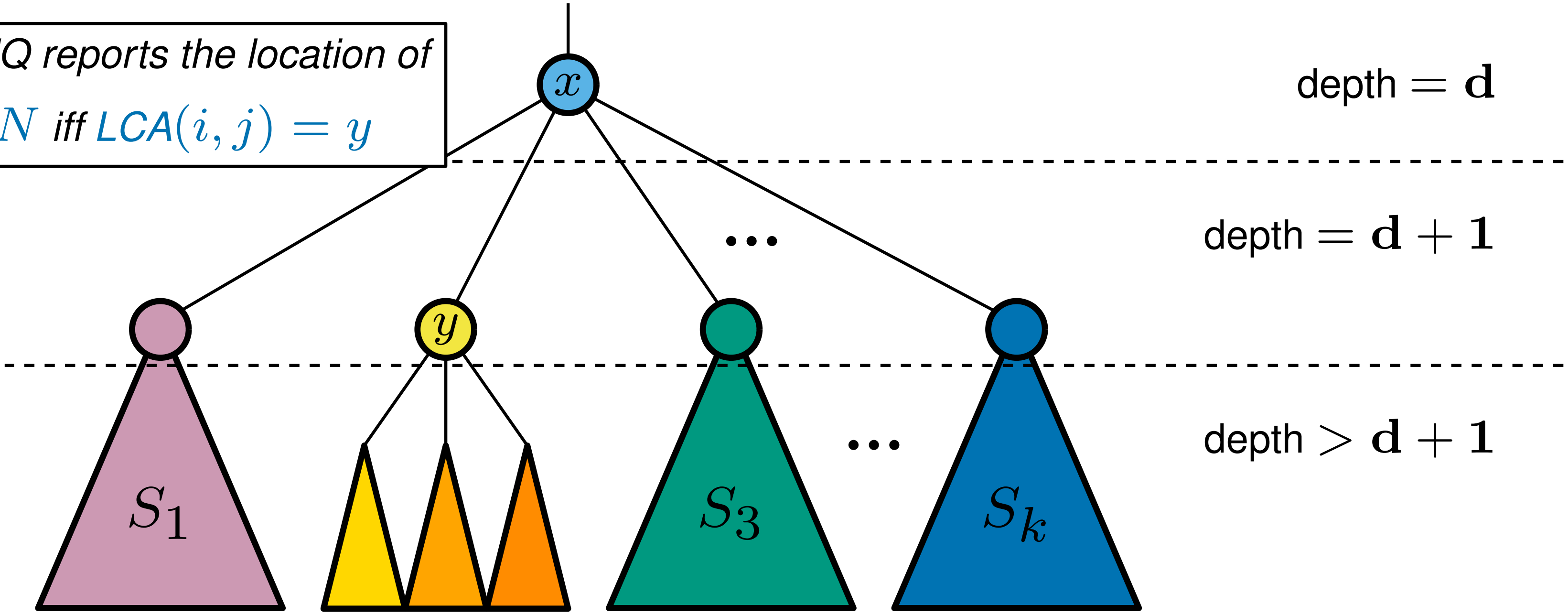
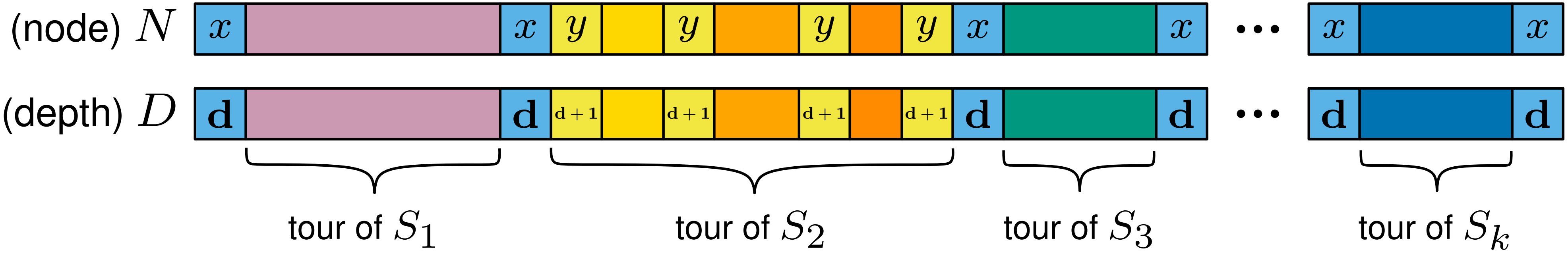

x
depth $= \mathbf{d}$
depth $= \mathbf{d+1}$
depth $> \mathbf{d+1}$
y
$\cdots$
S_1
S_3
$\cdots$
S_k
(node) N
x x y y y y x x $\cdots$ x x
(depth) D
$\mathbf{d}$ $\mathbf{d}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathbf{d}$ $\mathbf{d}$ $\cdots$ $\mathbf{d}$ $\mathbf{d}$
tour of S_1
tour of S_2
tour of S_3
tour of S_k

Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively...

Claim *the RMQ reports the location of some y in N iff LCA$(i,j) = y$*

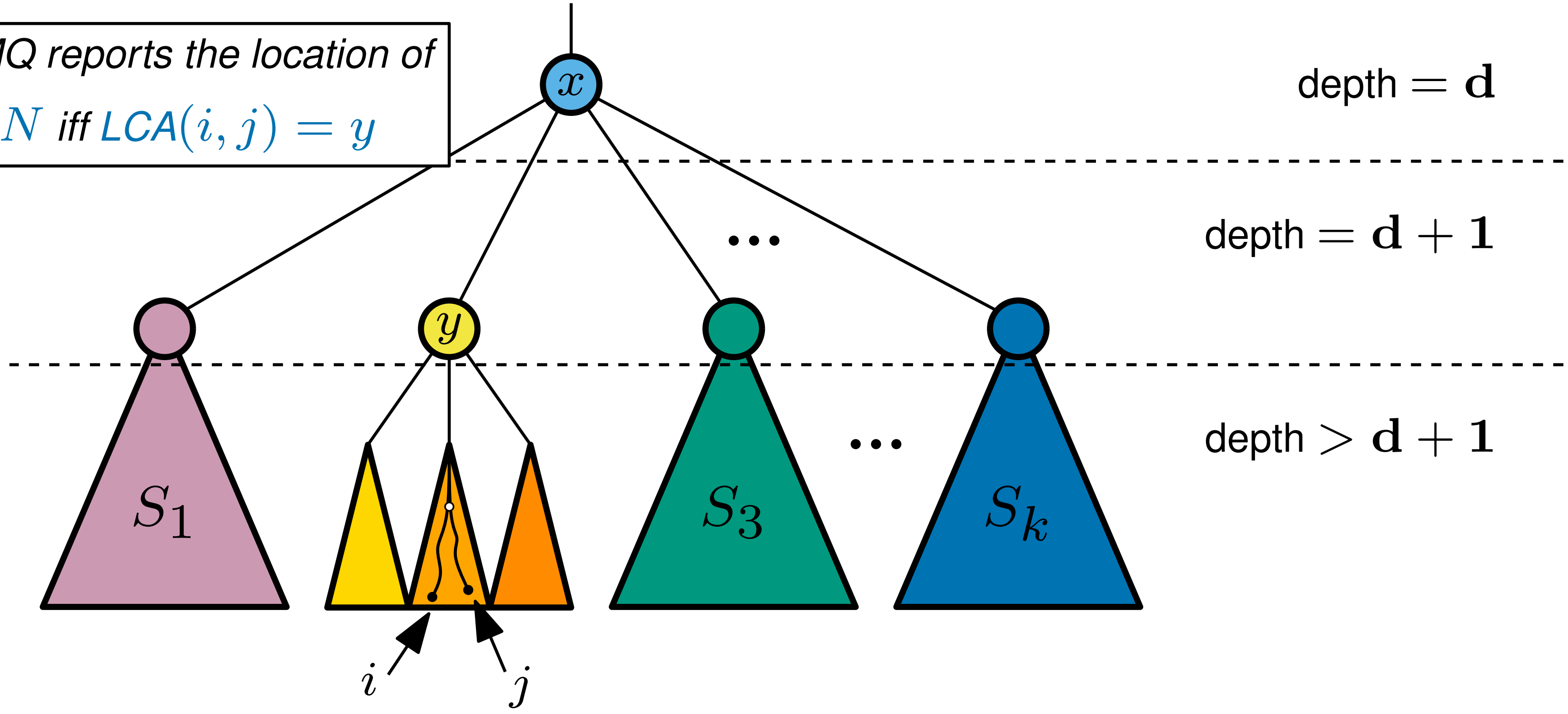




Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively. . .

Claim *the RMQ reports the location of some* y *in* N *iff* LCA$(i, j) = y$



(node) N	x		x	y		y		y		y	x		x	$\cdots$	x		x
(depth) D	$\mathbf{d}$		$\mathbf{d}$	$\mathrm{d+1}$		$\mathrm{d+1}$		$\mathrm{d+1}$		$\mathrm{d+1}$	$\mathbf{d}$		$\mathbf{d}$	$\cdots$	$\mathbf{d}$		$\mathbf{d}$

tour of S_1 tour of S_2 tour of S_3 tour of S_k

Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively. . .

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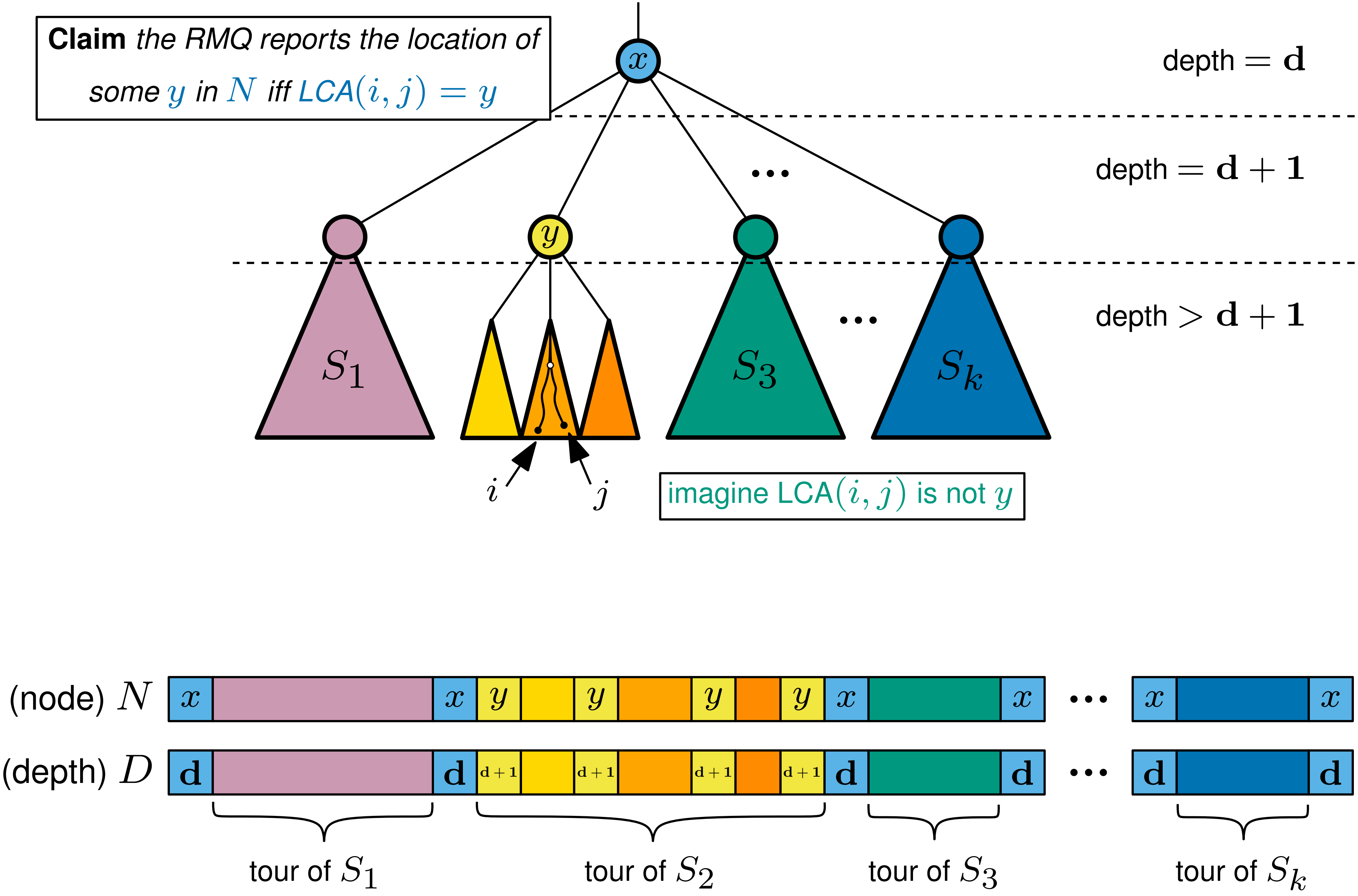

imagine LCA(i, j) is not y

Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively...

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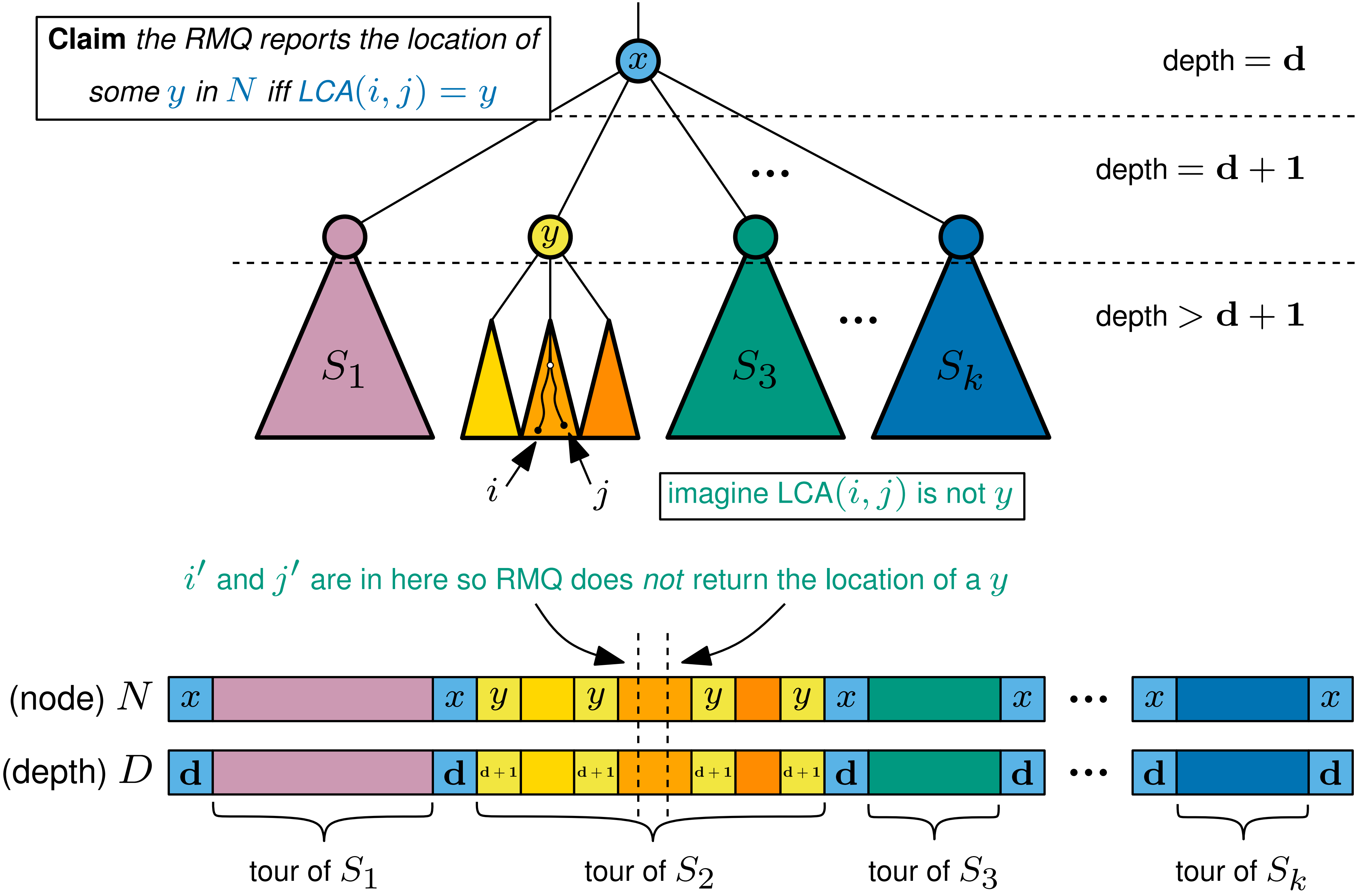

x
depth $= \mathbf{d}$
depth $= \mathbf{d+1}$
depth $> \mathbf{d+1}$
y
...
S_1
S_3
...
S_k
i
j
(node) N
x x y y y y x x ... x x
(depth) D
$\mathbf{d}$ $\mathbf{d}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathbf{d}$ $\mathbf{d}$ ... $\mathbf{d}$ $\mathbf{d}$
tour of S_1
tour of S_2
tour of S_3
tour of S_k

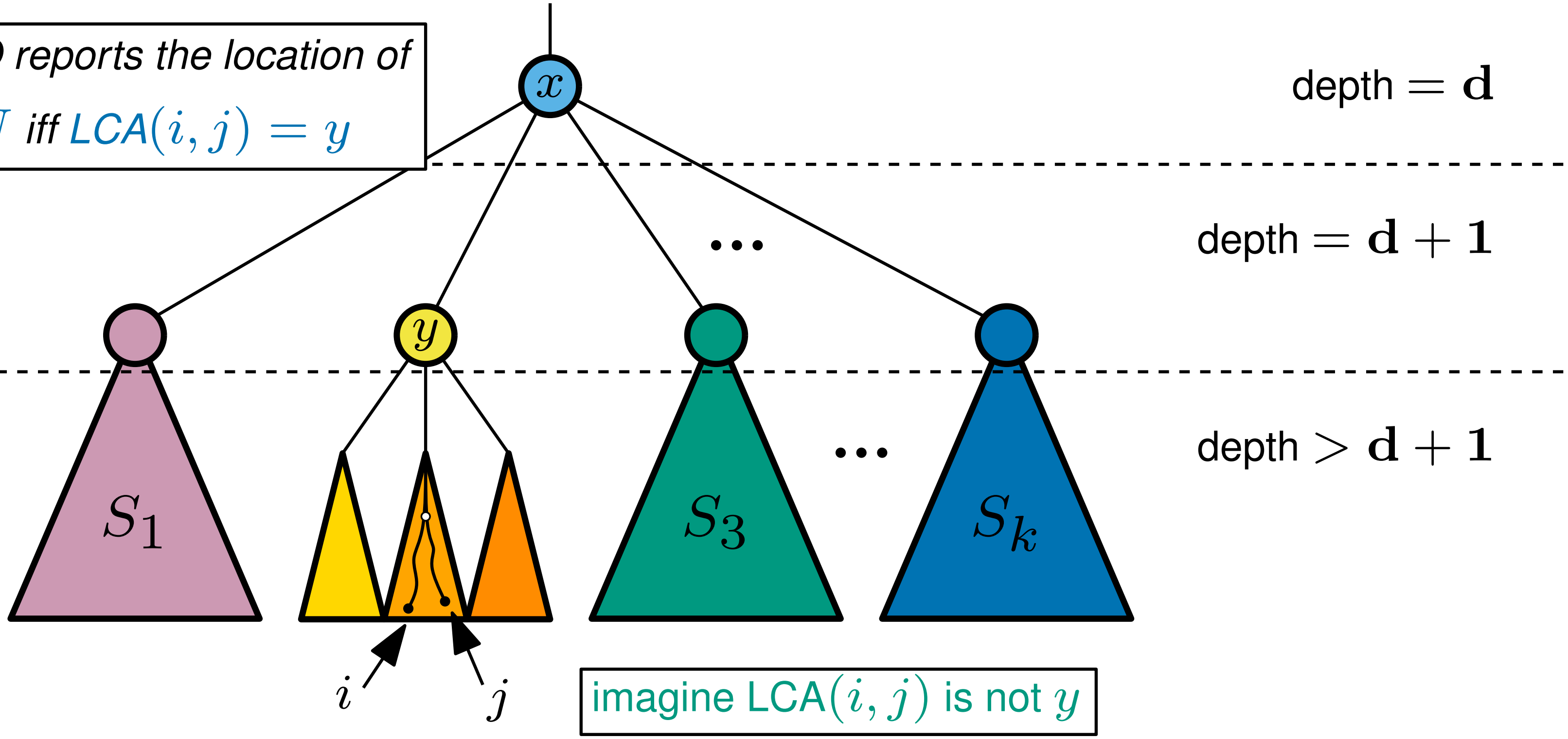
imagine LCA(i, j) is not y

i' and j' are in here so RMQ does *not* return the location of a y

Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively...

Claim *the RMQ reports the location of some y in N iff LCA$(i, j) = y$*



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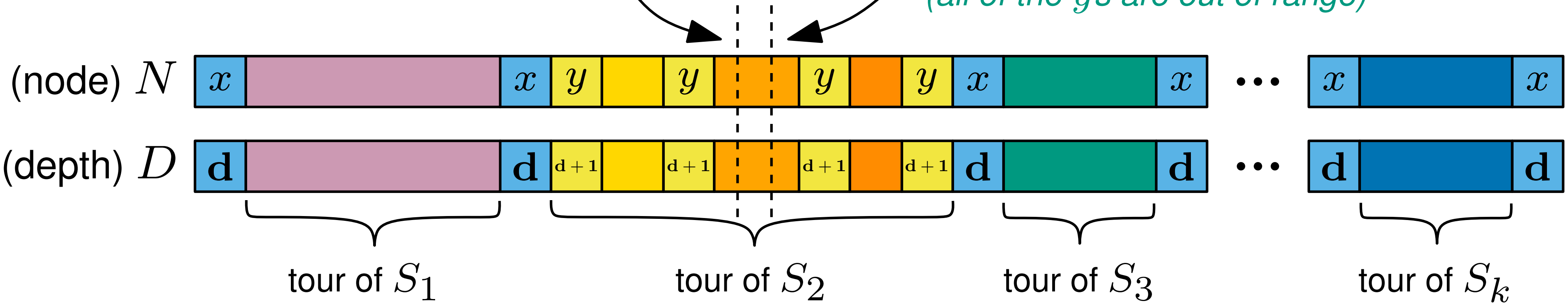
(all of the ys are out of range)



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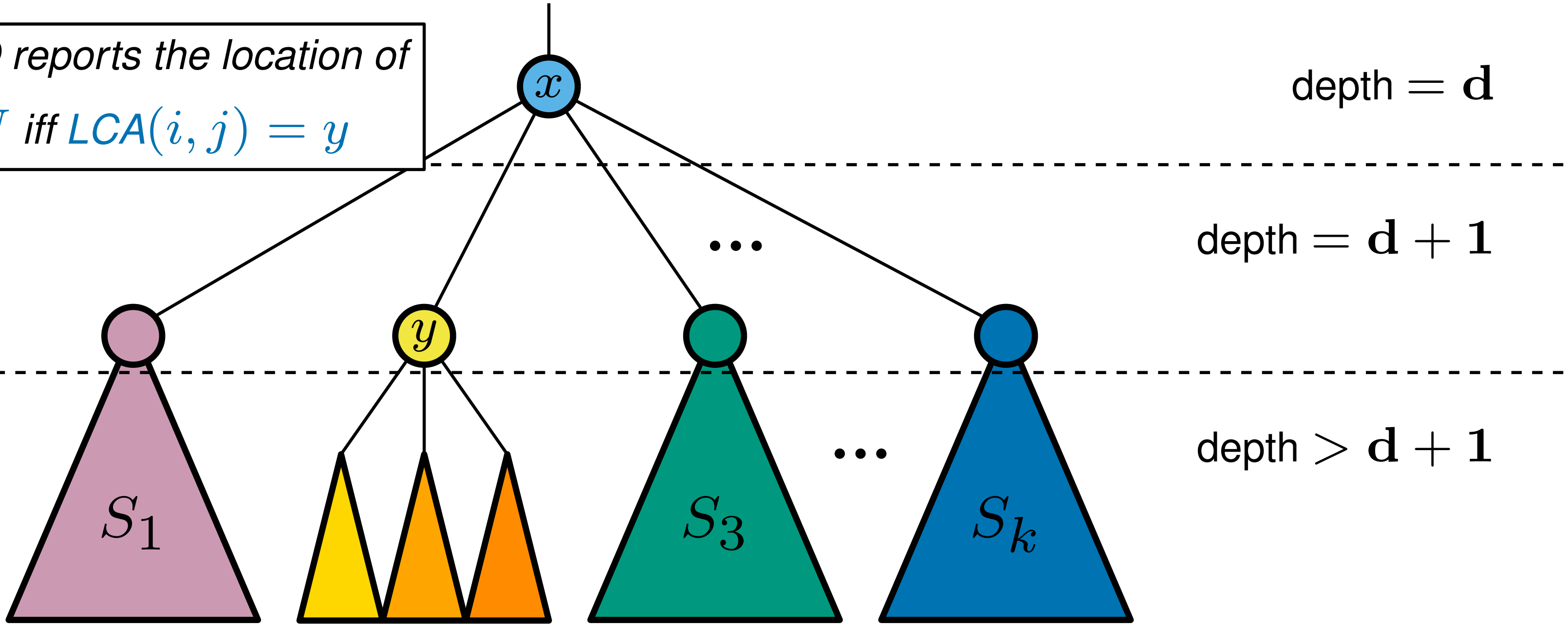

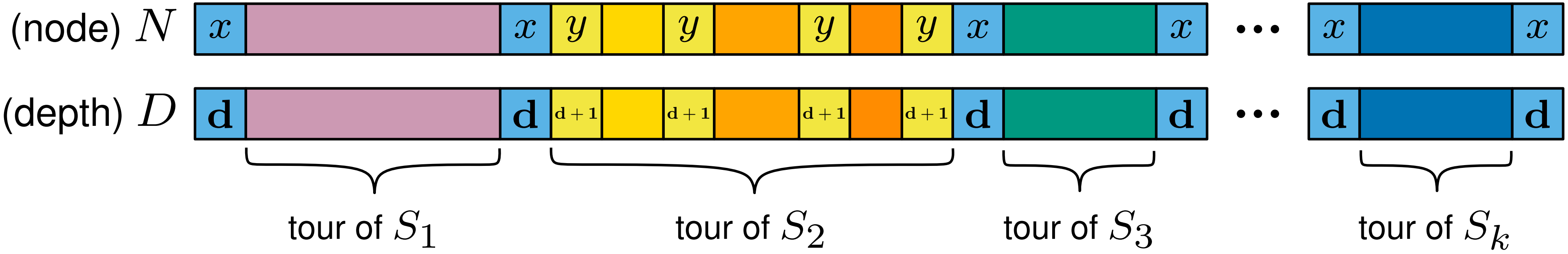

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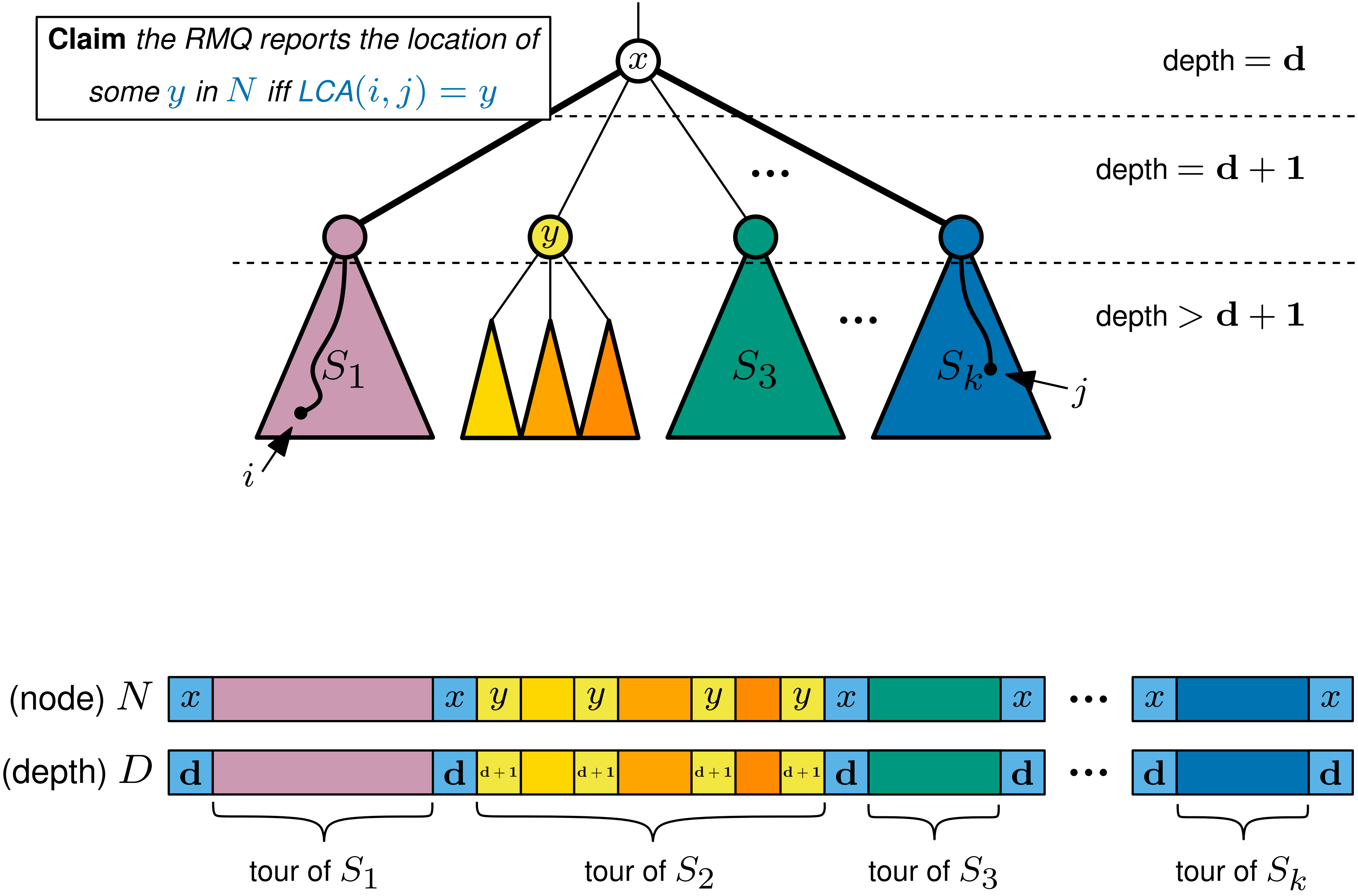

x
y
depth $= \mathbf{d}$
depth $= \mathbf{d+1}$
depth $> \mathbf{d+1}$
S_1
S_3
S_k
i
j
...
(node) N
x x y y y y x x ... x x
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$\mathbf{d}$ $\mathbf{d}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathrm{d+1}$ $\mathbf{d}$ $\mathbf{d}$ ... $\mathbf{d}$ $\mathbf{d}$
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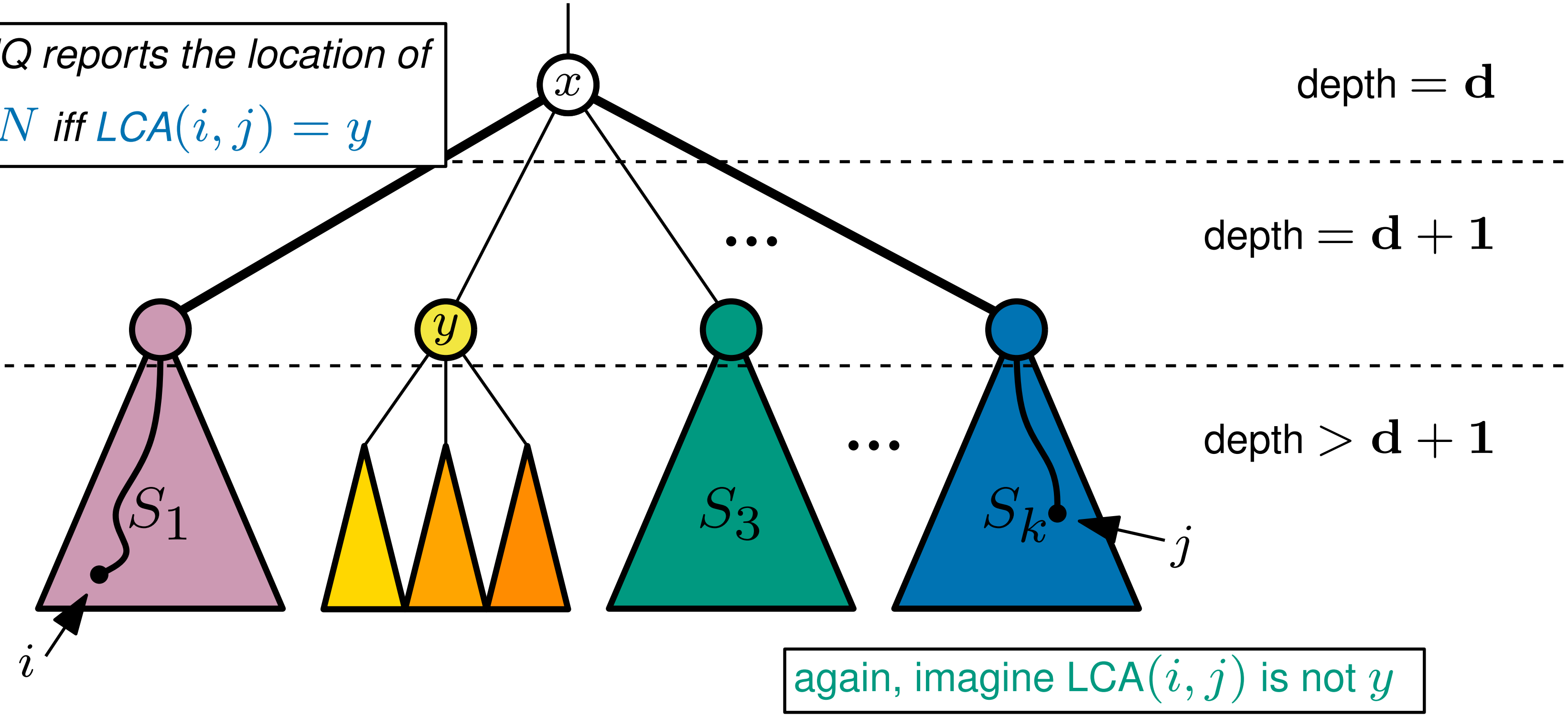

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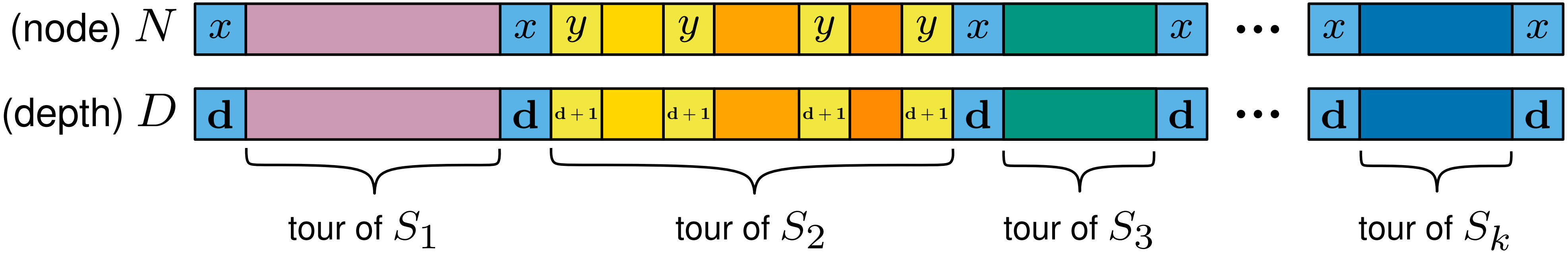

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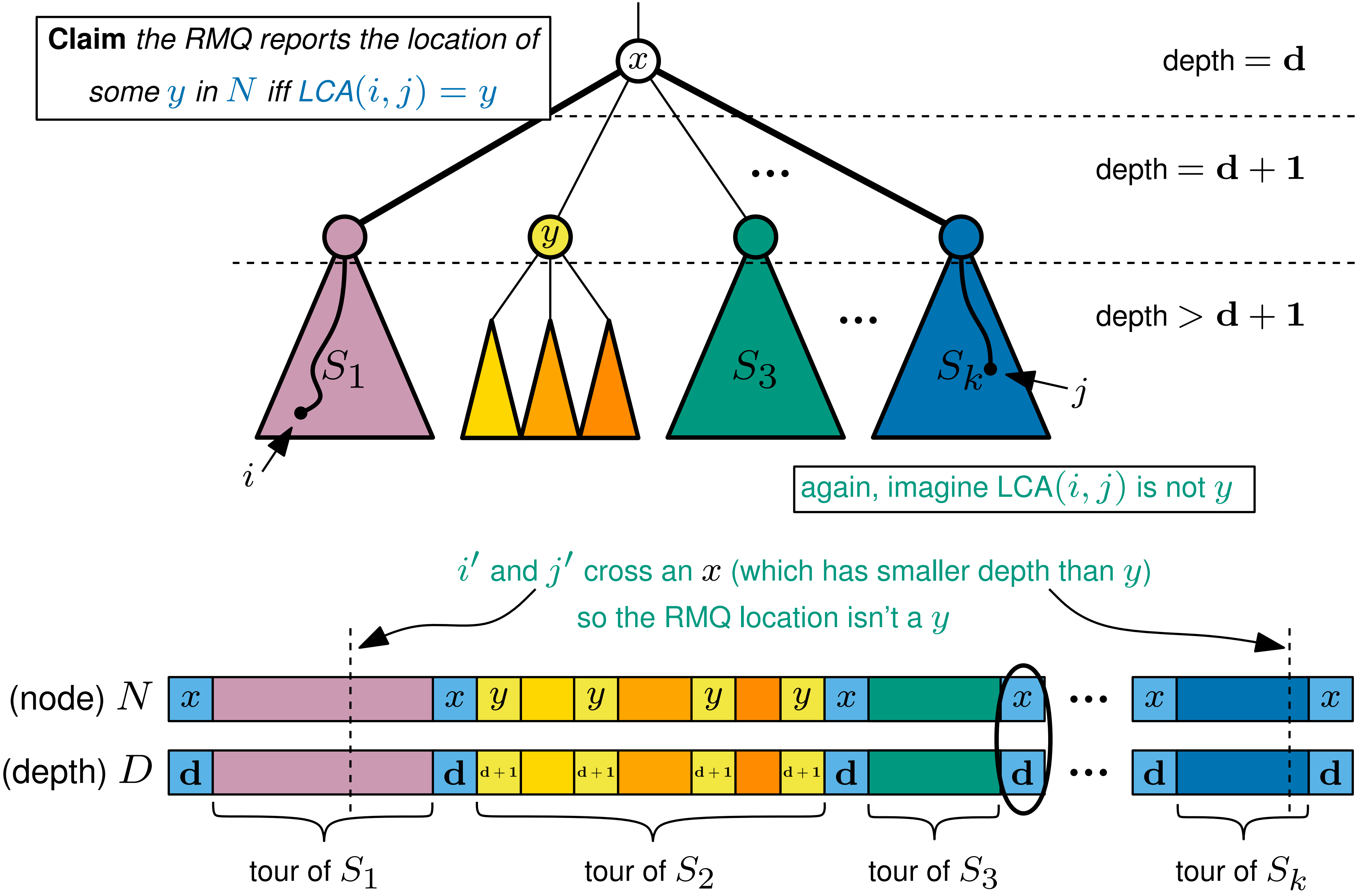


again, imagine LCA(i, j) is not y

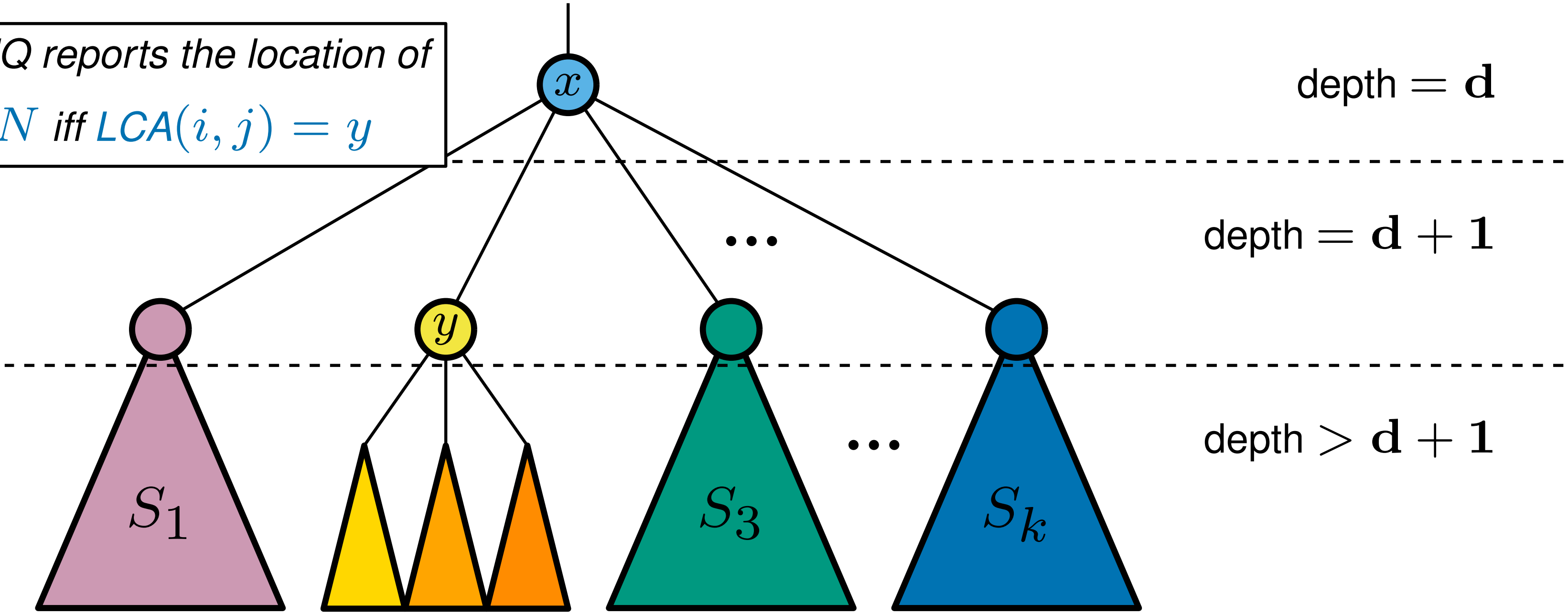
i' and j' cross an x (which has smaller depth than y)

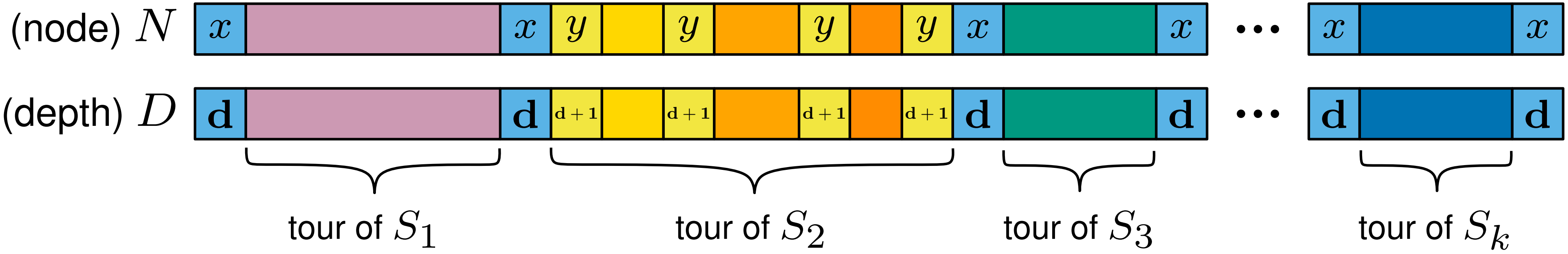
so the RMQ location isn't a y

Solving LCA using RMQ - correctness

We can also define a Euler tour of T recursively...

Claim *the RMQ reports the location of some* y *in* N *iff* LCA$(i,j) = y$





(node) N
x x y y y y x x ... x x
(depth) D
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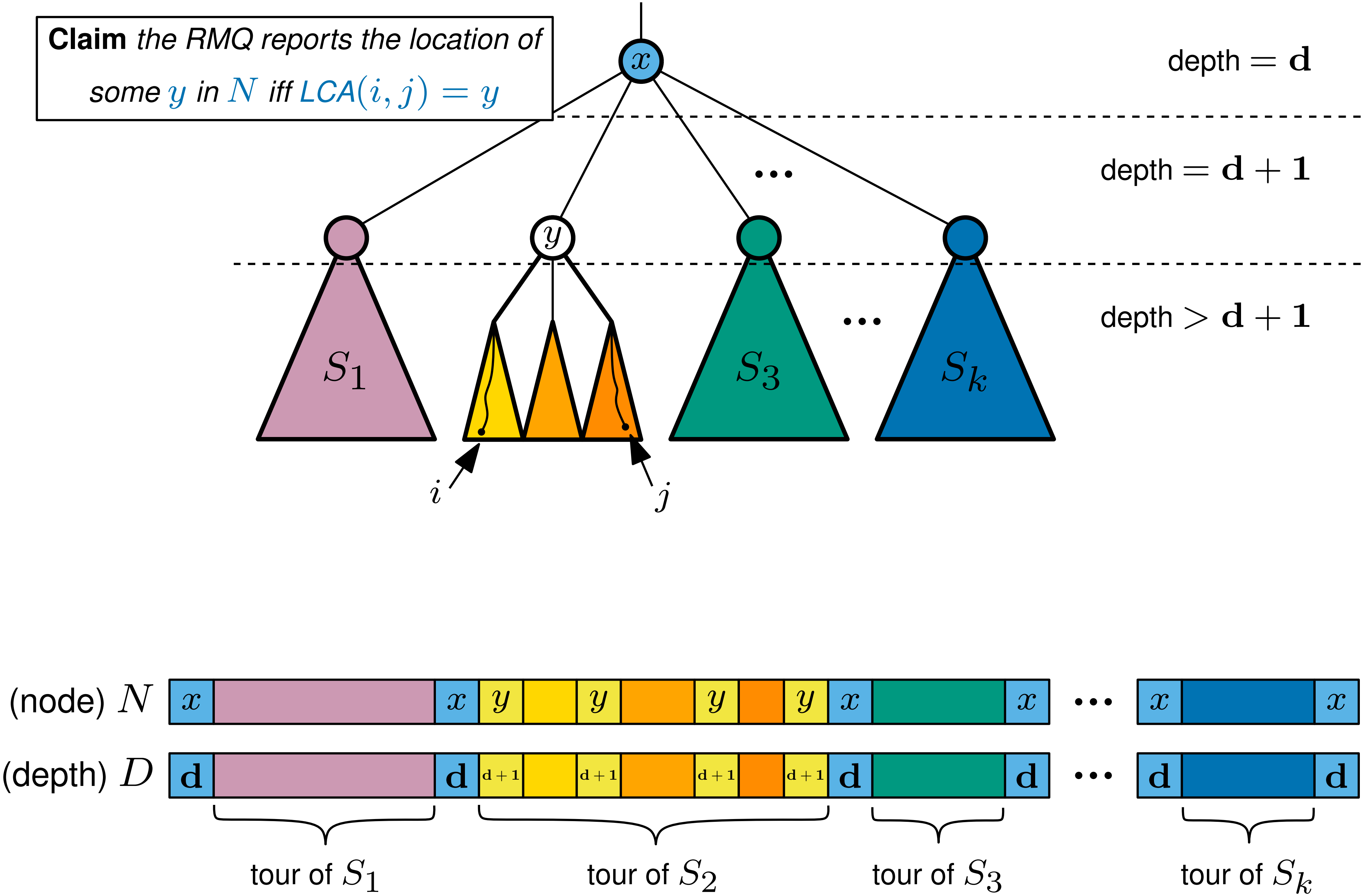
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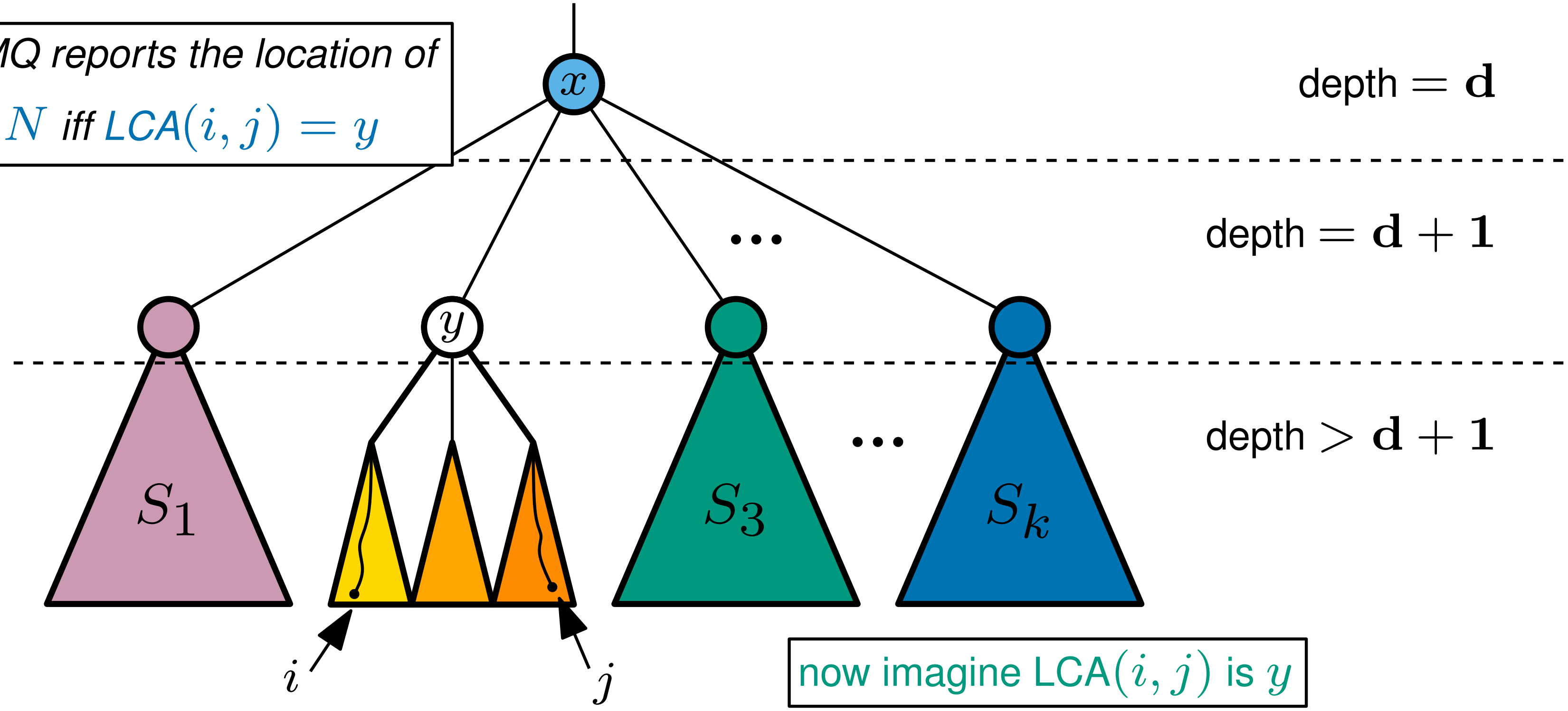

now imagine LCA(i, j) is y

(node) N	x		x	y		y		y		y	x		x	...	x		x
(depth) D	$\mathbf{d}$		$\mathbf{d}$	$\mathrm{d+1}$		$\mathrm{d+1}$		$\mathrm{d+1}$		$\mathrm{d+1}$	$\mathbf{d}$		$\mathbf{d}$	...	$\mathbf{d}$		$\mathbf{d}$

tour of S_1 tour of S_2 tour of S_3 tour of S_k

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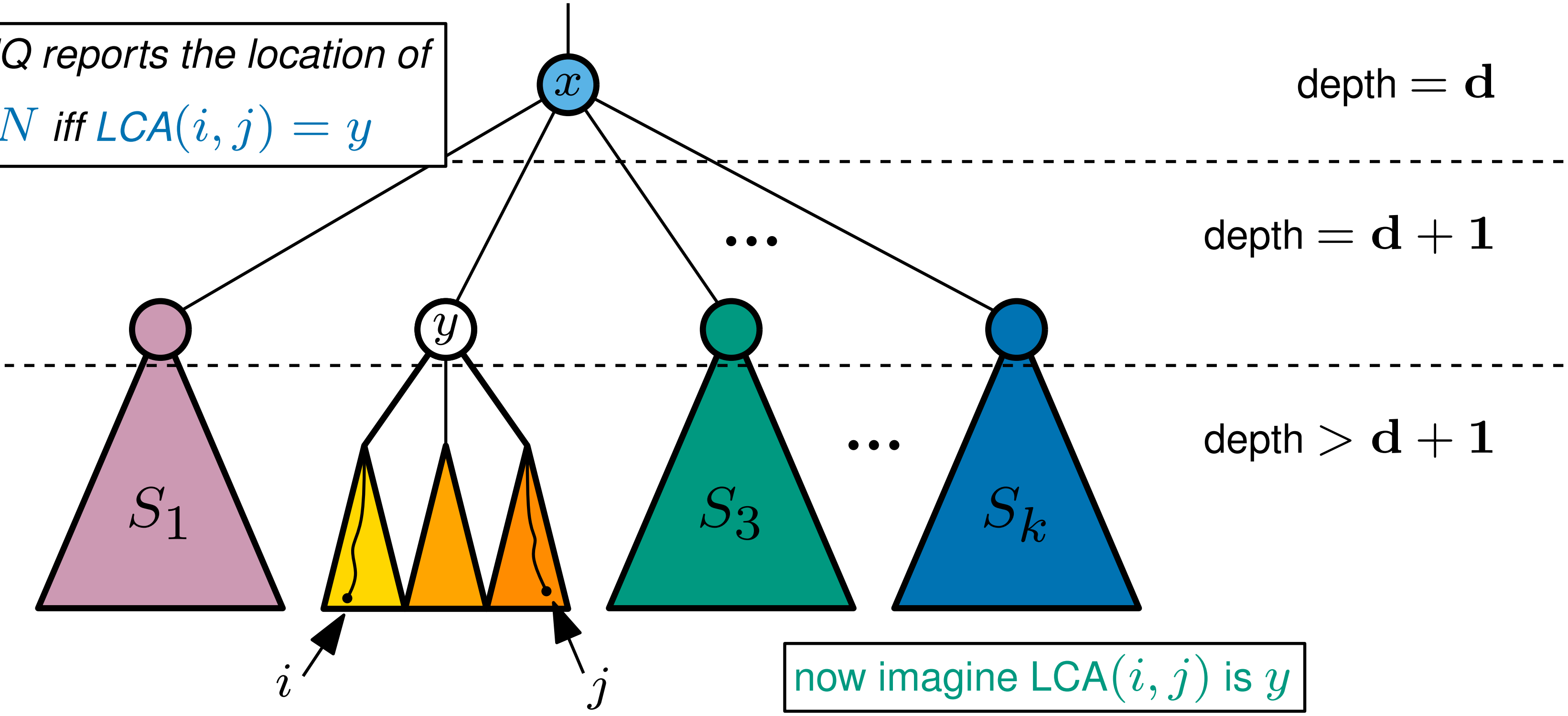

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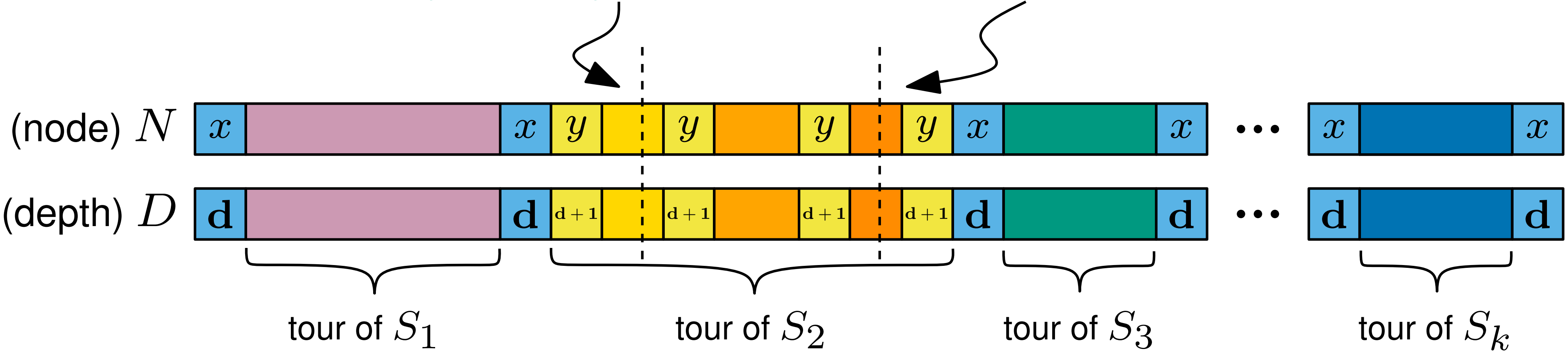

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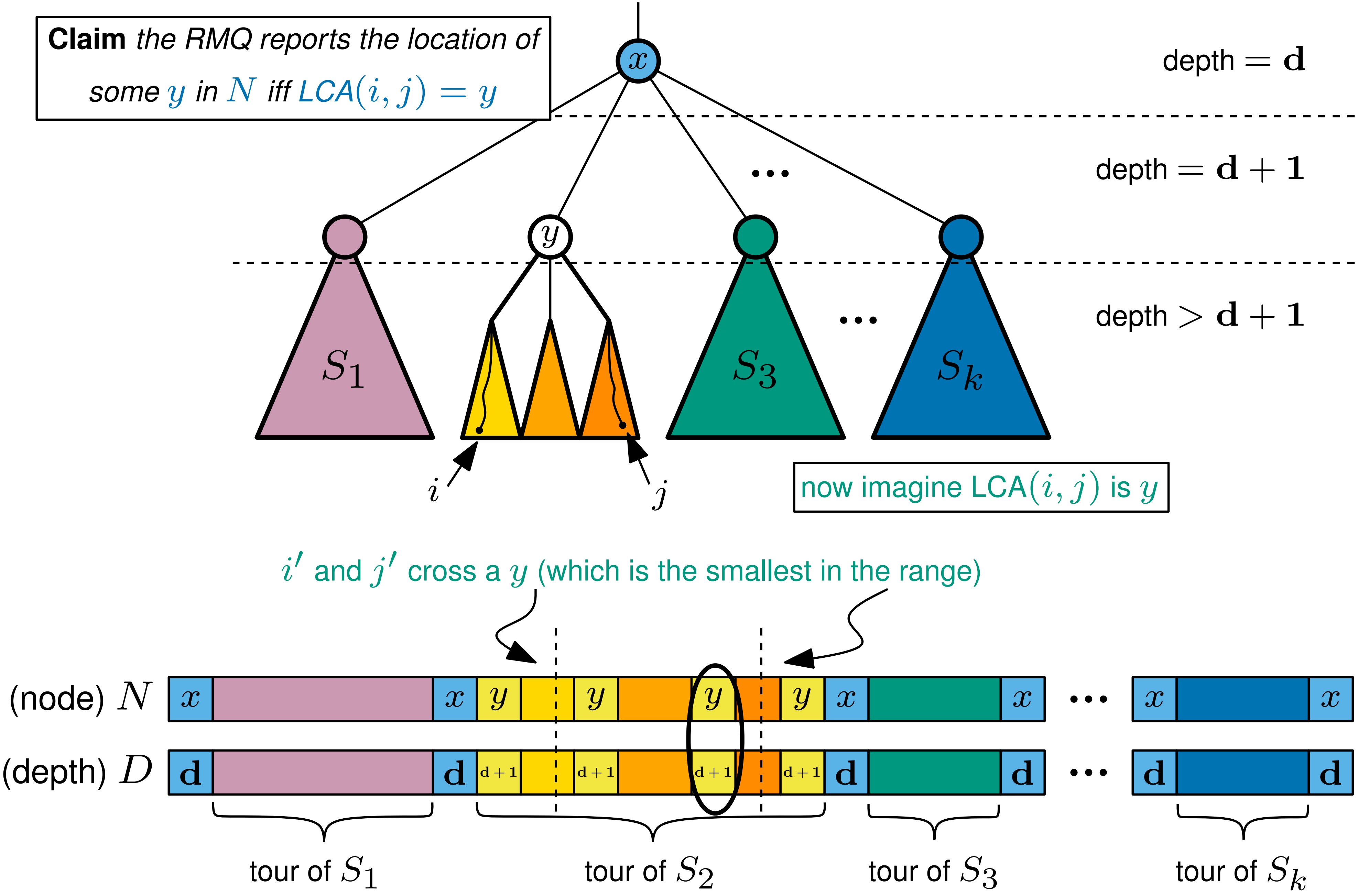

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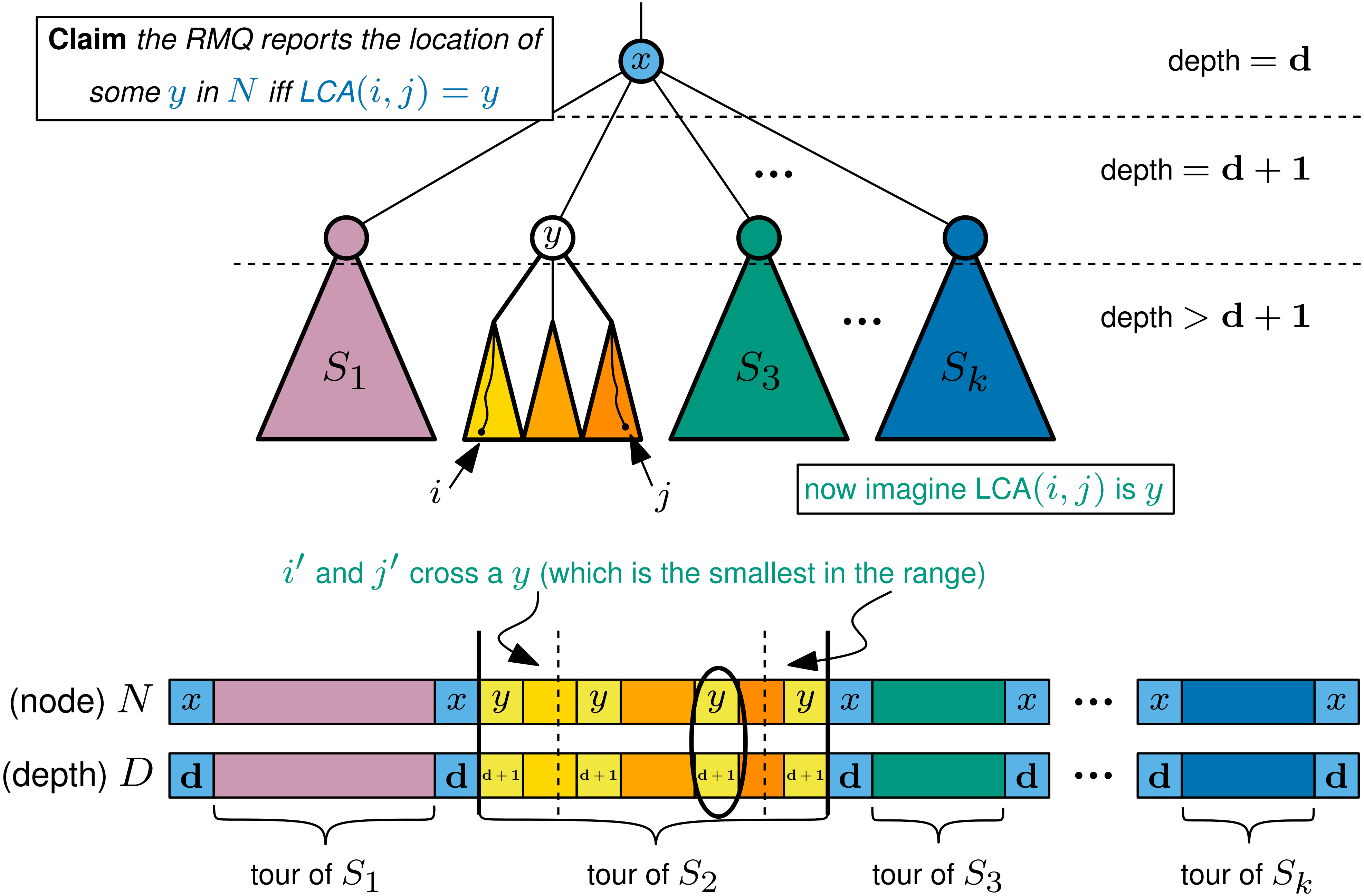

x
depth $= \mathbf{d}$
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depth $> \mathbf{d+1}$
y
S_1
S_3
S_k
i
j
(node) N
x x y y y y x x x x
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$\mathbf{d}$ $\mathbf{d}$ $\mathbf{d+1}$ $\mathbf{d+1}$ $\mathbf{d+1}$ $\mathbf{d+1}$ $\mathbf{d}$ $\mathbf{d}$ $\mathbf{d}$ $\mathbf{d}$
tour of S_1
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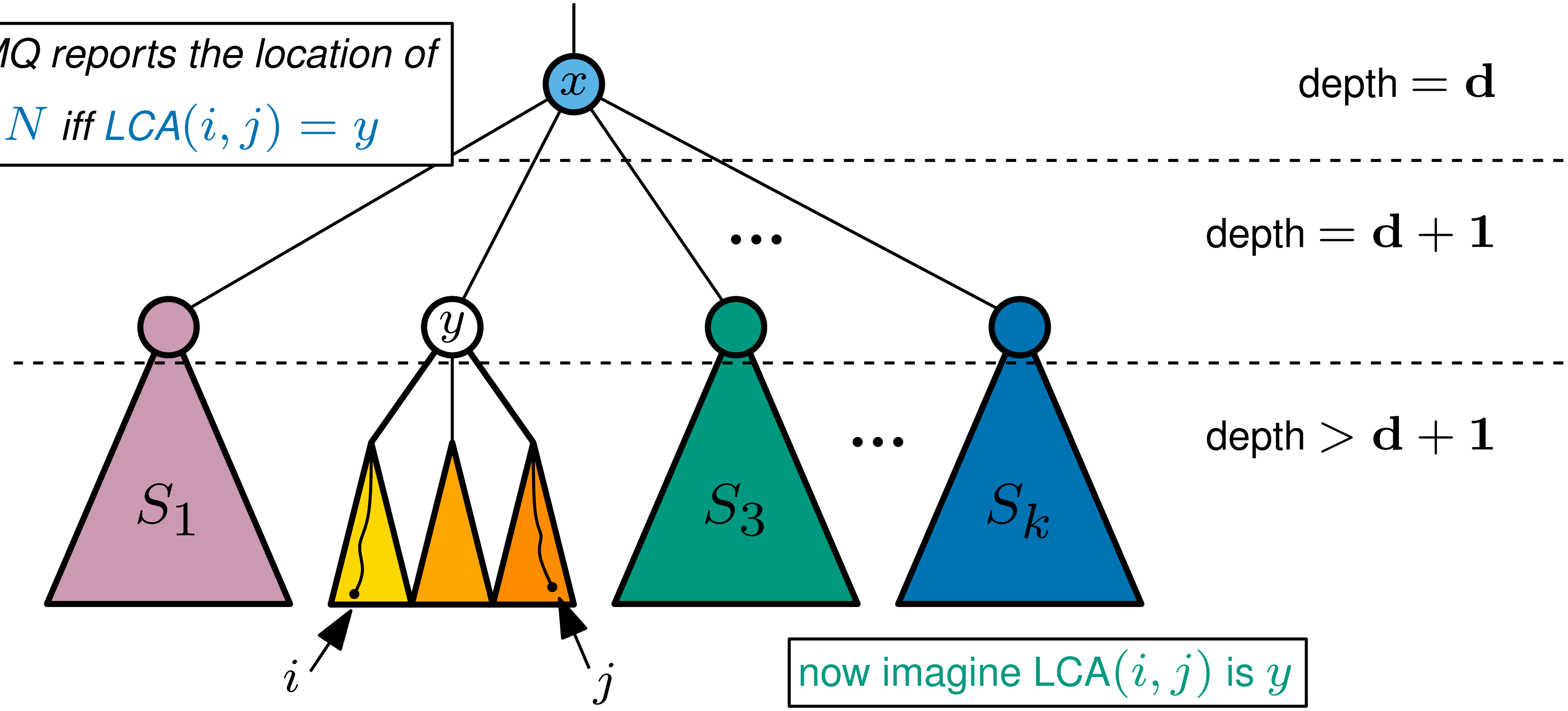

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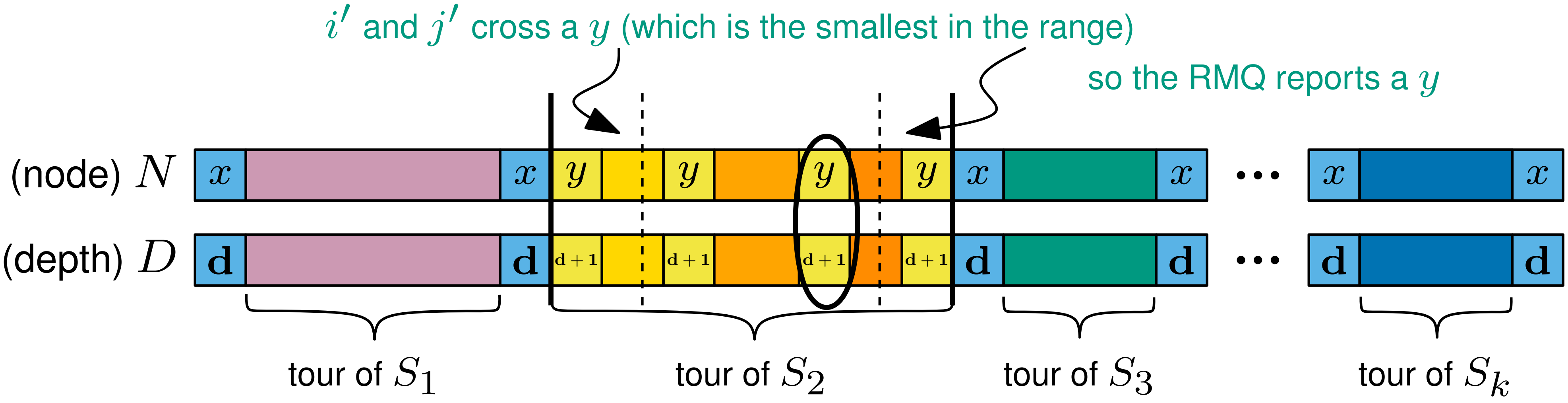

Ongoing Summary

We have seen an $O(n \log\log n)$ space, $O(n \log\log n)$ prep. time and $O(1)$ query time solution

for the Lowest Common Ancestor problem

which uses solution 3 for RMQ *from last lecture*

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We have seen an $O(n \log \log n)$ space, $O(n \log \log n)$ prep. time and $O(1)$ query time solution

for the Lowest Common Ancestor problem

which uses solution 3 for RMQ *from last lecture*

Can we do better?

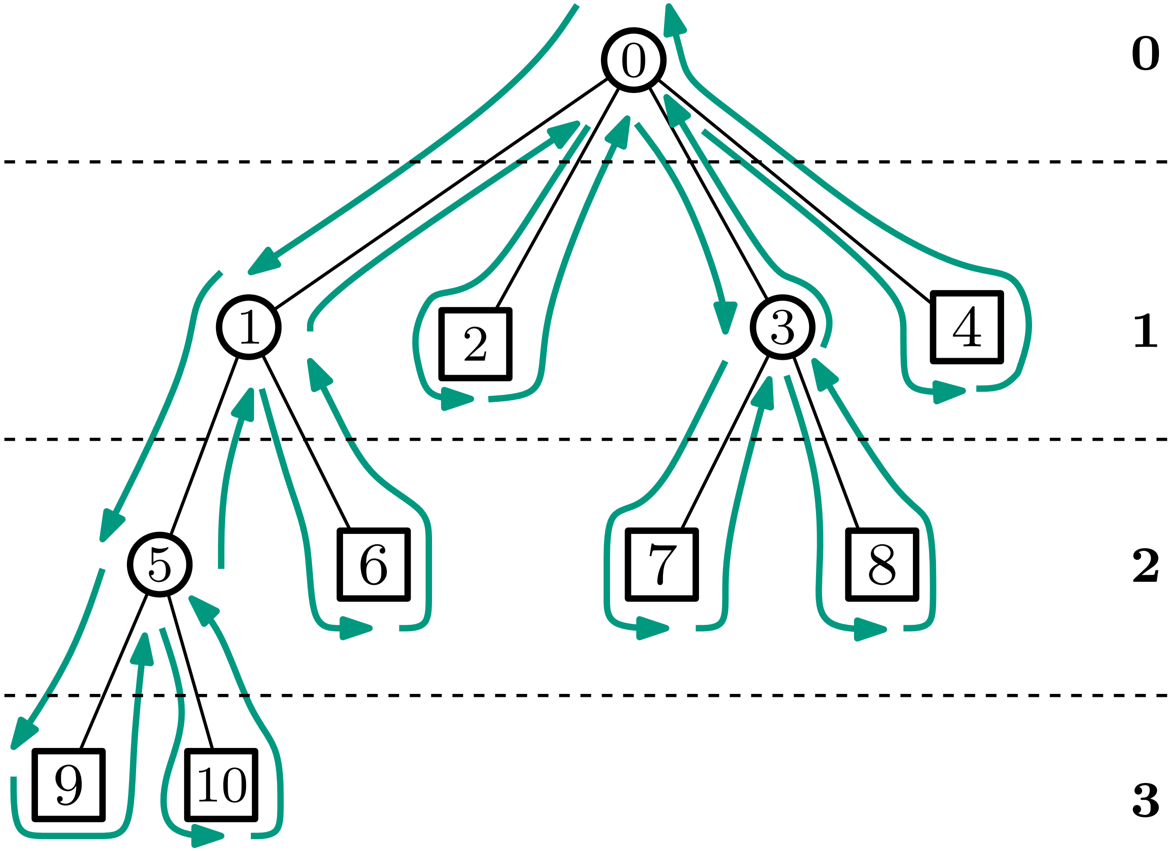
Solving LCAs using RMQs - efficiency

Preprocessing Summary

1. Construct N and D from T
2. Add a pointer from each node i to some $N[i'] = i$
3. Preprocess D for RMQs

Query Summary - LCA(i,j)

1. Find (any) i' st. $N[i'] = i$
2. Find (any) j' st. $N[j'] = j$
3. Compute RMQ(i', j') in D
4. LCA$(i, j) = N[$RMQ$(i', j')]$



(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Solving LCAs using RMQs - efficiency

Preprocessing Summary

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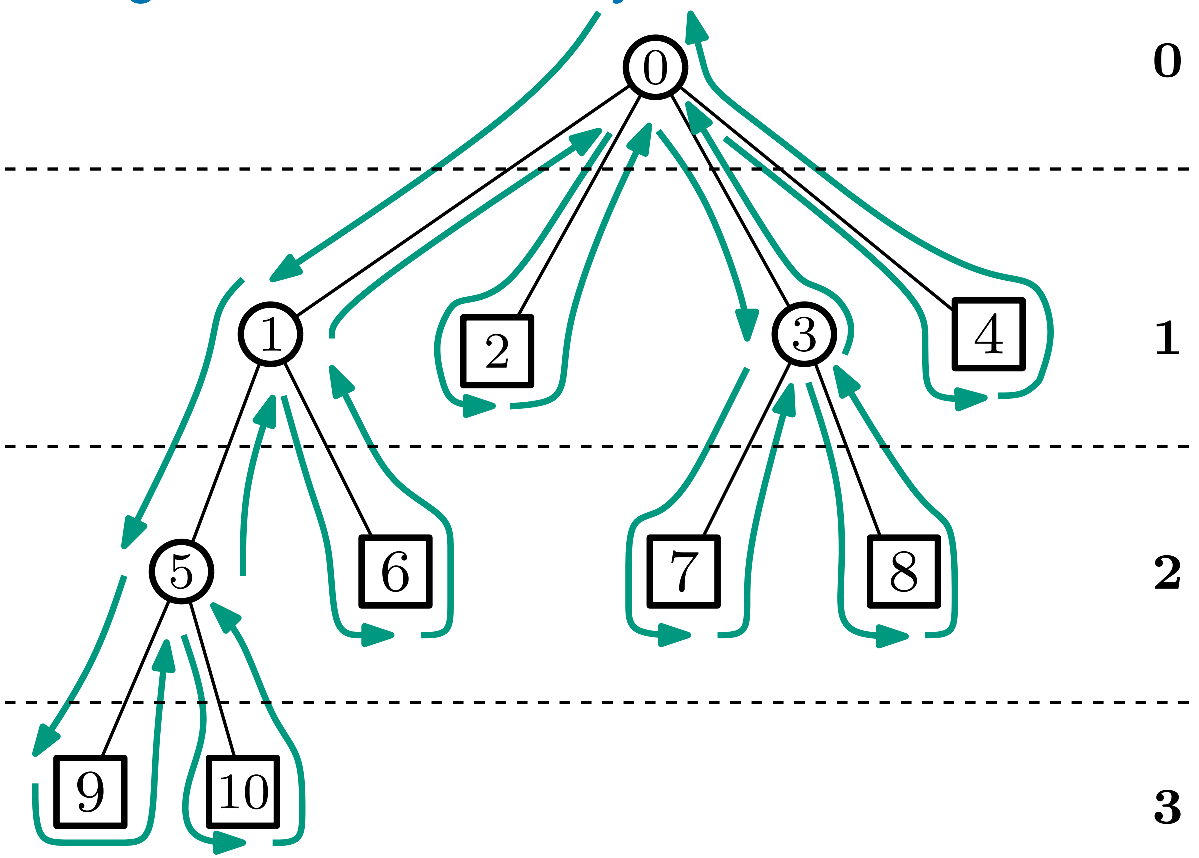

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(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

Notice anything interesting about D?

Solving LCAs using RMQs - efficiency

Preprocessing Summary

1. Construct N and D from T
2. Add a pointer from each node i to some $N[i'] = i$
3. Preprocess D for RMQs

Query Summary - LCA(i,j)

1. Find (any) i' st. $N[i'] = i$
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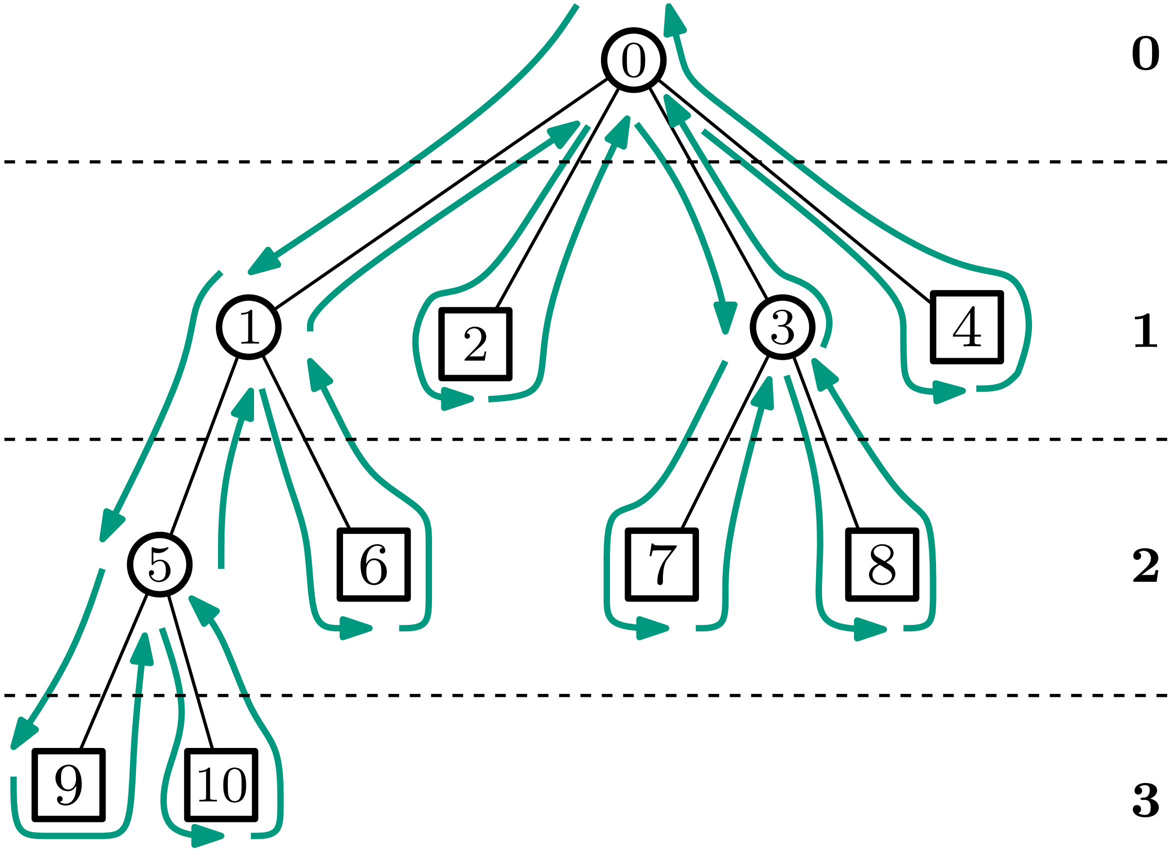


(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0
(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

$2n-1$

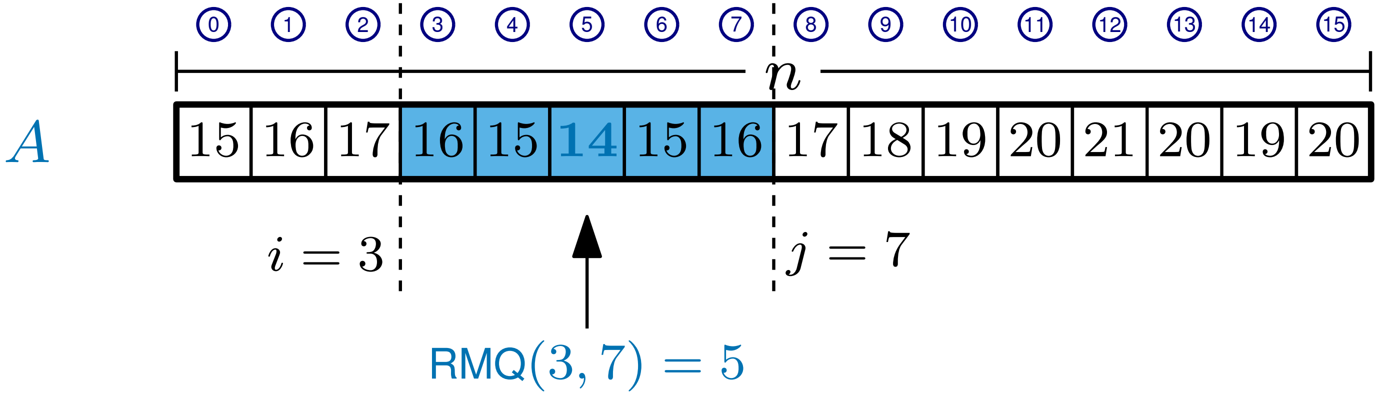
Notice anything interesting about D?

$$\boxed{D[i+1] = D[i] \pm 1}$$

± 1 Range minimum query

Preprocess an integer array A (length n) to answer range minimum queries...

where for all k, we have $A[k+1] = A[k] \pm 1$



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15	16	17	16	15	14	15	16	17	18	19	20	21	20	19	20

After preprocessing, a **range minimum query** is given by RMQ(i, j)

the output is the location of the smallest element in $A[i, j]$

(in a tie, report the leftmost)

e.g. RMQ$(3, 7) = 5$, which is the location of the smallest element in $A[3, 7]$

- Can we exploit this ± 1 property to get a more efficient RMQ data structure?

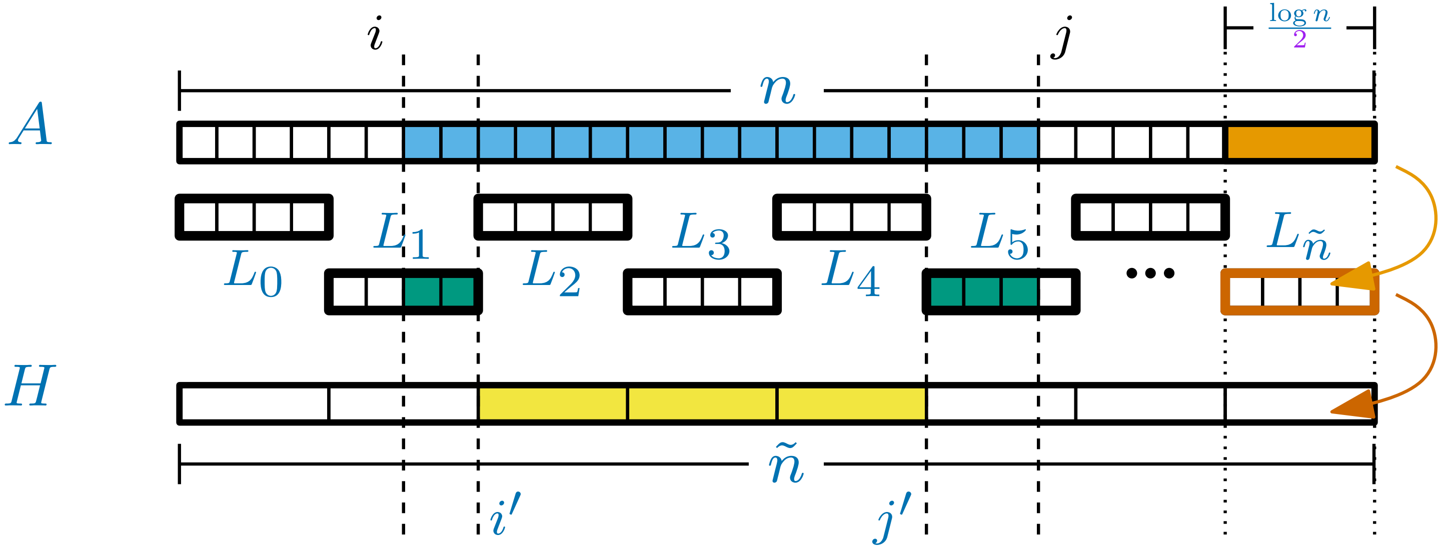
- Ideally we would like $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time

Low-resolution RMQ (again)

$$\tilde{n} = \frac{2n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



all of these go in here

min of **these** goes in here

Preprocess the array H $\left(\text{which has length } \tilde{n} = \frac{2n}{\log n}\right)$ to answer RMQs. . .

in $O(n)$ space/prep time

Preprocess each array L_i (which has length $(\log n)/2$) to answer RMQs. . .

in $O(\log n \log \log n)$ space/prep time

as there are $O(n/\log n)$ L_i arrays, we have $O(n \log \log n)$ total space/prep time

---

How do we answer a query in A in $O(1)$ time?

Do one query in H and one query in two different L_i and return the smallest

Low-resolution RMQ (again)

$$\boxed{\tilde{n} = \frac{2n}{\log n}}$$

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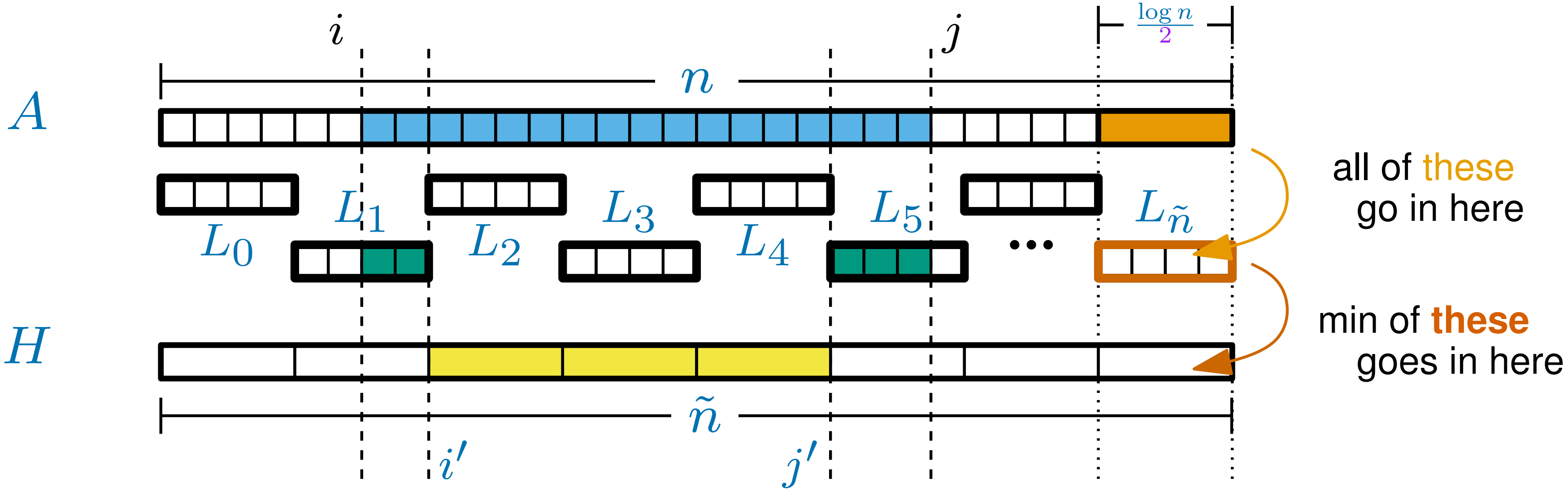

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Preprocess each array L_i (which has length $(\log n)/2$) to answer RMQs...

too big and slow! → in $O(\log n \log \log n)$ space/prep time

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How do we answer a query in A in $O(1)$ time?

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Counting ± 1 RMQ arrays

How many different ± 1 RMQ arrays like this. . . $\overset{L}{\underbrace{\boxed{\quad\quad\quad}}_{\frac{\log n}{2}}}$ are there?

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We say that $\overset{L_x}{\underset{\vdash\ \frac{\log n}{2}\ \dashv}{\boxed{}}}$ is equivalent to $\overset{L_y}{\underset{\vdash\ \frac{\log n}{2}\ \dashv}{\boxed{}}}$ iff for all (i, j): $\text{RMQ}_x(i, j) = \text{RMQ}_y(i, j)$

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L_x

0	1	2	3	4
16	15	14	15	14

is equivalent to

L_y

0	1	2	3	4
10	9	8	9	8

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16	15	14	15	14

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L_y

0	1	2	3	4
10	9	8	9	8

RMQ$_x(0, 2) =$ RMQ$_y(0, 2) = 2$

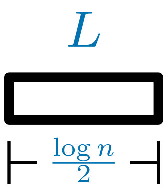
RMQ$_x(3, 4) =$ RMQ$_y(3, 4) = 4$

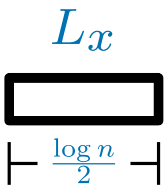
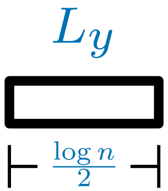
RMQ$_x(0, 4) =$ RMQ$_y(0, 4) = 2$

RMQ$_x(0, 1) =$ RMQ$_y(0, 1) = 1$

$\vdots$

Counting ± 1 RMQ arrays

How many different ± 1 RMQ arrays like this. . .  are there?

We say that  is equivalent to  iff for all (i, j): RMQ$_x(i, j) =$ RMQ$_y(i, j)$

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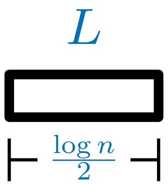
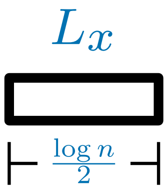
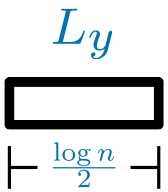
How many different ± 1 RMQ arrays like this...  are there?

We say that  is equivalent to  iff for all (i,j): RMQ$_x(i,j)$ = RMQ$_y(i,j)$

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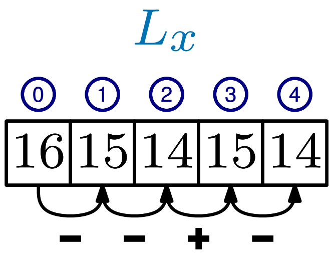

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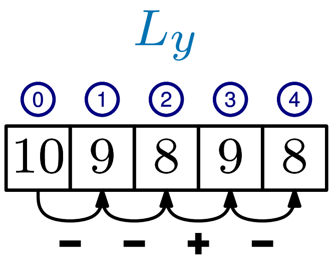

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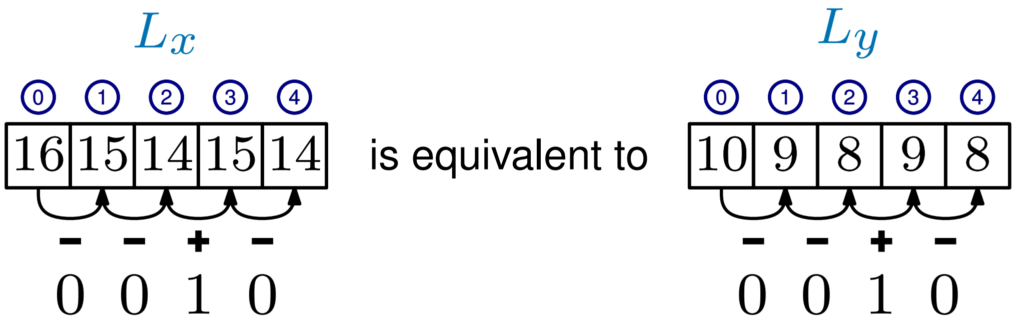

L_x

0	1	2	3	4
16	15	14	15	14

− − + −

0 0 1 0

is equivalent to

L_y

0	1	2	3	4
10	9	8	9	8

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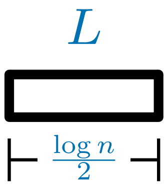
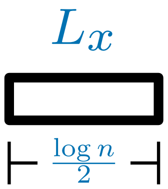
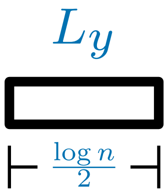
Counting ± 1 RMQ arrays

How many different ± 1 RMQ arrays like this…  are there?

We say that  is equivalent to  iff for all (i, j): RMQ$_x(i, j) =$ RMQ$_y(i, j)$

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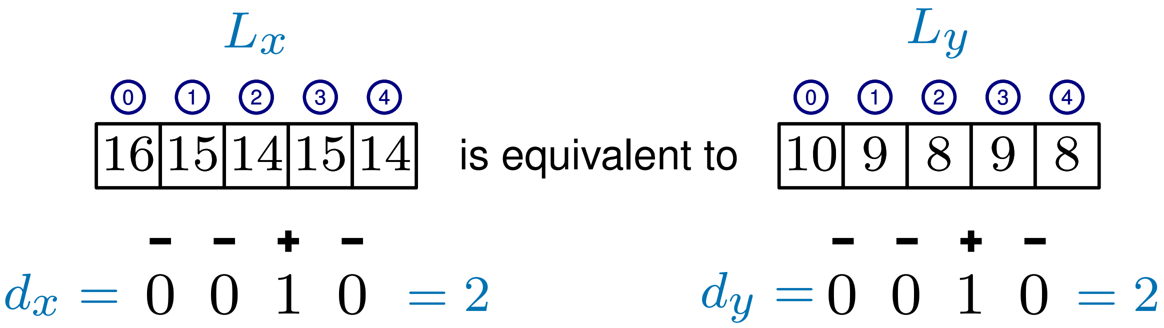

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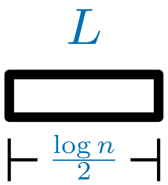
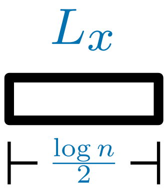
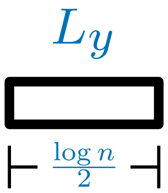
How many different ±1 RMQ arrays like this…  are there?

We say that  is equivalent to  iff for all (i, j): RMQ$_x(i, j) =$ RMQ$_y(i, j)$

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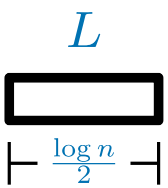
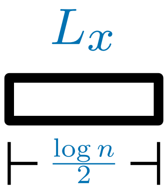
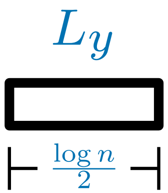
How many different ± 1 RMQ arrays like this...  are there?

We say that  is equivalent to  iff for all (i,j): RMQ$_x(i,j) =$ RMQ$_y(i,j)$

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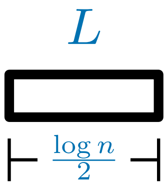
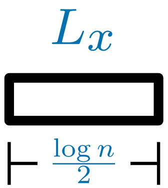
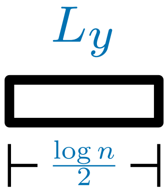
How many different ± 1 RMQ arrays like this...  are there?

We say that  is equivalent to  iff for all (i, j): RMQ$_x(i, j)$ = RMQ$_y(i, j)$

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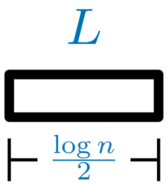
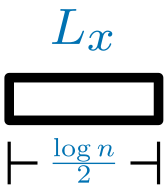
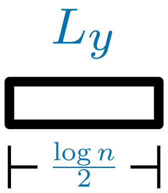
How many different ± 1 RMQ arrays like this...  are there?

We say that  is equivalent to  iff for all (i,j): RMQ$_x(i,j) =$ RMQ$_y(i,j)$

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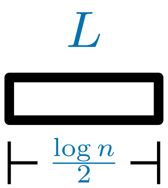
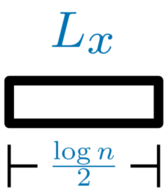
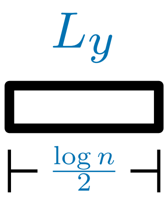
How many different ± 1 RMQ arrays like this...  are there?

We say that  is equivalent to  iff for all (i, j): RMQ$_x(i, j) =$ RMQ$_y(i, j)$

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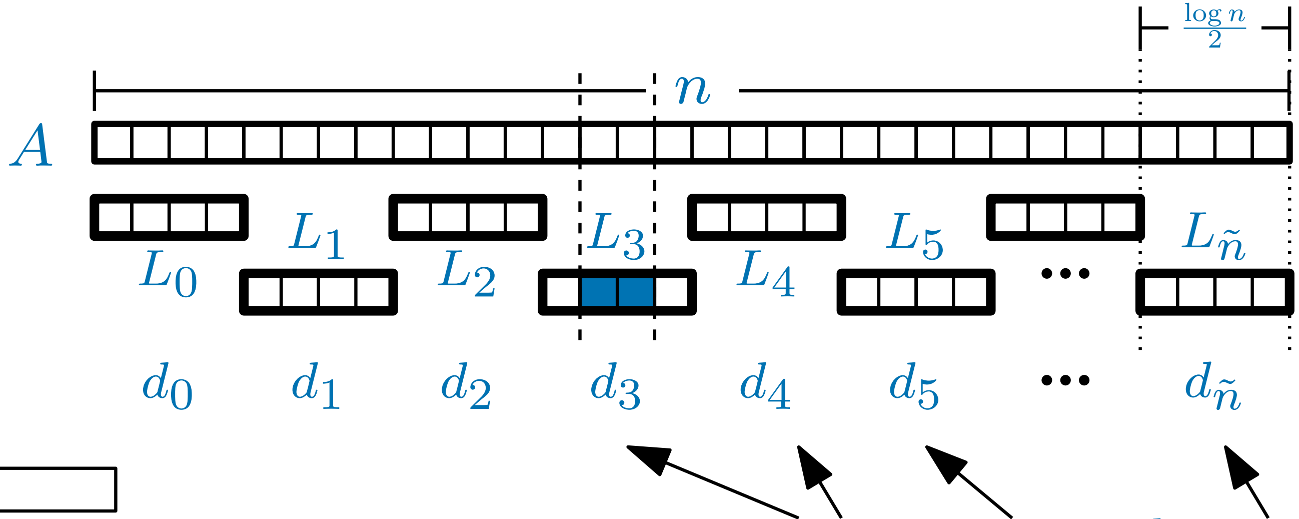
...this requires $O(\sqrt{n} \log^2 n) = O(n)$ total space and prep. time

RMQ on the L arrays in linear space

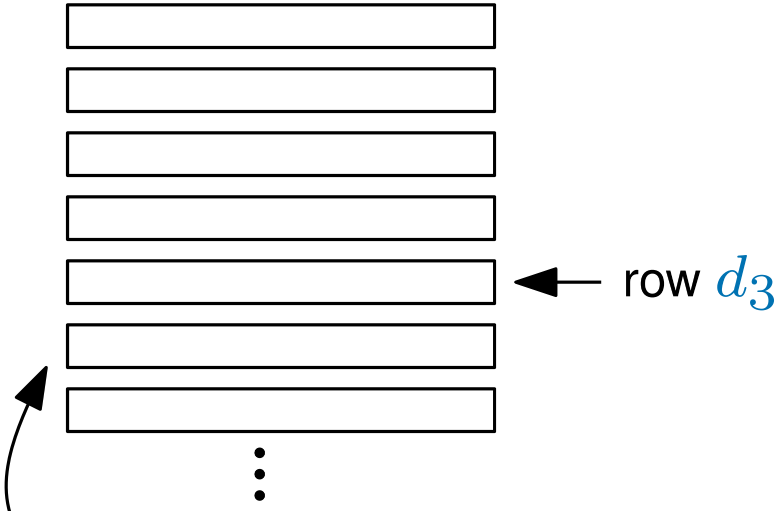
$$\boxed{\tilde{n} = \frac{2n}{\log n}}$$

Key Idea replace A with a smaller, *'low resolution'* array H



precompute the value of d_x for each L_x

in $O(n)$ total space and prep. time



Precompute all the RMQ answers for

every value $0 \leqslant d \leqslant \sqrt{n}$

in $O(n)$ total space and prep. time

To perform a query within some L_x

- Look up d_x
- Find the row d_x in the table
- Find the entry giving $\text{RMQ}_x(i, j)$

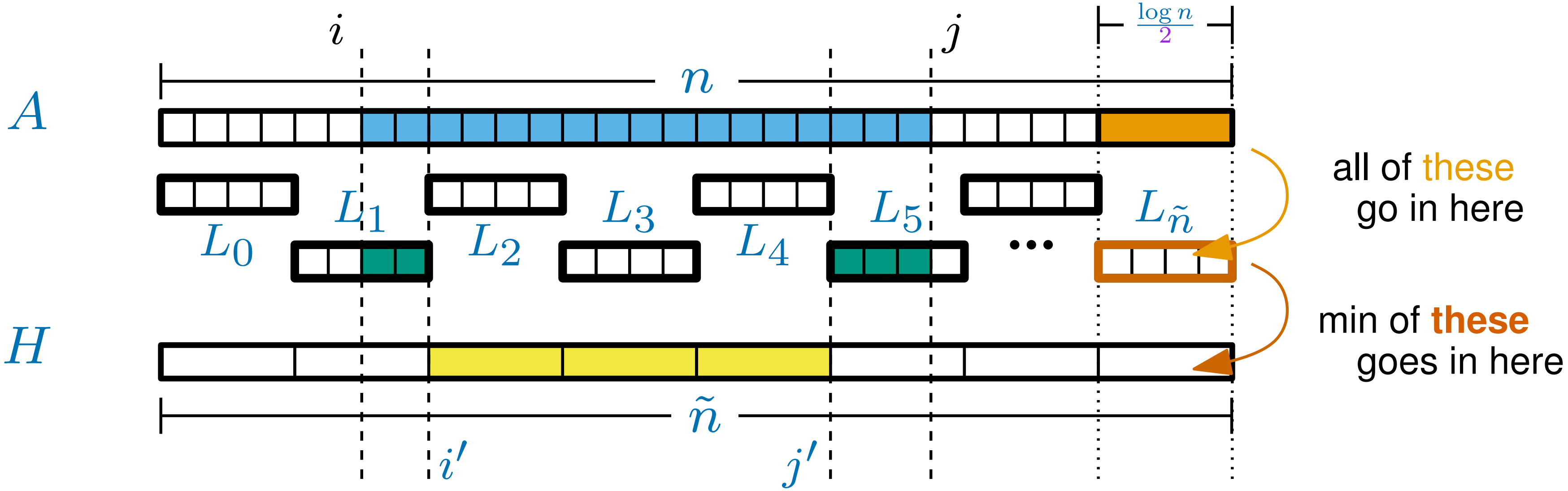
This takes $O(1)$ time

Optimal ± 1 RMQ

$$\tilde{n} = \frac{2n}{\log n}$$

Key Idea replace A with a smaller, *'low resolution'* array H

and many small arrays $L_0, L_1, L_2 \ldots$ *'for the details'*



Preprocess the array H to answer RMQs...

in $O(n)$ space/prep time

Preprocess each array L_i (which has length $(\log n)/2$) to answer RMQs...

build a complete table of answers

$O(n)$ total space/prep time

How do we answer a query in A in $O(1)$ time?

Do one query in H and one query in two different L_i and return the smallest

Ongoing Summary

We have seen an $O(n \log \log n)$ space, $O(n \log \log n)$ prep. time and $O(1)$ query time solution

for the Lowest Common Ancestor problem

which uses solution 3 for RMQ *from last lecture*

We have seen an $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time solution

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(but only for ± 1 inputs)

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which improves solution 3 for RMQ *from last lecture*

(but only for ± 1 inputs)

How does this affect our LCA solution?

Solving LCAs using RMQs

Preprocessing Summary

$O(n)$ 1. Construct N and D from T

$O(n)$ 2. Add a pointer from each node i to some $N[i'] = i$

$O(n)$ 3. Preprocess D for RMQs

Query Summary - LCA(i,j)

$O(1)$ 1. Find (any) i' st. $N[i'] = i$

$O(1)$ 2. Find (any) j' st. $N[j'] = j$

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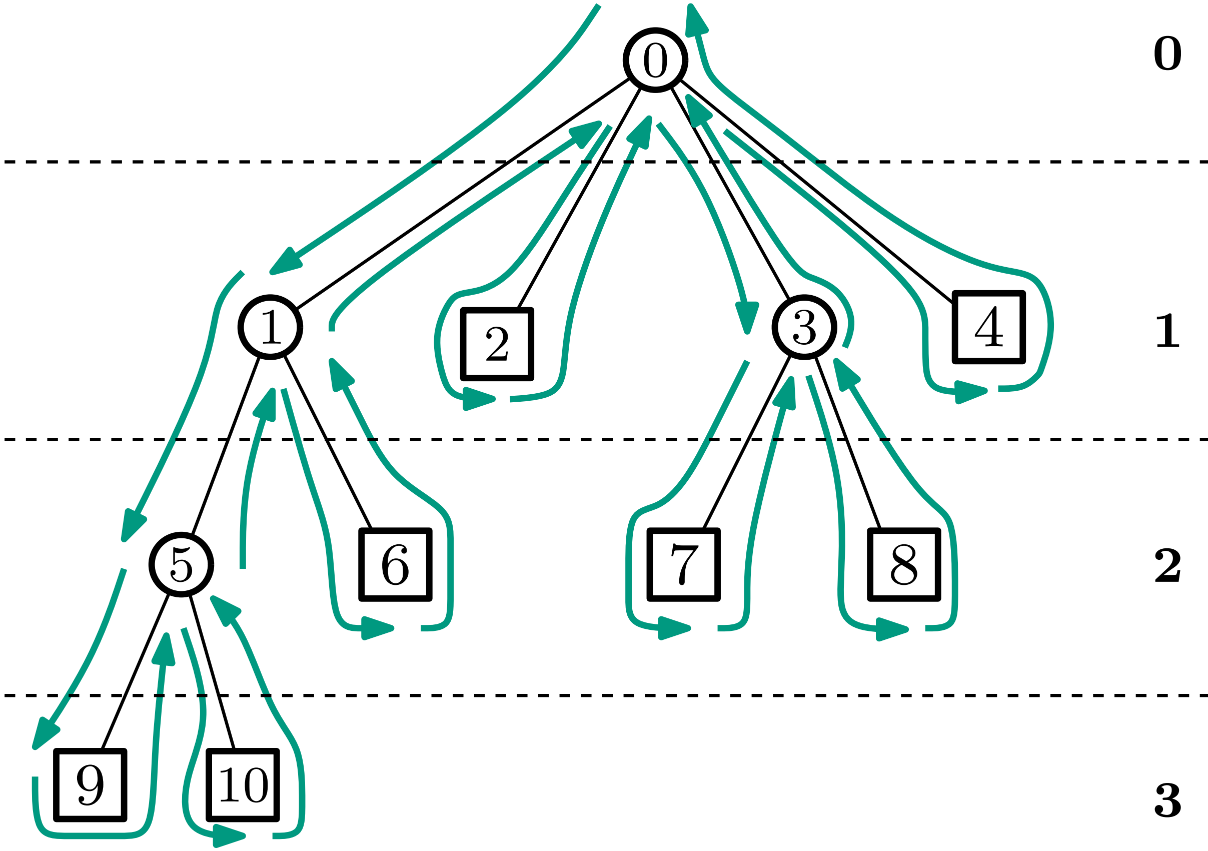
$O(1)$ 4. LCA$(i, j) = N[$RMQ$(i', j')]$



(node) N	0	1	5	9	5	10	5	1	6	1	0	2	0	3	7	3	8	3	0	4	0

$2n-1$

(depth) D	**0**	**1**	**2**	**3**	**2**	**3**	**2**	**1**	**2**	**1**	**0**	**1**	**0**	**1**	**2**	**1**	**2**	**1**	**0**	**1**	**0**

Solving LCAs using RMQs

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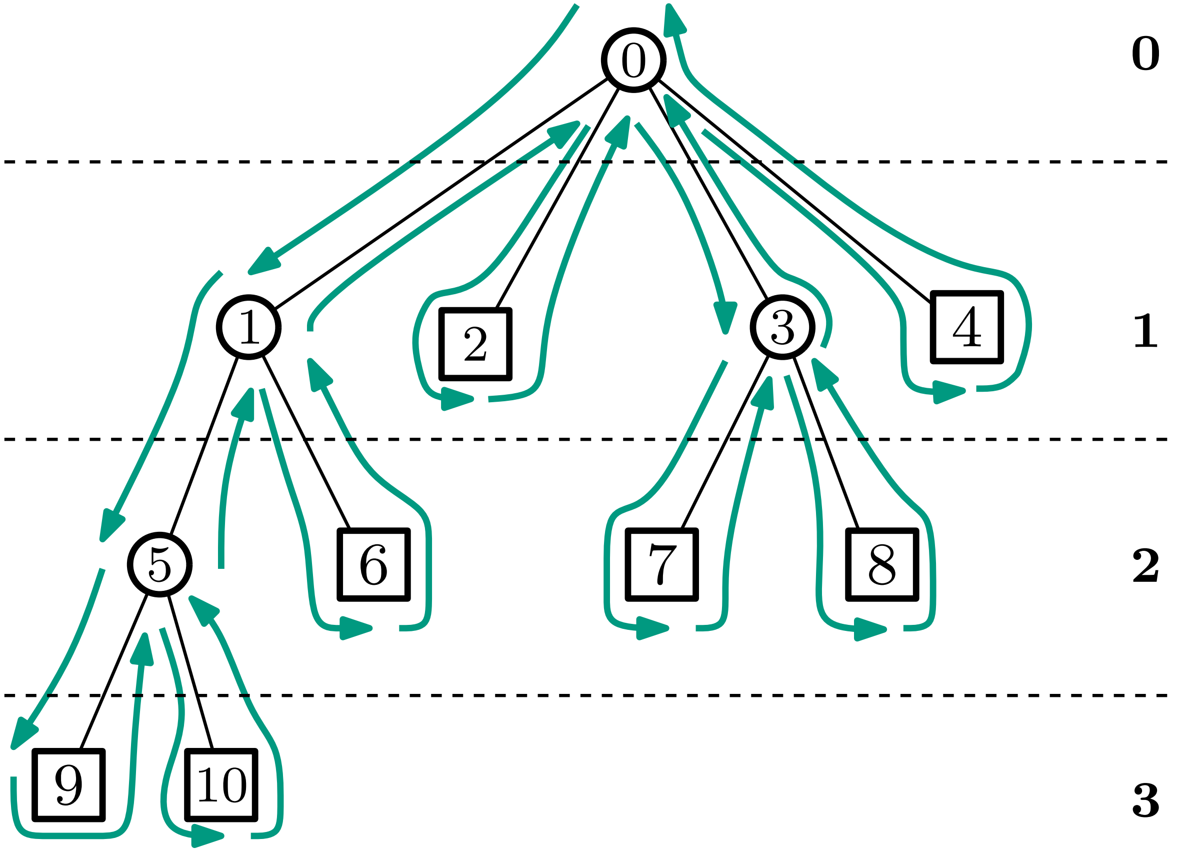

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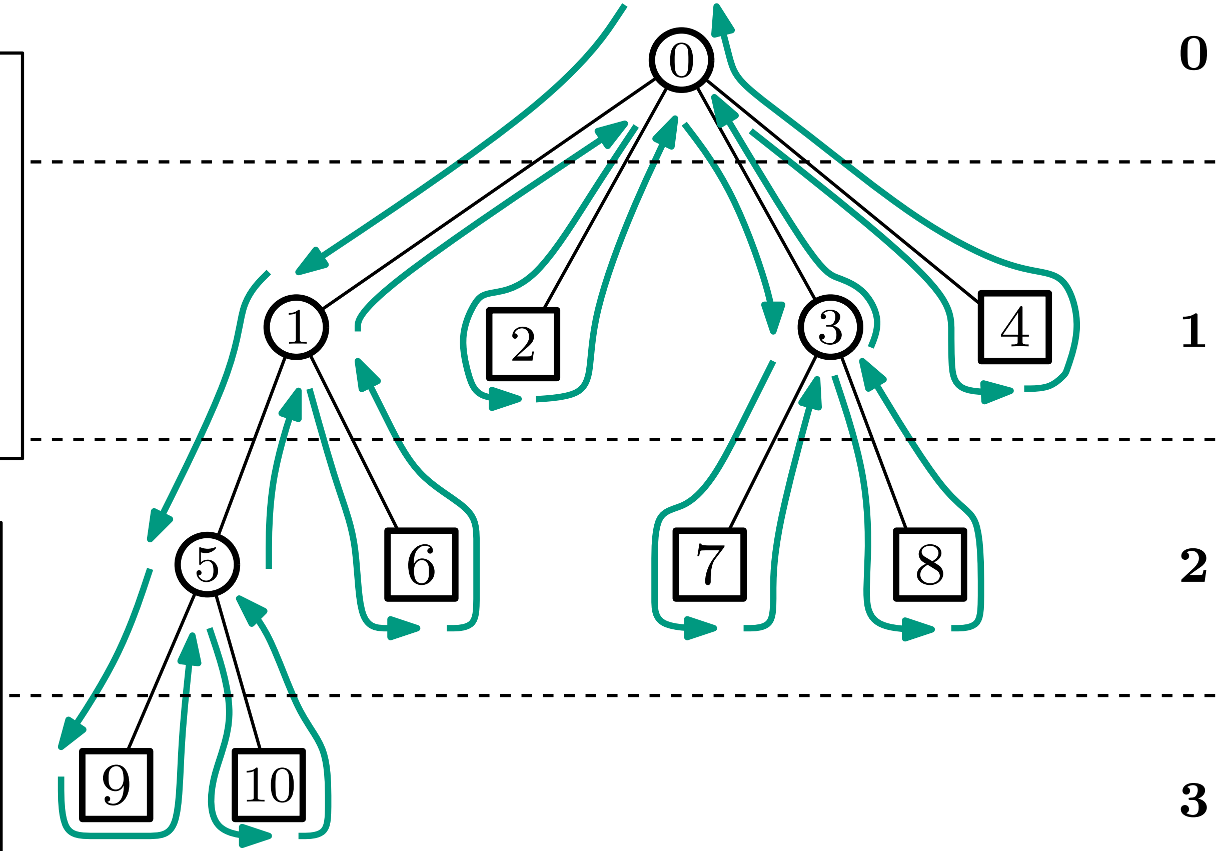

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This gives us $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time for the LCA problem

Solving LCAs using RMQs

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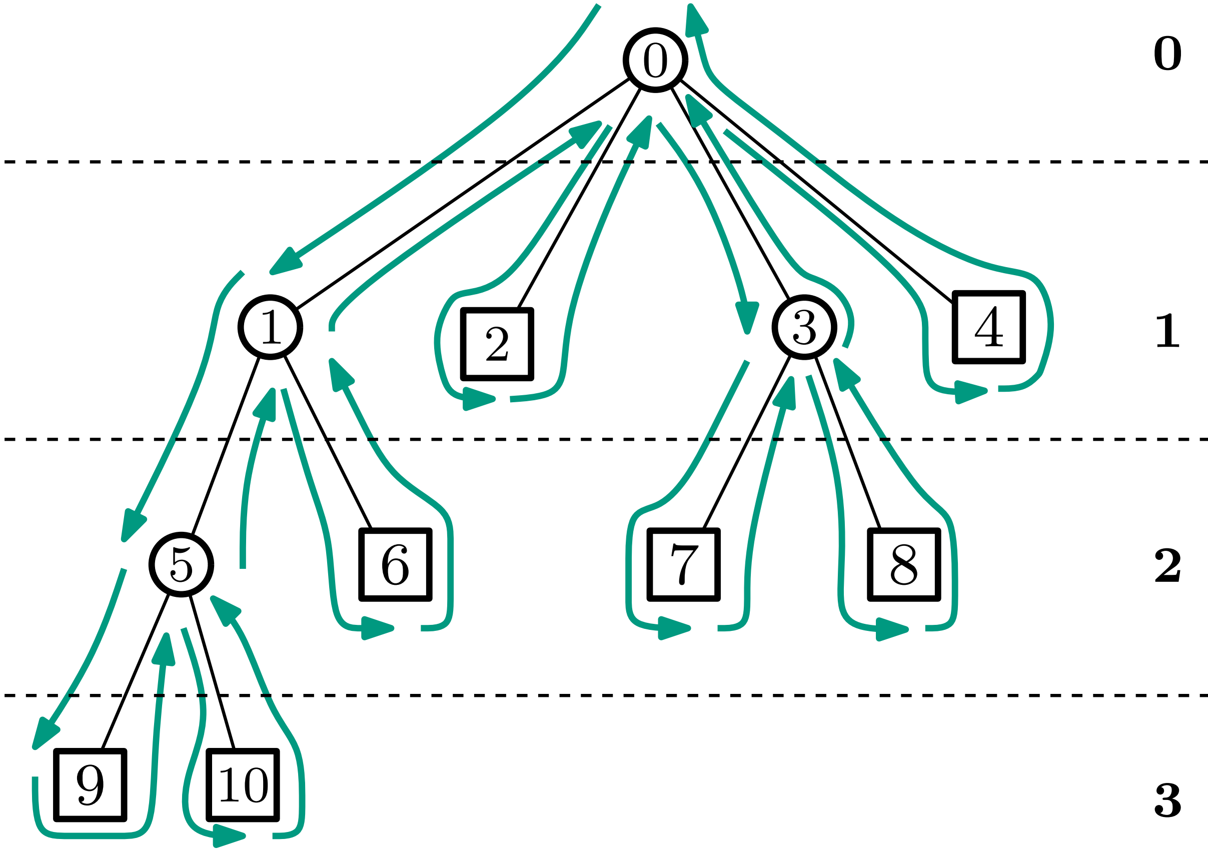

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$\longleftarrow 2n-1 \longrightarrow$

This gives us $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time for the LCA problem

by using the solution to ±1RMQ

Ongoing Summary

We have seen an $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time solution

for the ± 1 Range Minimum Query problem

which improves solution 3 for RMQ *from last lecture*

(but only for ± 1 inputs)

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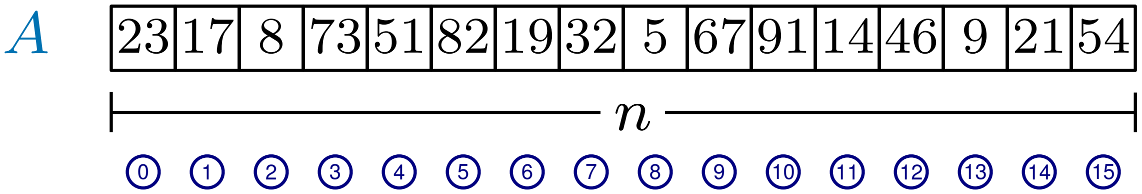
What about the general Range Minimum Query problem?

(when the inputs aren't ± 1)

Solving RMQs using LCAs

Build the Cartesian tree, T_A of the array A:



A															
23	17	8	73	51	82	19	32	5	67	91	14	46	9	21	54
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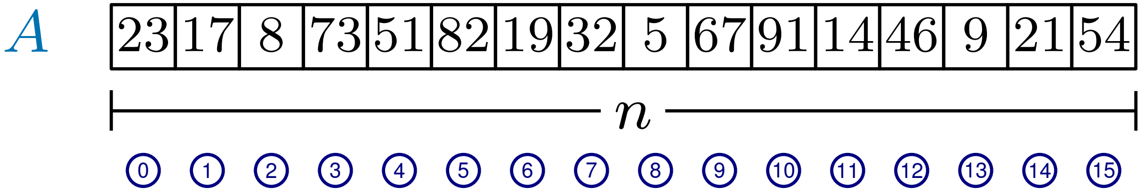

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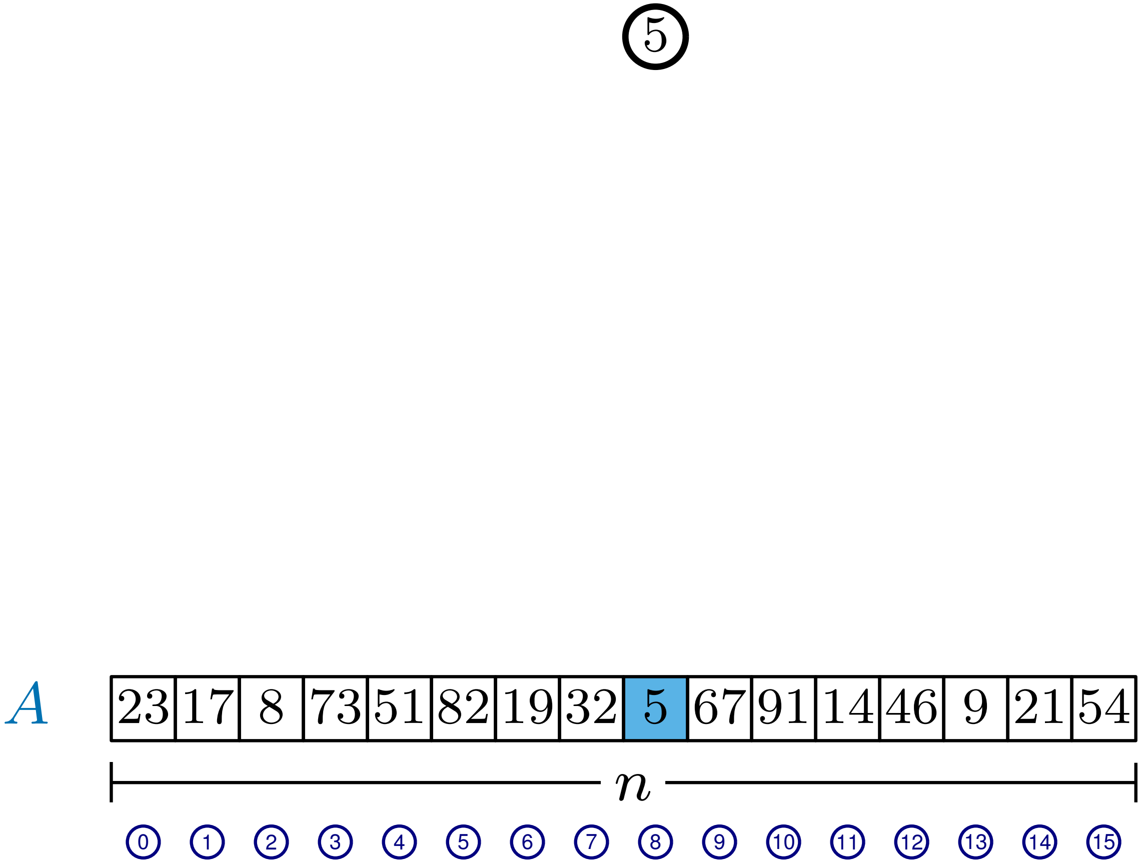

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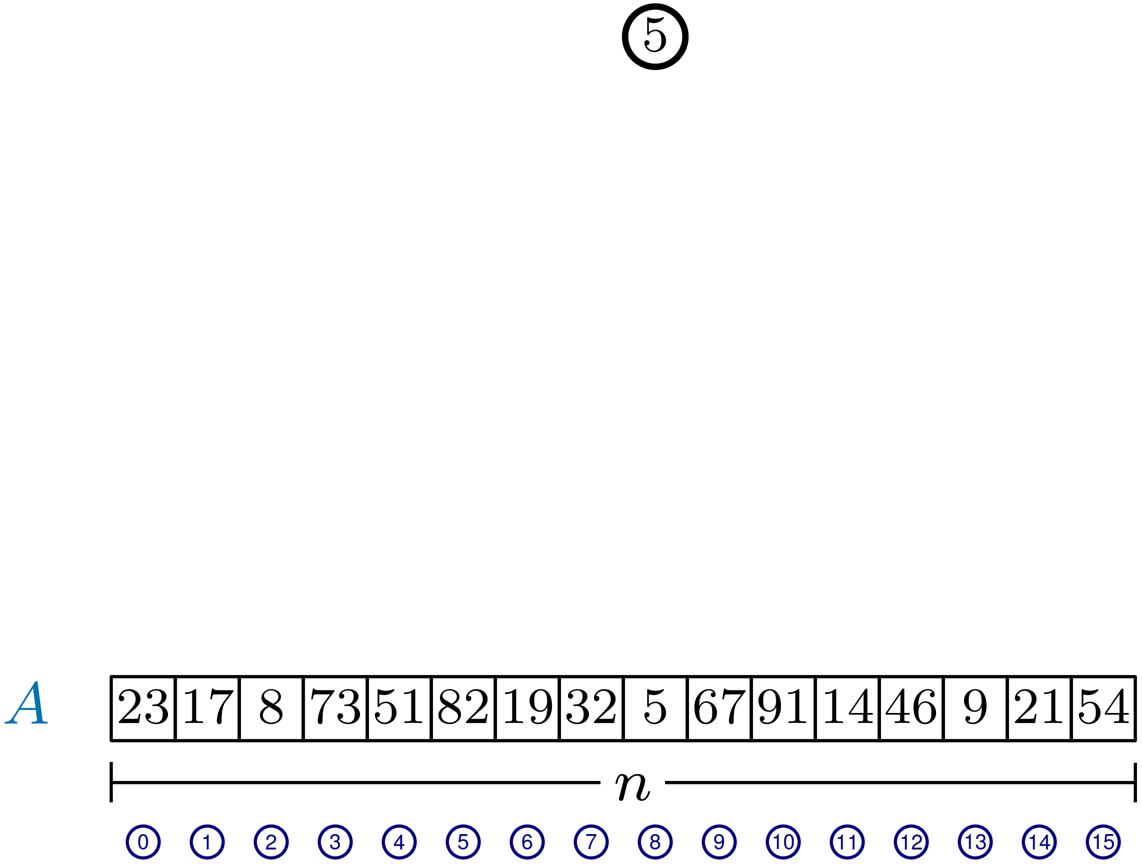

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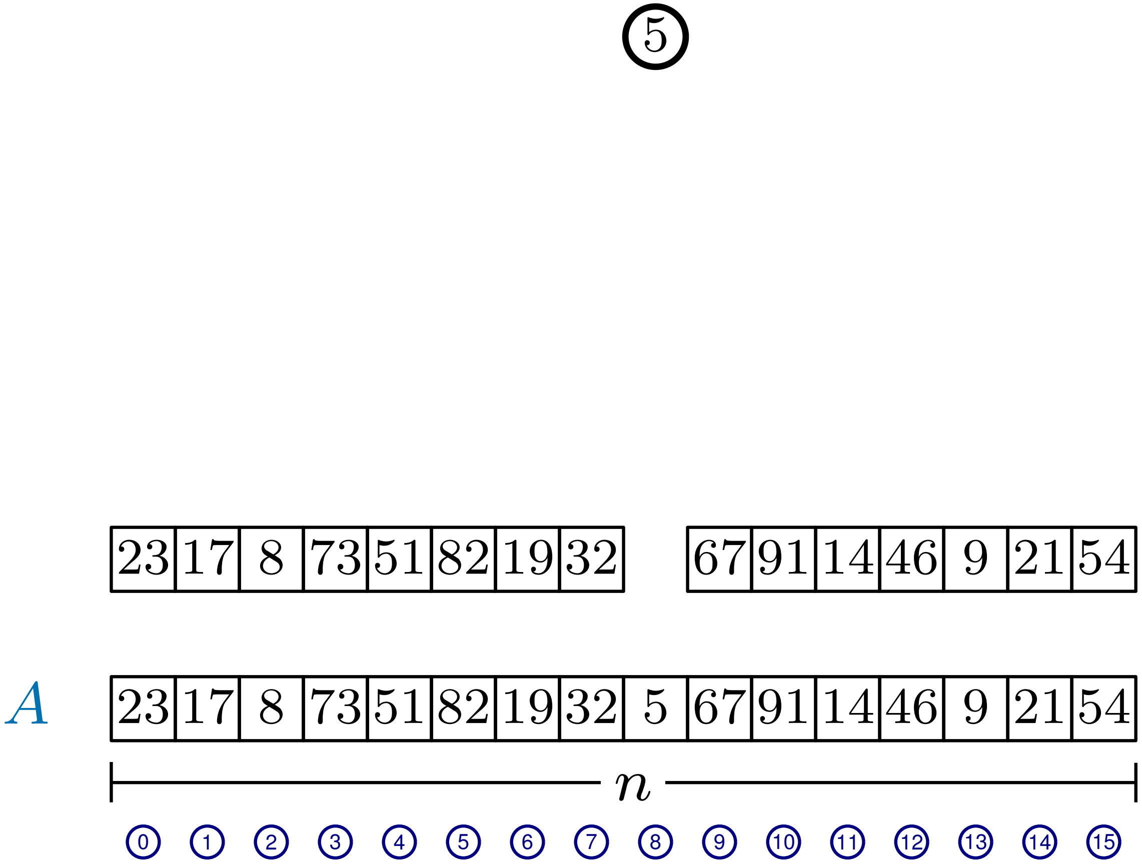

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Build the Cartesian tree, T_A of the array A:

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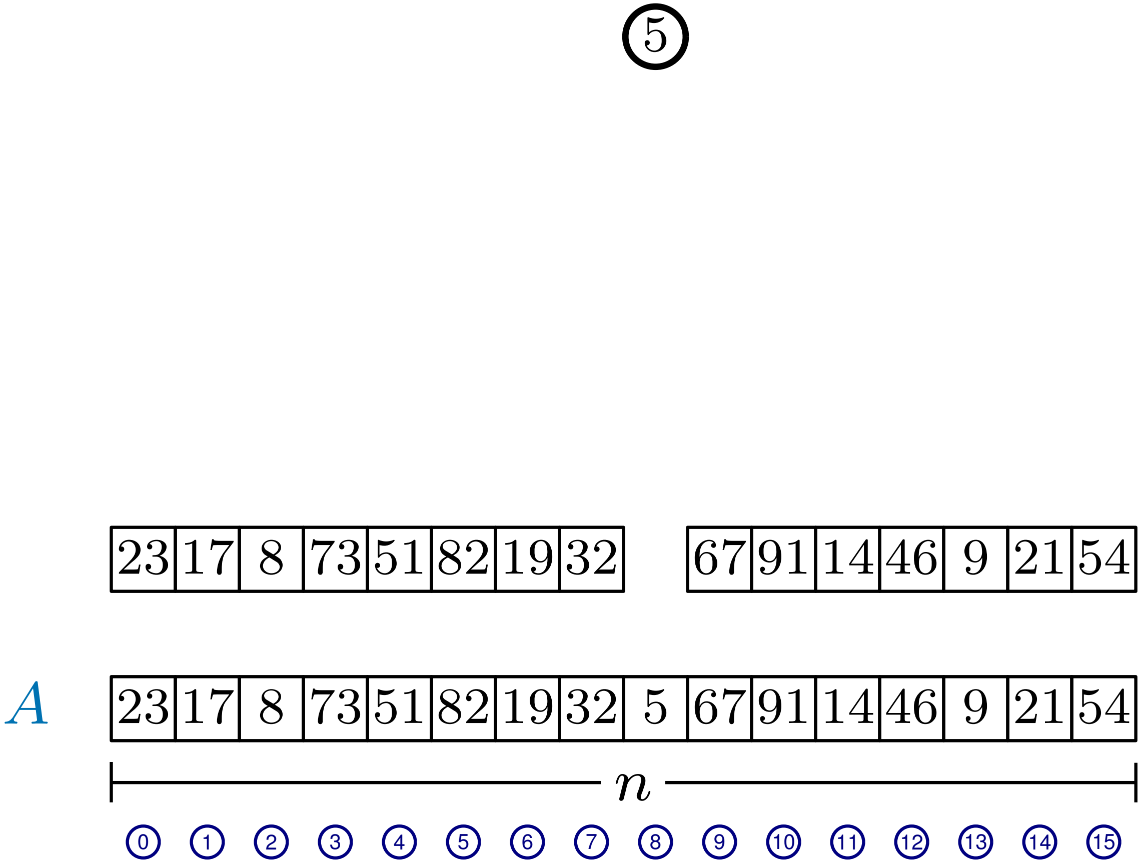

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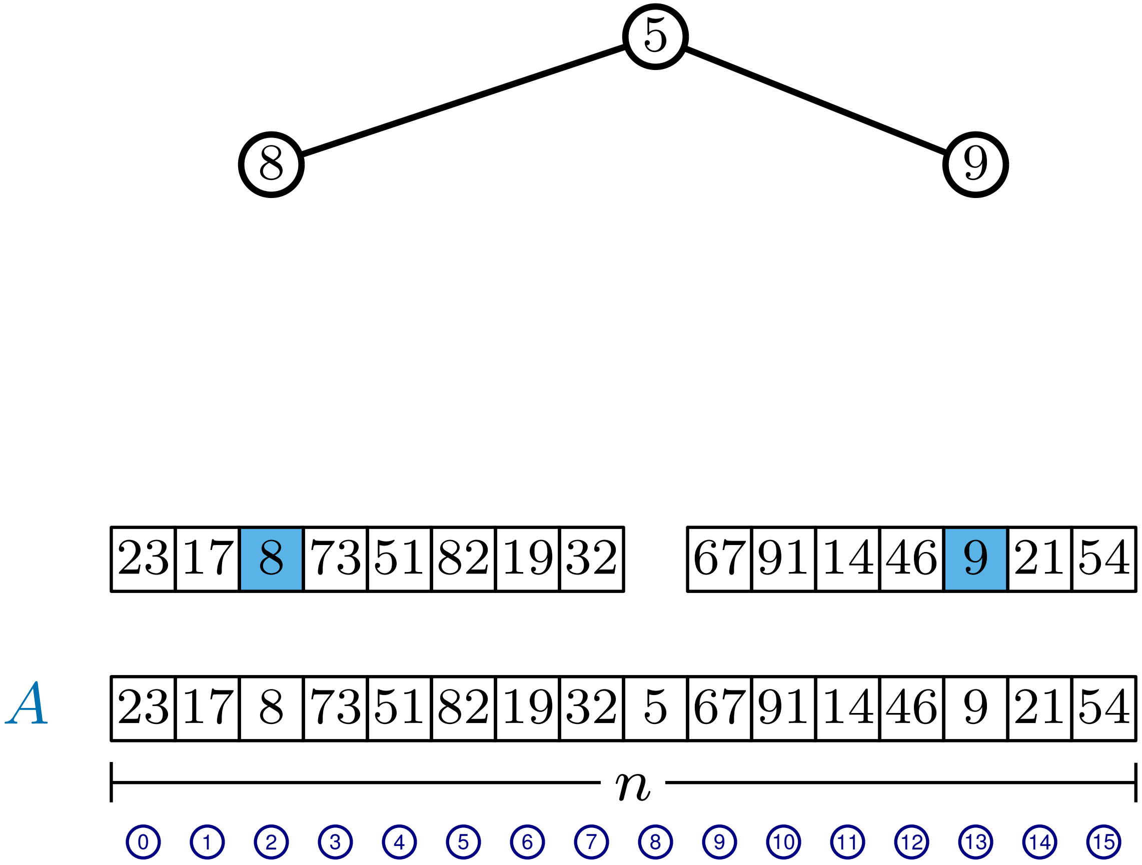

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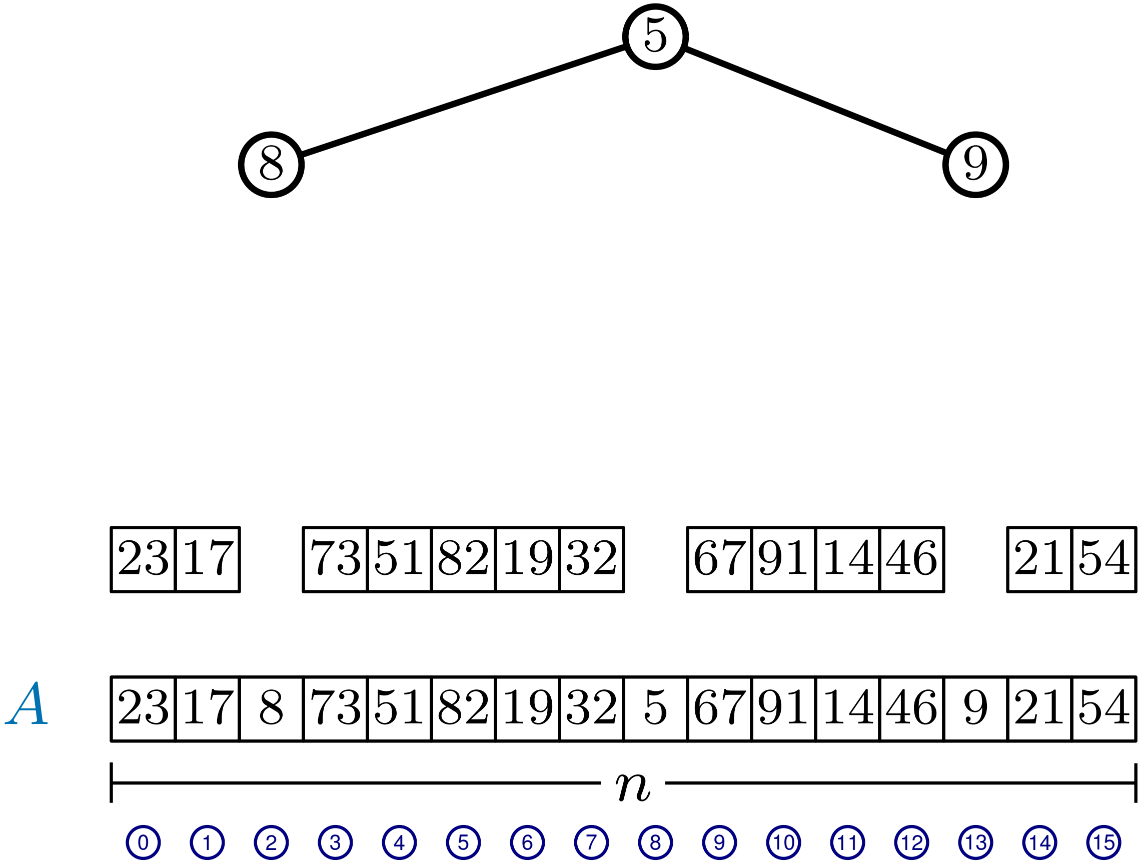

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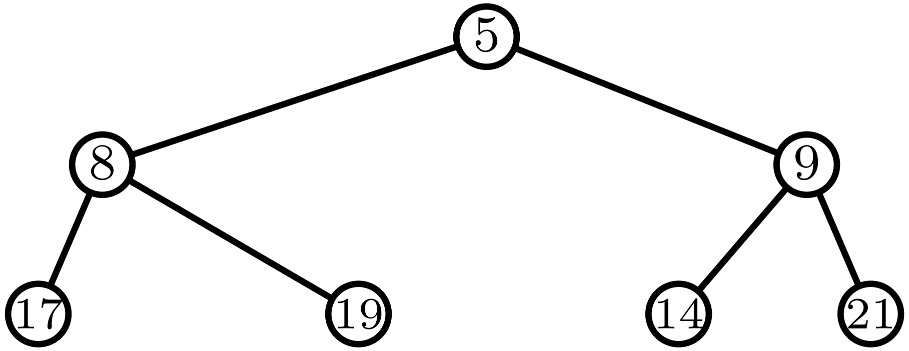

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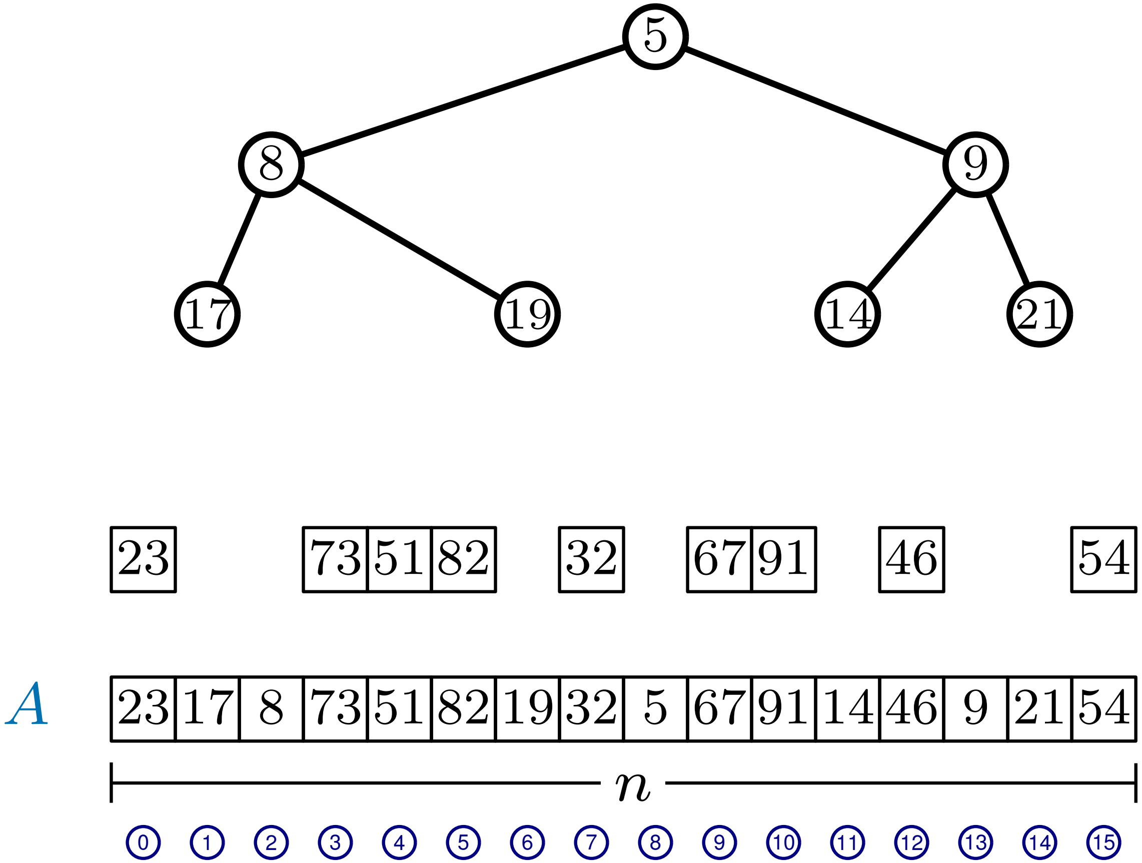

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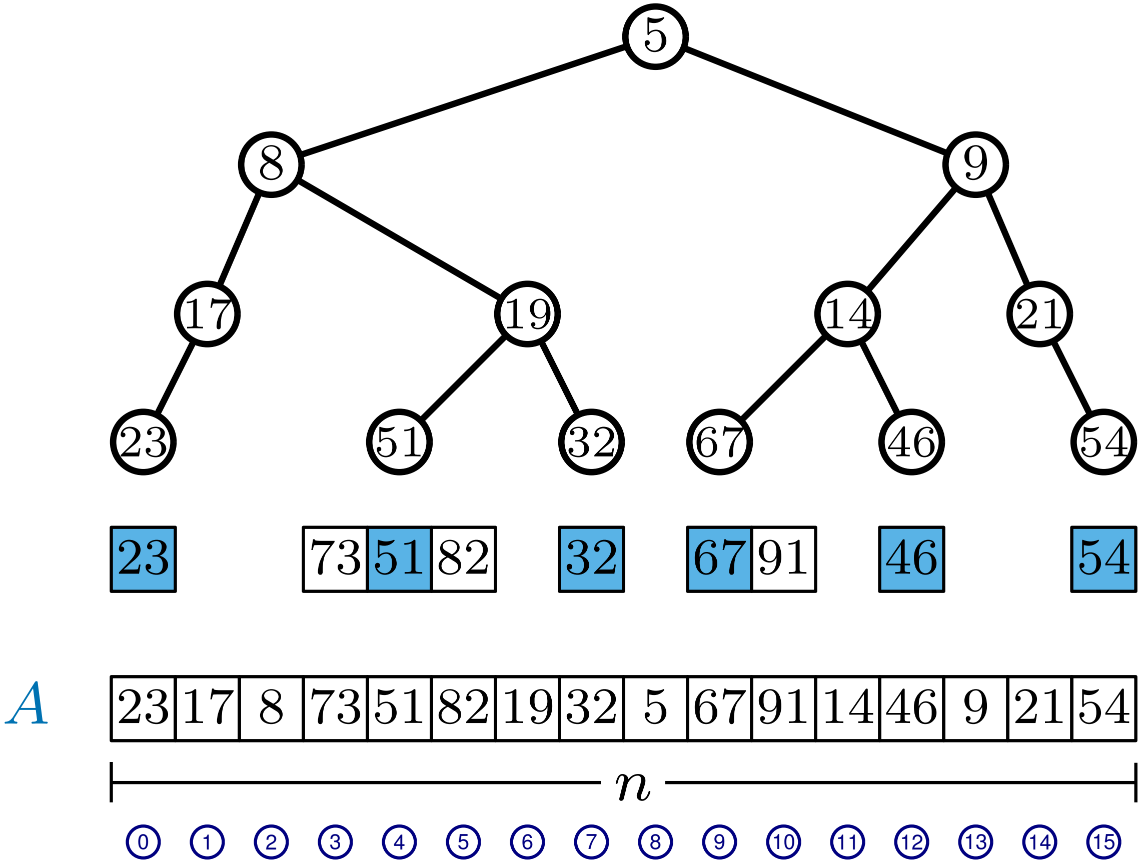

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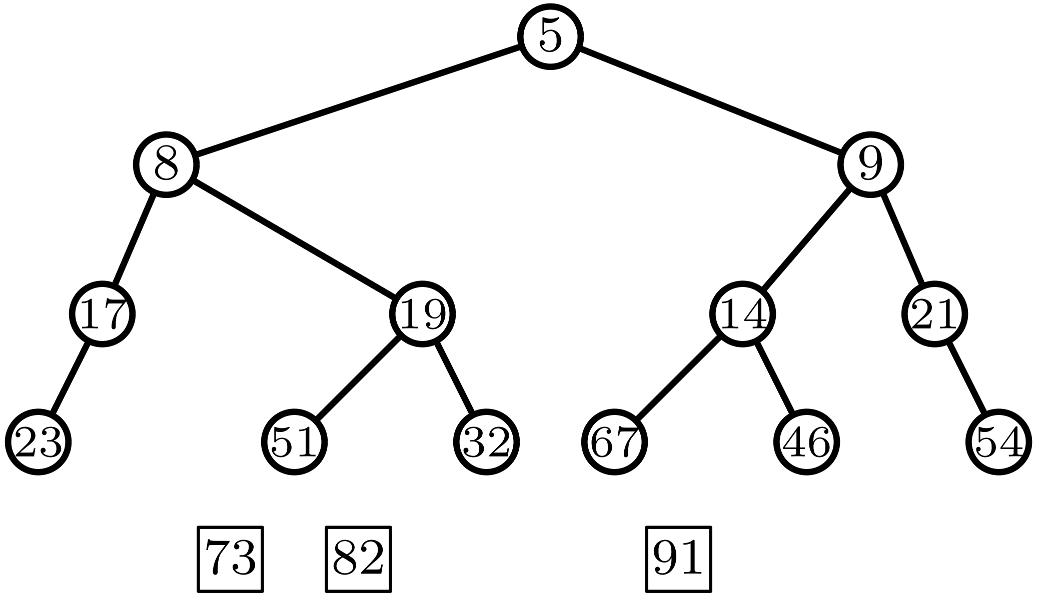

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$\underbrace{\hspace{10em}}_{n}$

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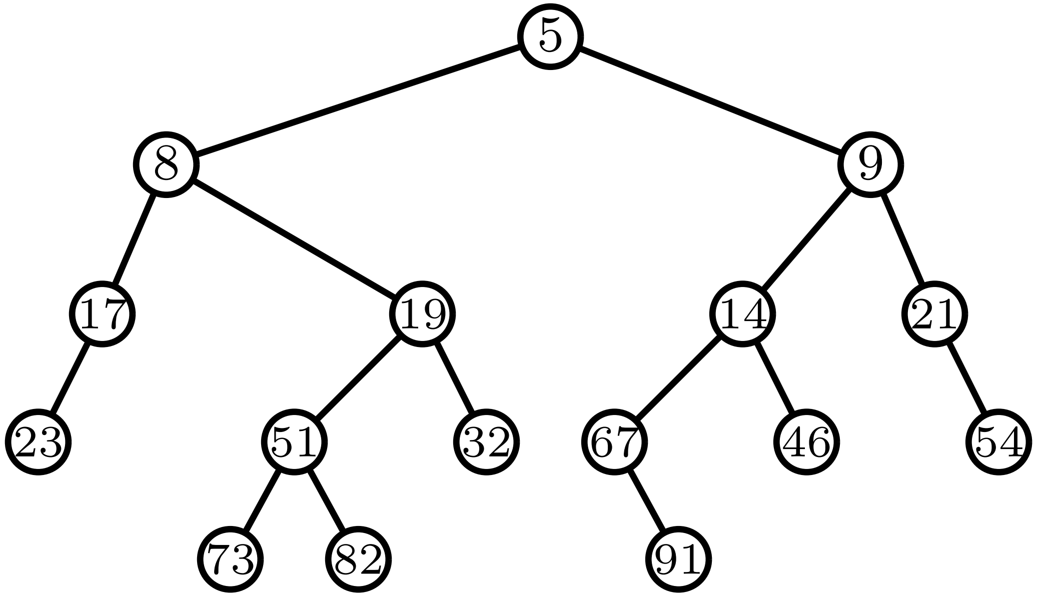

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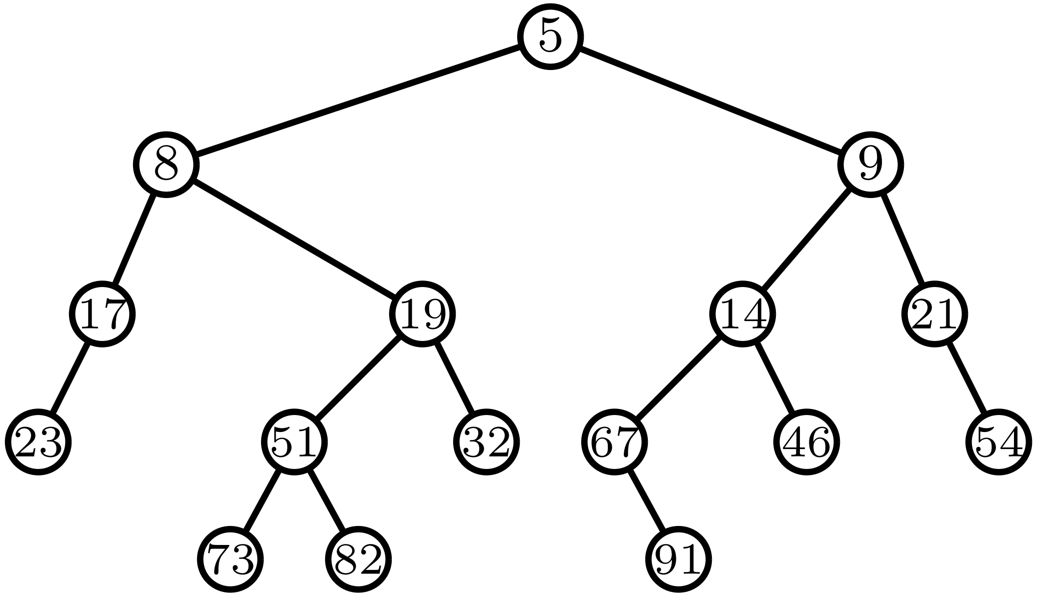
This process isn't very efficient. . .
a better one takes $O(n)$ time



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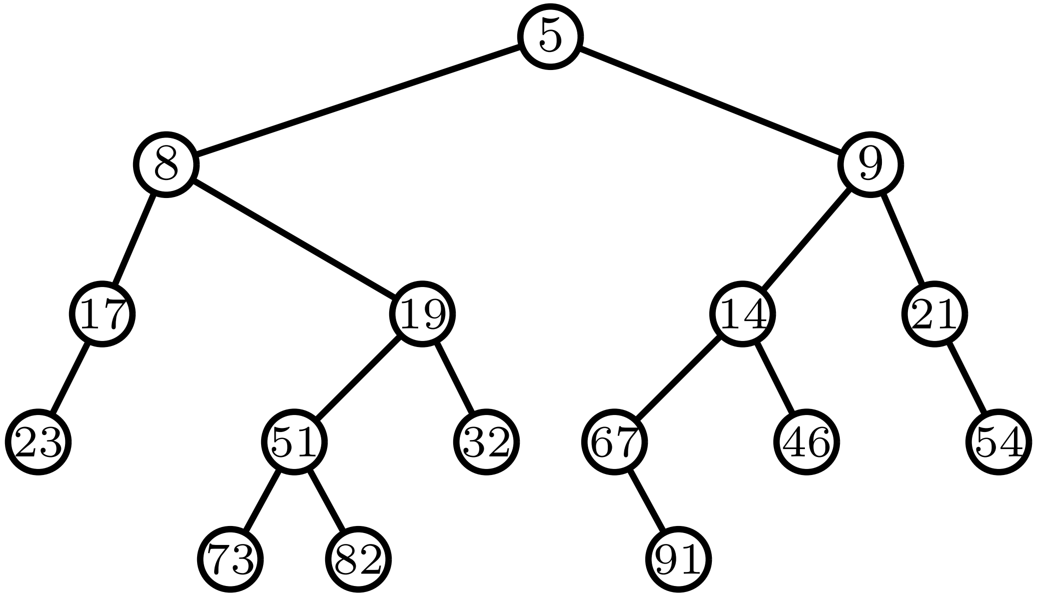

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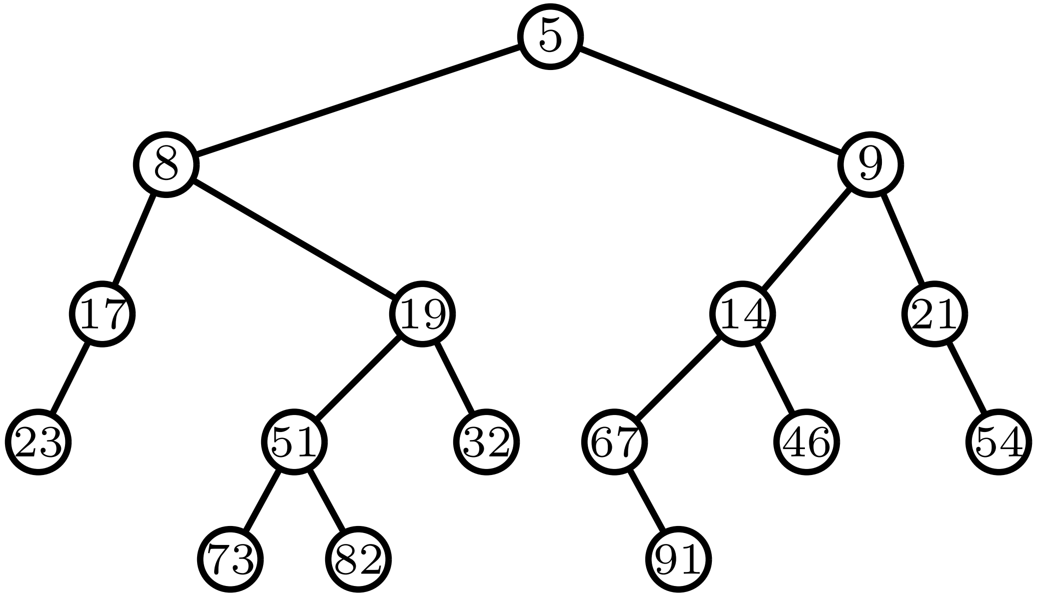

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Key Fact: The LCA in T_A equals the RMQ in A

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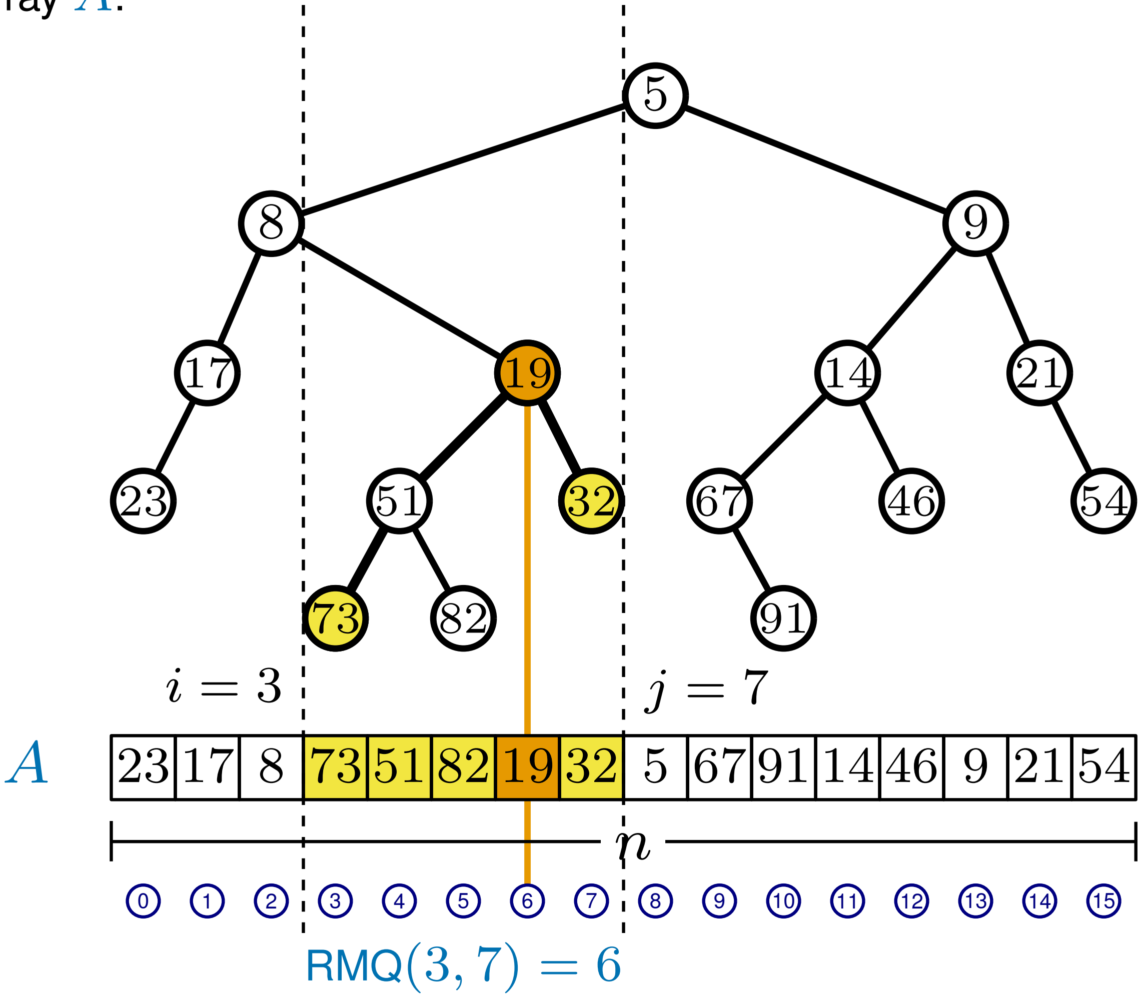

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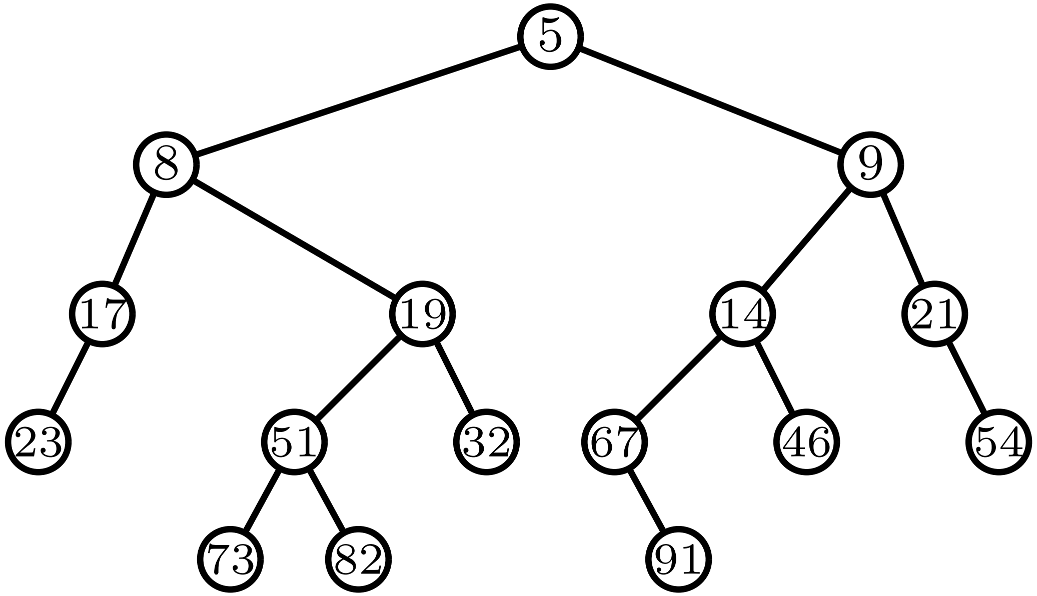

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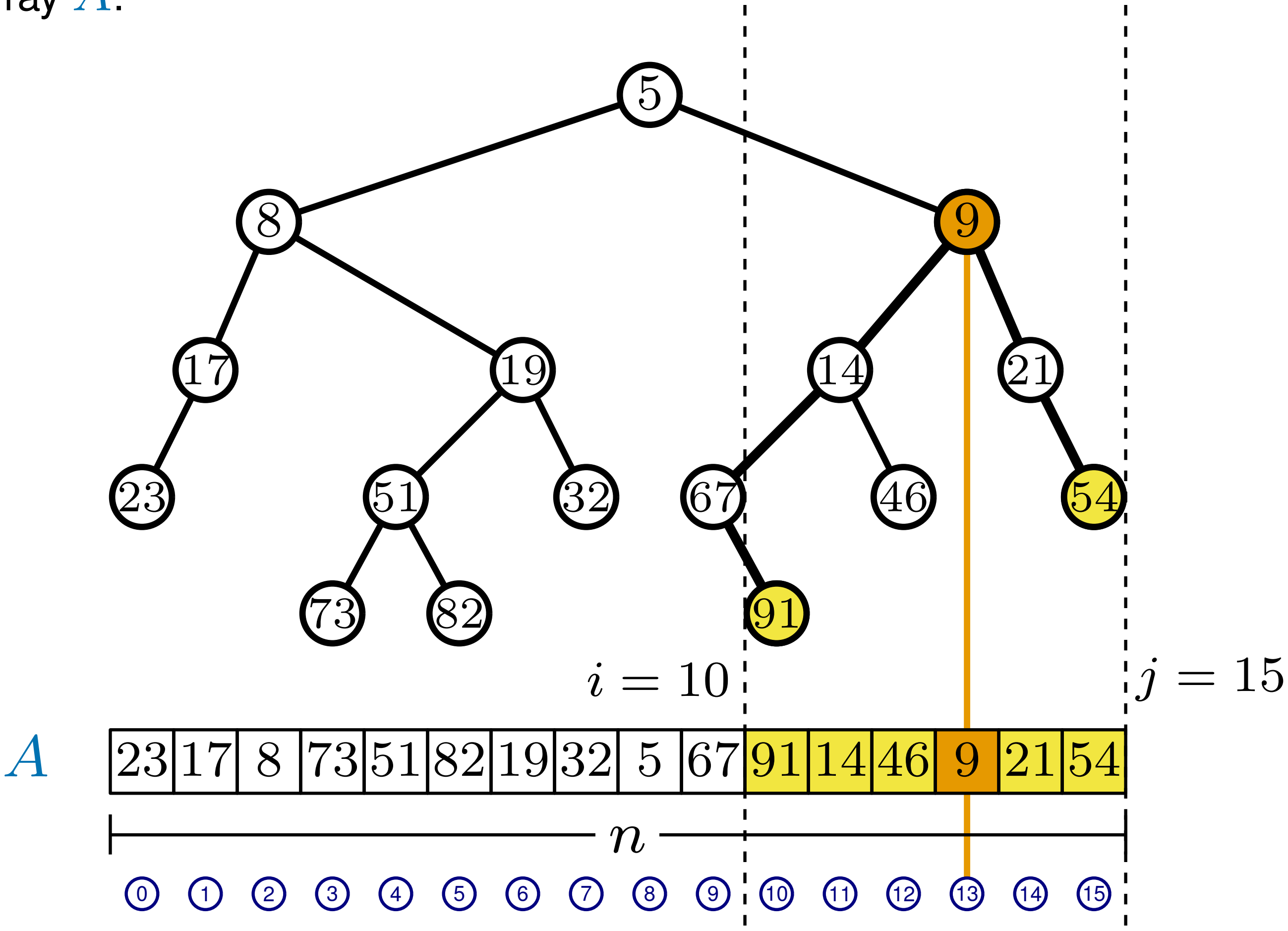

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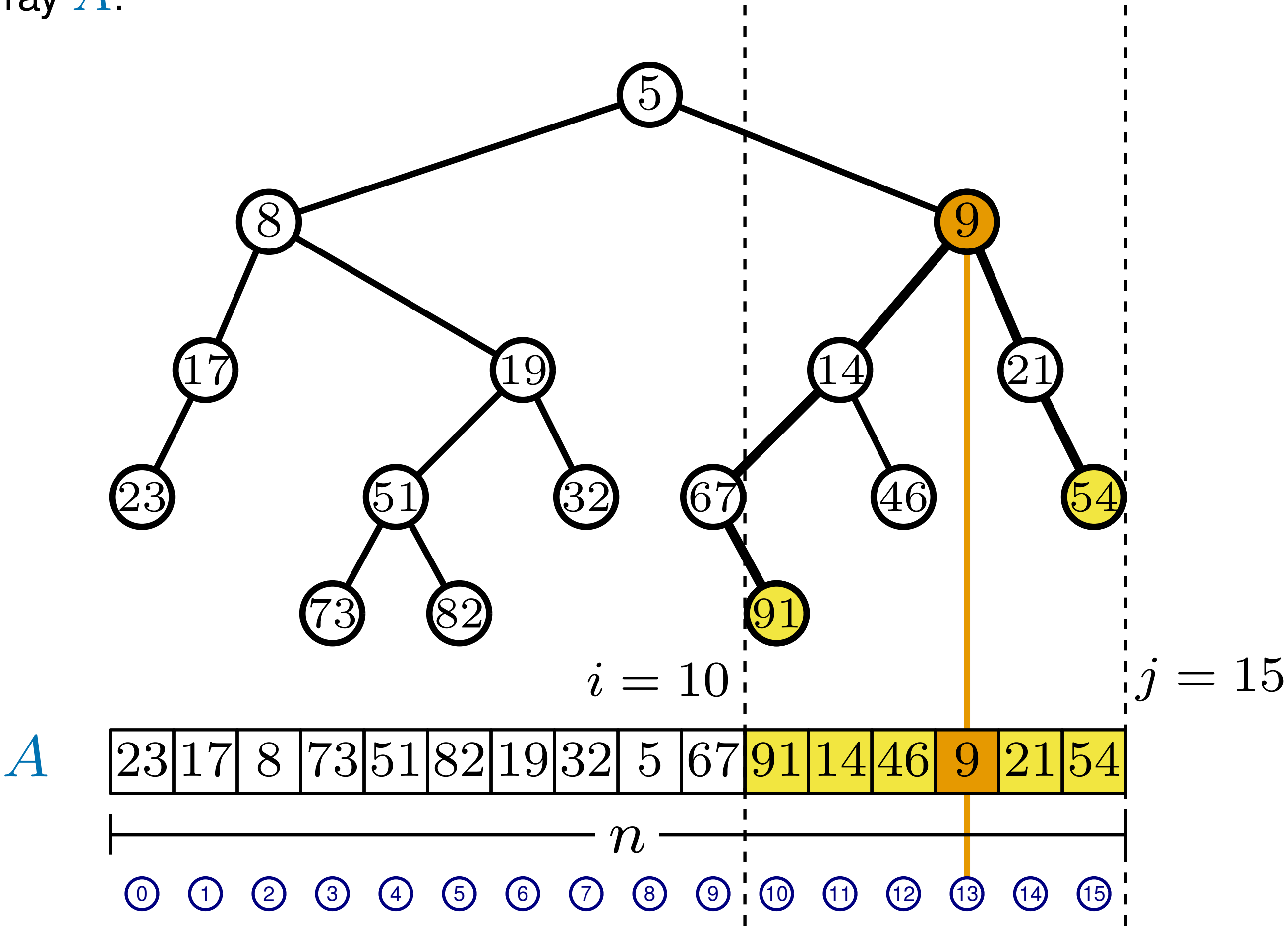

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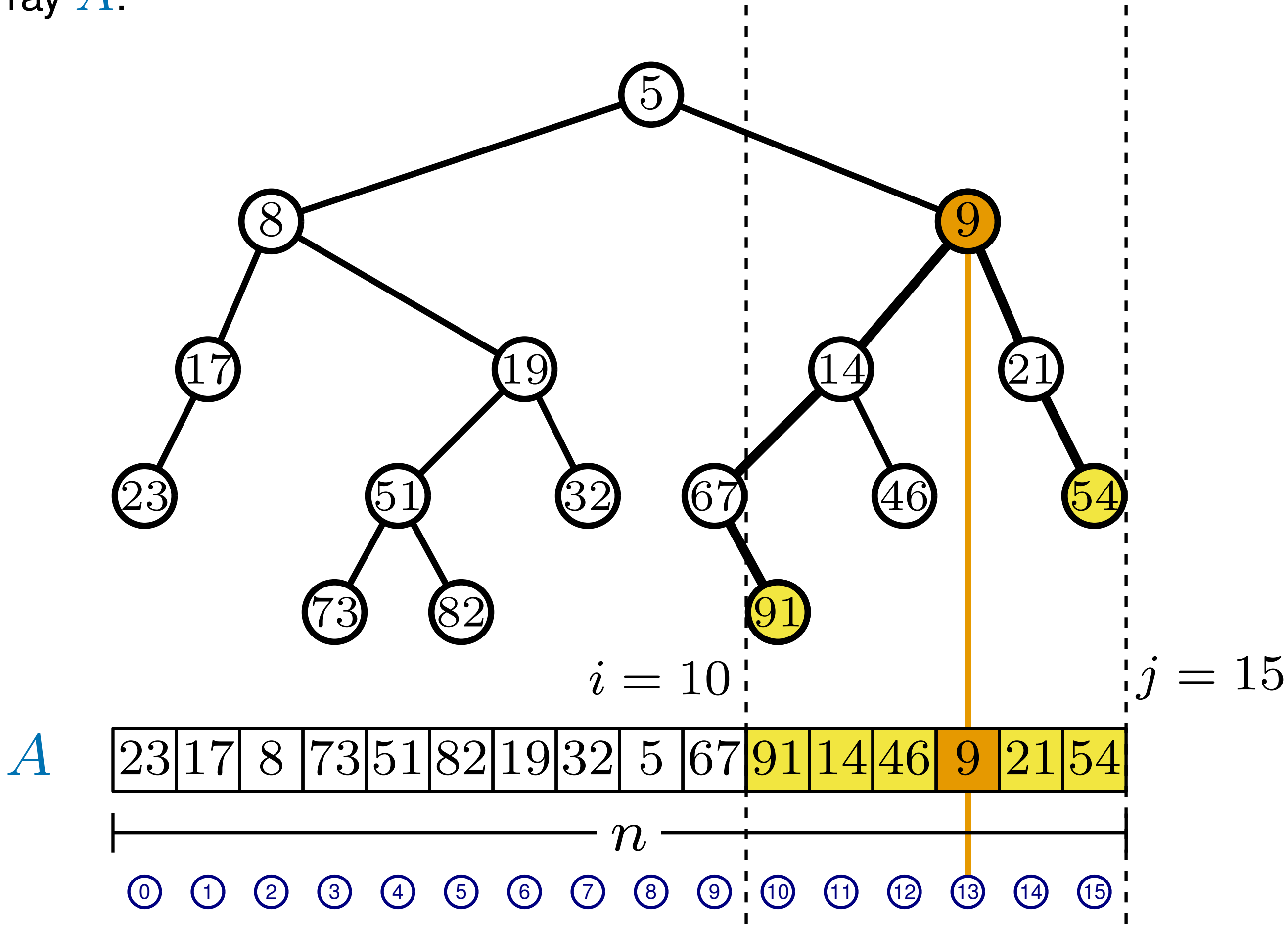

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Summary

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for the ± 1 Range Minimum Query problem

which improves solution 3 for RMQ *from last lecture*

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which uses the solution to ± 1RMQ

We have seen an $O(n)$ space, $O(n)$ prep. time and $O(1)$ query time solution

for the Range Minimum Query problem

which uses the solution to LCA

(which works for all inputs)